Distance from North Pole

Question: What is the expected distance of any point on Earth and the north pole? Take Earth radius 1.

Clarification: Shortest distance cuts through the sphere, instead of lying on surface.

Further thinking: Is this question same as choosing two random points on unit sphere and asking their expected distance?

Solution: Let θ be the angle between a point and z axis. Then the shortest distance cuts through the sphere from the point to the north pole is $\sqrt{2-2\cos(\theta)} = 2\sin\frac{\theta}{2}$.

Thus, the expected distance is $\frac{1}{4\pi}\int_0^\pi 2\sin\frac{\theta}{2} \cdot 2\pi\sin\theta d\theta = 4/3$.

Condition on the first point, the expected distance is $4/3$. Then since the first point is uniformly random, the expected distance of two random points is also $4/3$.

Shortest distance on surface: Then the shortest distance on surface from the point to the north pole is θ. Thus, the expected distance is $\frac{1}{4\pi}\int_0^\pi \theta \cdot 2\pi\sin\theta d\theta = \pi/2$.