The maths of question : calculate salary on nth day :-

$$F(n) = F(n-1) + F(n-2) + F(n-1) \times F(n-2)$$

$$F(n) = F(n-1)\{1 + F(n-2)\} + F(n-2)$$

$$F(n) = F(n-1)(1 + F(n-2)) + (1 + F(n-2)) - 1$$

$$(1 + F(n)) = (F(n-2) + 1)(F(n-1) + 1)$$

let $G(N) = 1 + F(n)$ || $\boxed{F(n) = G(N) - 1}$

then

$$\boxed{\begin{aligned} G(N-1) &= 1 + F(n-1) \\ G(N-2) &= 1 + F(n-2) \end{aligned}}$$

$$\boxed{G(N) = G(N-1) * G(N-2)}$$

$G(0) =$ let a

$G(1) =$ let b

$G(2) = a^1 * b^1$

$G(3) = a^1 b^2$

$G(3) = a^2 b^3$

$G(4) = a^3 b^4$

⋮

problem is that %. is on $a^{fib(n-1)}$ but $f(n-1)$ can exceed the limit easily & so lets get mod of it too.

$$G(N) = \left(a^{fib(n-1)} \times b^{fib(n)}\right) \% m$$

$$= \left(a^{fib(n-1)} \% m \times b^{fib(n)} \% m\right) \% m$$

Fermats law $\boxed{a^{p-1} \bmod p = 1}$

dividend

$$\boxed{fib(n-1) = k * (p-1) + fib(n-1) \% (p-1)}$$

simple division expansion

quotient divisor remainder

$$a^{fib(n-1)} \% m = a^{fib(n-1) \% (p-1) + k*(p-1)}$$

$\boxed{\text{where } p \text{ is } m}$ so let put that

$$= \left[a^{k*(m-1)} \times a^{fib(n-1) \% (m-1)} \right] \% m$$

$$= \left[a^{k*(m-1)} \% m \times a^{fib(n-1) \% (m-1)} \% m \right] \% m$$

$\boxed{1 \text{ acc to fermets}}$

$$\left(a^{fib(n-1) \% (m-1)}\right) \% m$$

now since we are taking mod of factorial, we can easily calculate this value using matrix exponentiation and modular exponentiation.

$$\boxed{b^{fib(n)} \% m = \left(b^{fib(n) \% (m-1)}\right) \% m}$$