Congestion Relief in Downlink OFDMA Cellular System Using Inter-Cell Relay

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Abstract: This paper proposes the concept of inter-cell relay for downlink orthogonal frequency division multiple access (OFDMA) cellular systems, which uses multi-hop to relay calls from overloaded cells to light-load neighboring cells. It is shown that when using inter-cell relay, the number of calls in the congestion cell can be significantly increased. The congestion cell is divided into two parts. One is called non-relay area (NRA), in which a call directly communicates with the base station (BS) of a congested cell. The other is called relay area (RA), in which a call communicates with the BS of a neighboring cell through a relay station (RS). The two parts have different user-call densities. By adjusting the densities of two parts, we will maximize the number of supported calls inside a congested cell. The results show the benefits gained from inter-cell relay in congestion relief, which can reduce cell congestion by fully utilizing the available resources in the neighboring cells.

Keywords: OFDMA; cellular system; congestion relief; optimization problem; inter-cell relay

I. INTRODUCTION

OFDMA has been one of the promising solutions of multiple accessing for 4G wireless communication due to its high spectral efficiency and inherent anti-fading robustness. The transmission bandwidth can be more efficiently used in OFDMA system because it is divided into a series of orthogonal subcarrier sets which are allocated for different users. The interference in cellular network arises in the form of intra-cell interference and inter-cell interference (ICI). The orthogonal among subcarriers avoids the intra-cell interference, but ICI remains a key issue in OFDMA cellular network. In downlink, the signals from the neighboring cells are ICIs for the target user in the reference cell. ICI is a major factor to affect the performance of OFDMA cellular system. The increase of ICI leads to the decrease of signal to interference ratio (SIR) and then reduces the performance of the OFDMA cellular cell system.

To enhance the performance of the OFDMA cellular system furthermore, the relay

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In this paper, we propose a inter-cell relay method to solve the congestion problem and optimize the maximum number of calls in congested cell by using the variation of densities.

technology is introduced [1]. Relays have been one of the key technologies in LTE-A where RSs are deployed in place to forward the radio signals from BS to mobile terminal (MT). The deployment of RSs can shorten the distance between transmitter and receiver in every hop, reduce the path loss of signals and then improve the communication quality. A prominent approach is to take advantage of multi-hop wireless relay[2-3]. Accordingly, the concept of multi-hop cellular networks (MCNs) has been introduced to overcome weaknesses such as dead spots, congestion and load imbalance [4]. However, a problem has been aroused that the wireless resource management becomes more complex [5-6]. The resource allocation in OFDMA cellular network involves many factors such as power allocation, subcarriers allocation and modulation mode selection.

The analysis about interference in OFDMA multi-hop cellular network based on signal to interference ratio (SIR) and signal to interference plus noise ratio (SINR) is performed in [7]-[12]. With the log-normal shadowing and Rayleigh fading, the statistical ICI models for uplink and downlink cellular OFDMA networks transmitting multiple quadrature amplitude modulation (QAM) symbols per hop are revealed in [7] and [8] respectively. The result shows that the statistics of the average ICI for the uplink transmission are more significant than that for downlink transmission under the traditionally assumed Gaussian distribution. In [9], a lognormal distribution is used to approximate the uplink ICI from each cell using statistical parameters determined by analysis. The analytical expression of the uplink ICI is derived using two different schemes and system performance in terms of spectral efficiency. In [10], the interference in downlink OFDMA cellular system is divided into two groups by propagation delays. One is the co-channel interference, the other is the inter-carrier interference. And it quantitatively evaluated the total amount of interference. It is derived in [11] that the closed-form expressions for SINR distributions using the fixed relays in OFDMA cellular system. The analytical results can be applied to calculate the average cell system capacity. In [12], to multiplex data from different users, two types of mapping strategies named interleaved frequency division multiple access (IFDMA) and localized frequency division multiple access (LFDMA) are used in uplink OFDMA system. It obtains explicit SIR expressions for the two types of mapping strategies respectively and analyzes the effect of carrier frequency offset of multiple users on SIR. In [13], the concept of using multi-hop communication to transfer users from an overload cell to an inter-cell relay of a light-load neighboring cell is proposed. The result shows that the number of users in a congested cell can be significantly increased by using inter-cell relay. They designed two congestion mitigation methods based on network conditions to maximize the number of users in a crowded cell. The distribution-based method determines the number of extra hops for inter-cell relay based on user distribution. On the other hand, the delay-sensitive method supposes that the number of extra hops for inter-cell relay is limited by user quality of service requirement. By limiting the number of extra hops, the method decides the number of inter-cell relayed users and the number of users connected to the target BS. In [14], they proposed a new network planning framework for OFDMA cellular system based on traffic congestion avoidance and co-channel interference. The network planning is formulated as a non-linear multi-objective optimization problem which subject to resource constraints and minimum interference at single cell under heterogeneous traffic. In [15], Salem et al. made a survey of the current literature on OFDMA cellular networks enhanced with decode-and-forward relay and proposed their link to earlier literature in non-OFDMA cellular networks. In [16], Rawi *et al.* proposed a new network planning framework for traffic congestion avoidance. This network planning is aimed to minimum the interference and related resource constraints. It is formulated as multi-objective problem. This problem is represented by maximizing the throughput of each cell without damaging the remaining cells, which results in uniform throughput in the whole network. In [17], cross-layer congestion control is proposed. To avoid co-channel interference, OFDMA sub-channel separation problem is considered. They resolve this problem by dividing it into two sub-problems. One is a transmission control protocol (TCP) layer congestion control for traffic allocation. The other is a weighted scheduling for OFDMA sub-channel assignment and power control. In [18], complete partitioning (CP) policy is proposed to overcome congestion problem. The scheme divides the available bandwidth according to the CP structure while considering the user signal noise ratio (SNR). The results show that each traffic category plays an important role in ensuring quality of service (QoS) assurance services.In this paper, we analyze how to resolve the congestion problem by using inter-cell relay technology. Firstly, the congestion cell is divided into two parts according to the width of annulus: NRA and RA. It is assumed that a call within the NRA communicates with the BS directly, and a call within the RA communicates with the BS of a neighboring cell through a RS. Secondly, we decompose the communication into two time-slots,and then we analyze the required subcarriers for different time-slots in different parts. Finally, we propose an optimization problem to maximize the calls in the congestion cell.

