

Getting fair result from unfair coin

Question: We have a weighted coin which shows a Head with probability p , ($0.5 < p < 1$). How do we get a fair toss from this? That is, how do we toss this coin in such a way that we can have probability of winning = losing = 50%?

Solution: Let $win = \{head, tail\}$ and $lose = \{tail, head\}$. Then $P(win) = P(lose)$.

Use naive rejection, toss the weighted coin twice. If not $\{head, tail\}$ or $\{tail, head\}$, discard the result. Thus, $P(win) = P(lose) = 0.5$.

Follow up: I proposed a faster method, "lets keep tossing the coin to form a sequence of H 's and T 's . I win if HT appears before TH " . Was I bluffing?

Solution: Yes. Note if the first toss is H , then I always win; and if the first toss is T , then I always lose.