Color Switches

Question: You are given an urn with 100 balls (50 black and 50 white). You pick balls from urn one by one without replacements until all the balls are out. A black followed by a white or a white followed by a black is "a color change". Calculate the expected number of color changes if the balls are being picked randomly from the urn.

Solution: For $1 \leq i \leq 99$, let $X_i = 1$ if the color of i^{th} ball is different from color of $(i+1)^{\text{th}}$ ball and 0 otherwise. Let Y be number of color changes. Then $Y = \sum_{i=1}^{99} X_i$.

Note $E(X_i) = P(X_i = 1) = 50/99$. Thus, $E(Y) = \sum_{i=1}^{99} E(X_i) = 99 \times 50/99 = 50$.