The rest of the paper is organized as follows. In section 2, the analytical model of OFDMA cellular network system is proposed. In section 3, the expressions for the ISR are given, including its mean and variance. Based on the *ISR* distribution and cumulative distribution function (CDF), the overall subcarriers required are obtained. Subcarrier requirements analysis is provided in section 4. Section 5 describes the optimization problem and section 6 provides the simulation results. The last presents a conclusion.

II. SYSTEM MODEL

The OFDMA cellular system model with the cell radius D is shown in figure 1. At the center of the cells, the base stations (BSs) equipped with omni-directional antennas are deployed. And the relay stations (RSs) are uniform distribution at the edge of the target cell. The center target BS is marked as BS_0 and the six neighboring cells in first layer are numbered as 1,...,6 respectively. A reasonable approximation has been verified that all the integrals in the hexagonal unit can be performed on the circular unit using the polar coordinates in the network model. The coordinate system is established with BS0 as the origin. The coordinates of the neighboring *BS*s are also confirmed.

To increase the utilization rate of the spectrum of OFDMA two-hop relay cellular system, the frequency reuse factor is set to be 1. It means that all cells use the same spectrum resource. In this paper the center cell and its first layer neighboring cells are considered, and the reference BS of the center cell and its first layer neighboring *BS*s are all in downlink transmission. In OFDMA two-hop relay cellular system, the intra-cell interference is omitted, but the existence of the ICI is inevitable. The ICI depends on the location and channel

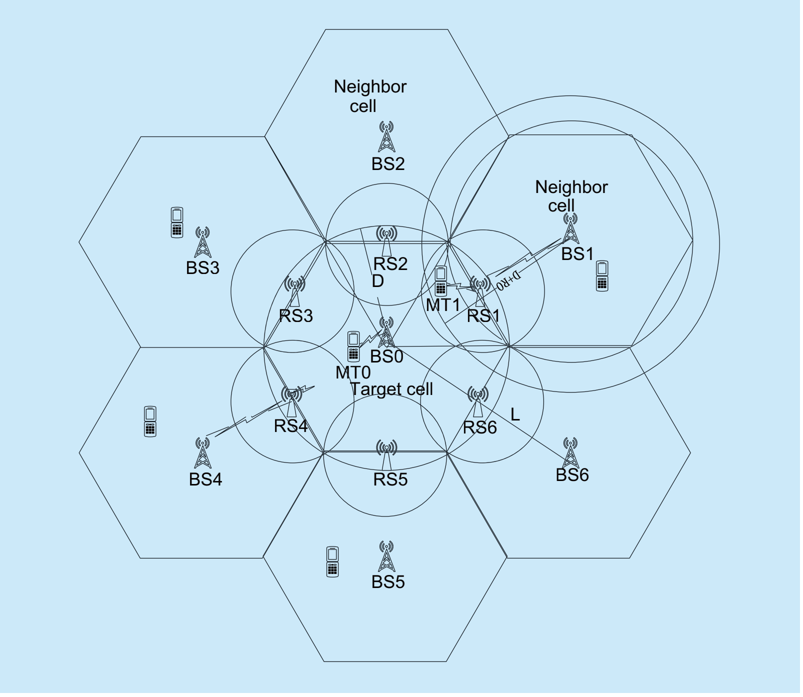

Fig. 1. *The OFDMA cellular system model.*

characteristics of the interfered station, such as path loss, shadow and fading. In this paper, the shadow and fading are ignored, and the fixed transmission power is considered. That means when the transmitter transmits at power P_{transmit} to a receiver at r, the received power is $P_{\text{receive}} = P_{\text{transmit}} r^{-\beta}$, where β is the path loss exponent.

