Wilson's Theorem (and p is prime)

$$(p-1)! \bmod p \equiv -1 \text{ or } (p-1)$$

Now when you want to take out mod of a factorial.

Note that for $(n!)\%p$

if $n \geq p$

$$(1 \times 2 \times 3 \cdots \times p \times \cdots n)\%p \equiv 0$$

but for $n < p$

$$(1 \times 2 \times 3 \cdots n)\%p.$$

now extend this to $(p-1)$

$$(1 \times 2 \times 3 \times \cdots n \times (n+1) \cdots \times (p-1))\%p.$$

$$n! \bmod p \times (n+1) \bmod p \times \cdots (p-1) \bmod p \equiv -1$$

↗ wilson's applied.

$$n! \bmod p \equiv (n+1)^{-1} \bmod p \times \cdots (p-1)^{-1} \bmod p \times -1$$

Fermets says $a^{-1} \bmod p \equiv a^{p-2} \bmod p$

↗ mod expo application

$$n! \bmod p \equiv (n+1)^{p-2} \bmod p \times (n+2)^{p-2} \bmod p \cdots (p-1)^{p-2} \bmod p \times -1$$