

Chuck a Luck

Question: In this gambling game, a player can buy a ticket for \$1 on any number from 1 to 6. Three identical and unfair dice are rolled. If the booked number appears on 0, 1, 2 or 3 dice, player wins \$0, \$1, \$2 or \$3 respectively, without returning the original \$1. What is expected money you can win after buying a ticket for \$1?

Solution: Let X be the booked number, for $i = 1, 2, 3$, Y_i be the number on the i^{th} unfair dice and Z be how much money the player can win each time. Then $Z = \mathbb{I}_{Y_1=X} + \mathbb{I}_{Y_2=X} + \mathbb{I}_{Y_3=X}$.

$$\begin{aligned} E(Z) &= \sum_{i=1}^6 E(Z \mid X = i) \cdot P(X = i) \\ &= \sum_{i=1}^6 E(\mathbb{I}_{Y_1=X} + \mathbb{I}_{Y_2=X} + \mathbb{I}_{Y_3=X} \mid X = i) \cdot P(X = i) \\ &= \sum_{i=1}^6 \sum_{j=1}^3 E(\mathbb{I}_{Y_j=X} \mid X = i) \cdot P(X = i) \\ &= \sum_{j=1}^3 \sum_{i=1}^6 E(\mathbb{I}_{Y_j=i} \mid X = i) \cdot P(X = i) \\ &= \sum_{j=1}^3 \sum_{i=1}^6 P(Y_j = i) \cdot P(X = i) \\ &= \sum_{j=1}^3 \sum_{i=1}^6 P(Y_j = i) \cdot \frac{1}{6} \\ &= \sum_{j=1}^3 \frac{1}{6} \\ &= 0.5. \end{aligned}$$