Messing with Envelops

Question: There are n letters and n envelopes. Your servant puts the letters randomly in the envelopes so that each letter is in one envelope and all envelopes have exactly one letter. (Effectively a random permutation of n numbers chosen uniformly). Calculate the expected number of envelopes with correct letter inside them.

Solution: Let $X_i = \mathbb{I}_{\{i^{th} \text{ envelop with correct letter}\}}$ for $1 \leq i \leq n$.

Let Y be the number of envelops with correct letter. Then $E(Y) = E(\sum_{i=1}^{n} X_i) = \sum_{i=1}^{n} E(X_i) = \sum_{i=1}^{n} P(X_i = 1)$.

Note $P(X_i = 1) = 1/n$. Then $E(Y) = 1$.