Distinct Number Draws

Question: Given the set of numbers from 1 to $n : \{1, 2, 3, \ldots, n\}$. We draw n numbers randomly (with uniform distribution) from this set (with replacement). What is the expected number of distinct values that we would draw?

Solution: Let $X_i = \mathbb{I}_{\{\text{number i is drew at least once}\}}$ for $1 \leq i \leq n$. Let Y be number of distinct values. Then $Y = \sum_{i=1}^{n} X_i$.

Thus, $E(Y) = \sum_{i=1}^{n} E(X_i) = \sum_{i=1}^{n} P(X_i = 1)$. $P(X_i = 1) = 1 - P(X_i = 0) = 1 - (\frac{n-1}{n})^n$.

Thus, $E(Y) = n(1 - (\frac{n-1}{n})^n)$.