Question: You are given two identical eggs and you have access to a building with N floors labeled from 1 to N. You know that there exists a floor f where $0 \leq f \leq N$ such that any egg dropped at a floor higher than f will break, and any egg dropped at or below floor f will not break. In each move, you may take an unbroken egg and drop it from any floor x (where $1 \leq x \leq N$). If the egg breaks, you can no longer use it. However, if the egg does not break, you may reuse it in future moves. Find the minimum number of moves that you need to determine with certainty what the value of f is.

Solution: First suppose we are given only one egg. Then the only way to determine f is dropping it from floor one and if it does not break then floor two and so on until it breaks. Now we are given two eggs, we can use the first egg to narrow down the range for f.

Let n_1 be the floor that the first egg is dropped from at first try. If it breaks, we drop the second egg starting from floor 1 to floor $n_1 - 1$. In the worst case, we need $1 + n_1 - 1$ drops. If it does not break, suppose we drop it from floor $n_1 + n_2$. If it breaks, we drop the second egg starting from floor $n_1 + 1$ to floor $n_1 + n_2 - 1$. In the worst case, we need $2 + n_2 - 1$ drops. Thus, let $n_1, n_1 + n_2, \ldots, n_1 + n_2 + \ldots + n_k$ be floor numbers the first egg is dropped from. Thus, $n_1 + n_2 + \ldots + n_k = N$. For $1 \leq i \leq k$, in the worst case, we need $i + n_i - 1$ drops.

Let m be number of drops that we need to determine f. Then the problem is equivalent to finding minimum m such that $m \geq i + n_i - 1$ for $1 \leq i \leq k$.

Adding those k inequalities, we obtain

$$k \cdot m \geq \sum_{i=1}^{k} i + n_i - 1 = \frac{(1+k) \cdot k}{2} + N - k = \frac{k \cdot (k-1)}{2} + N.$$

Dividing both sides by k,

$$m \geq \frac{k-1}{2} + \frac{N}{k} \geq 2\sqrt{\frac{k}{2} \cdot \frac{N}{k}} - \frac{1}{2} = \sqrt{2N} - \frac{1}{2}.$$

Thus, $m = \left\lceil \sqrt{2N} - \frac{1}{2} \right\rceil$.