Drunk Ant

Question: An ant is standing on one corner of a cube and can only walk on the edges. The ant is drunk and from any corner, it moves randomly by choosing any edge! What is the expected number of edges the ant travels, to reach the opposite corner?

Solution: Denote the vertices of the cube by A, B, C, D, E, F, G, H.

By symmetry, vertices are divided into 4 groups: $\{A\}, \{B, D, E\}, \{C, F, H\}, \{G\}$.

Let x_1, x_2, x_3 be the expected number of edges from A, B, C to G respectively.

By the first step analysis,

$$\begin{cases} x_1 = x_2 + 1 \\ x_2 = (x_1 + 2x_3)/3 + 1 \\ x_3 = 2x_2/3 + 1 \end{cases} \tag{1}$$

$x_1 = 10, x_2 = 9, x_3 = 7$.