Pattern on Snowflakes

Question: Snow-particles are falling on the ground one after another. A particular snowflake turns out to be of type "Stellar Dendrite" with probability p if its previous particle was also Stellar Dendrite, and with probability q if previous one was something else. If a snowflake is picked from ground, what is the probability that it is Stellar Dendrite?

PS: Although no two snowflakes are alike, yet there are various crystalline structures to categorize their interesting shapes. The image depicts the most popular shape, called Stellar Dendrites, which means star-like particles with tree-like branches.

Solution: Let $Curr$ be type of current snowflake and $Prev$ be type of previous snowflake, and let 1 be Stellar Dendrite and 0 otherwise. Then $P(Curr = 1|Prev = 1) = p$ and $P(Curr = 1|Prev = 0) = q$.

$$
\begin{aligned}
&P(Curr = 1)\\
=&P(Curr = 1 \mid Prev = 1) \cdot P(Prev = 1) + P(Curr = 1 \mid Prev = 0) \cdot P(Prev = 0)\\
=&pP(Prev = 1) + qP(Prev = 0)
\end{aligned}
$$

Now what is $P(Prev = 1)$?

Suppose $P(Prev = 1) = P(Curr = 1)$. Then $P(Curr = 1) = q/(1 - p + q)$.

Consider the recursive relation: $P_{n+1} = pP_n + q(1 - P_n) = (p - q)P_n + q$.

Note $\lim P_n = q/(1 - p + q)$.

If assume the snow falls for enough long time, then $P(Curr = 1) = q/(1 - p + q)$.