**Random point on disk**

**Question:** $x$ and $y$ are two random points selected uniformly between 0 and 1. Using them, create a point uniformly random in the disk of radius 1. (uniform means that the probability density is constant)

**Solution:** Think of $x$ and $y$ as coordinate variables on $xy-$plane. Then $x$ and $y$ being uniformly random is equivalent to $1dxdy$ being the area form.

Let $r = f(x)$ and $\theta = g(y)$. Then the point in disk is given by $(p, q) \stackrel{\text{def}}{=} (r\cos(\theta), r\sin(\theta)) = (f(x)\cos(g(y)), f(x)\sin(g(y)))$. If $p$, $q$ are randomly uniformly, $\frac{1}{\pi}dpdq = dxdy$. Thus, $\frac{1}{\pi}f'(x)f(x)g'(y) = 1$. Thus, $g'(y)$ and $f'(x)f(x)$ are constant.

Since $\theta = g(y)$ is from 0 to $2\pi$, take $g'(y) = 2\pi$. Then $f'(x)f(x) = 1/2$. Thus, $f(x) = \sqrt{x}$. Hence, the point $(\sqrt{x}\cos(2\pi y), \sqrt{x}\sin(2\pi y))$ is uniformly random.