

Witches at the coffee shop

Question: Two witches make a nightly visit to an all-night coffee shop. Each arrives at a random time between 0:00 and 1:00. Each one of them stays for exactly 30 minutes. On any one given night, what is the probability that the witches will meet at the coffee shop?

Solution: Let X, Y be the arrival time for two witches.

Then $P(\text{meet}) = P(|X - Y| \leq 0.5) = \int_{\{|x-y| \leq 0.5, 0 \leq x, y \leq 1\}} dx dy = 1 - 0.5 \times 0.5 = 0.75$.