

## Collecting Lucky coupons

**Question:** A soda company is holding a contest where everyone who collects one each of  $N$  different coupons wins some prize. You get a coupon with each purchase of a soda, and each coupon is equally likely. Whats the expected number of soda bottles you have to buy in order to collect all the coupons?

**Solution:** Let  $X_n$  be number of soda bottles to collect  $n$  coupons.

Let  $Y$  be the number of soda bottles to collect  $n^{\text{th}}$  coupon after collecting  $n - 1$  coupons. Thus,  $Y \sim \text{Geom}(\frac{N-n+1}{N})$  and  $X_n = X_{n-1} + Y$ .

Then  $E(X_n) = E(X_{n-1}) + E(Y) = E(X_{n-1}) + \frac{N}{N-n+1}$ . Thus,  $E(X_N) = \sum_{n=1}^N \frac{N}{N-n+1} = \sum_{n=1}^N \frac{N}{n}$ .