Expected Breakup Length

Question: A stick is broken into 3 pieces, by randomly choosing two points along its unit length, and cutting it. What is the expected length of the middle part?

Solution: Let $X, Y \sim \text{Unif}(0,1)$. Then the length of the middle part is $\mid X - Y \mid$.

Then the expected length is

$$\int_0^1 \int_0^1 \mid x - y \mid dxdy = \int_0^1 \int_0^x (x-y)dydx + \int_0^1 \int_0^y (y-x)dxdy = 2 \times 1/6 = 1/3.$$