$$\overset{M}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \times \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix}$$

(2×2) $\quad$ (2×1)

$$\begin{bmatrix} af(n) + bf(n-1) \\ cf(n) + df(n-1) \end{bmatrix} = \begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix}$$

$a f(n) + f(n-1) * b = f(n+1)$

for fibonacci we know $\boxed{a=b=1}$

$c f(n) + d f(n-1) = f(n)$

$\boxed{d=0,\ c=1}$

thus $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Now see $\quad M \times \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+1) \\ f(n) \end{bmatrix}$

$$M \times M \times \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+2) \\ f(n) \end{bmatrix}$$

(i.e. M^2)

$\vdots$

$$M^K \times \begin{bmatrix} f(n) \\ f(n-1) \end{bmatrix} = \begin{bmatrix} f(n+k) \\ f(n+k-1) \end{bmatrix}$$

and if we want $f(8)$ we can multiply by M^8 as well as M^7 $\quad$ $M^8 \to \begin{bmatrix} f(9) \\ f(8) \end{bmatrix}$, $\quad$ $M^7 \to \begin{bmatrix} f(8) \\ f(7) \end{bmatrix}$ $\quad$ lets do M^7 not M^8.

For multiplying matrices

$A^n = A^{n/2} \times A^{n/2}$ if $n =$ even

$A^n = (A^{n/2} \times A^{n/2}) \times A$ if $n =$ odd.

$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $n = 5$.

$f(A, 5)$ ← f does not return anything as the changes are always reflected.

↓

$f(A, 2)$ → since n is even first do $A \times A$ now A contains A^2

↓

$f(A, 1)$

↓

1 and 0 are base cases we just return and A remains intact

when we come here first thing that happens is $A^2 \times A^2 = A^4$ and we a check for odd n also does $A^4 \times A = A^5$.

$f(A, 6)$

↓

$f(A, 3)$ → $A \times A = A^2$, $A^2 \times A = A^3$.

↓

$f(A, 1)$ → A remains intact

$A^3 \times A^3$

$\Rightarrow A^6$

[redacted]

final result.