

The Blind Archer

Question: A very sharp, consistently skillful blind archer aimed for the center of a circular board and shot 2 arrows. He is expected to hit the aim, but doesn't hit it for sure. The archer is told that his first shot is better than second. He tried one more shot. What is the probability that this 3rd shot is the best shot among 3? (i.e., Probability that 3rd arrow lands closer to center than his first two shots?)

Solution: Let X_i be score of i^{th} shot. Then

$$\begin{aligned} P(X_3 > X_1 | X_1 > X_2) &= \frac{P(X_3 > X_1 > X_2)}{P(X_1 > X_2)} \\ &= \frac{P(X_3 > X_1 > X_2)}{P(X_3 > X_1 > X_2) + P(X_1 > X_3 > X_2) + P(X_1 > X_2 > X_3)} \end{aligned}$$

Since the archer is consistently skillful, $P(X_3 > X_1 > X_2) = P(X_1 > X_3 > X_2) = P(X_1 > X_2 > X_3) = 1/3$. Thus, $P(X_3 > X_1 | X_1 > X_2) = 1/3$.