

Greed for an ACE

Question: What is the expected number of cards that need to be turned over in a regular 52-card deck in order to see the first ace?

Solution: First note 52 cards are divided into two groups: 4 aces and 48 others. Let $X_i \stackrel{\text{def}}{=} \mathbb{I}_{\{i^{\text{th}} \text{ card is turned over before any ace}\}}$ for $1 \leq i \leq 48$.

Then the expected number of cards that need to be turned over is $\sum_{i=1}^{48} E(X_i) + 1$.

Since $E(X_i) = P(X_i = 1) = 1/5$, the final answer is $48/5 + 1 = 53/5$.