A scheme to decide which MT's calls are relayed to neighboring BSs needs to be supported. A reasonable choice is the calls from MTs near the cell boundary. For instance, in figure 1 MT_1 is considered close enough to cell boundary, and if it is within the transmission range of the RS_1 of the neighboring cell area, its call can be relayed to the neighboring BS_1. The distance between these MTs and the neighboring BS must be less than $D + R_0$, where R_0 is the width of annulus. We assume that R_0 represents the communication distance of RS. Therefore, a call from the MT located in a cross-region between the congested cell and a disc centered at a neighboring BS with inner disc D and outer disc $D + R_0$, can be relayed to the neighboring BS. MT_S that are near to the target cell directly communicate with BS_0. To simplify calculations, the target cell is divided into six sectors each.

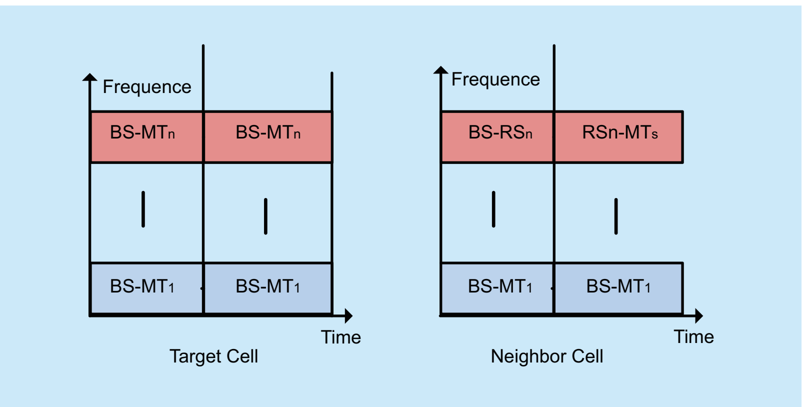

Fig. 2. *Time slot allocation.*

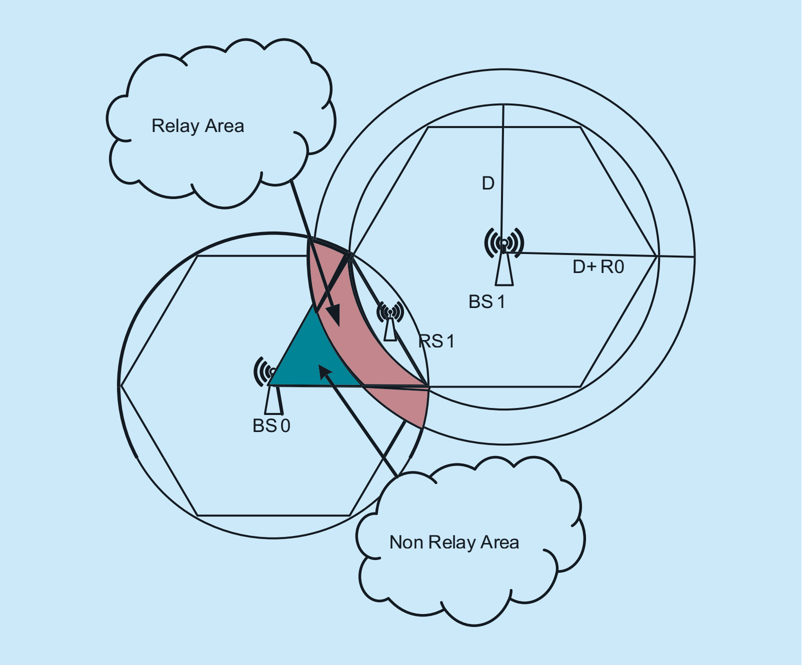

Fig. 3. *The Non-relaying area and relay area.*

In this system, each frame is divided into two time slots (TSs), as shown in figure 2. The load conditions of different cells in each TS are different. We assume that only the target cell is overloaded, and the remaining six neighboring cells are light loaded. To the neighboring cell, the first TS is the transmission of the BS to the inter-cell RS, or the time of direct transmission of the BS to the MT. The second TS is the transmission between RS and MT. To the target cell, the BS to MT transmission in the second TS is the same as the first TS. Therefore, for the target cell, the interference of BS to MT transmission link may be the interference of the neighboring BS in the first TS, or the interference from the RS in the second TS. However, the interference of indirect transmission link between RS and MT is generated from the target BS.

III. Interference Analysis in Downlink Transmission

In this section, we will analyze the interference in downlink transmission. In NRA, where the MTs communicate with BS inside the congestion cell directly, which we can consider as a single hop. In RA, where the MTs communicate with the neighboring BS through the RS. NRA and RA are showed as figure 3.

The area of NRA can be expressed as follows

$$NRA = 6\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_{0}^{D} r dr d\theta \qquad .$$

$$given\sqrt{r^2+L^2-2rL\cos\theta} > D+R_0 \quad (1)$$

And the area of RA can be given by

$$RA = 6\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_{D}^{D+R_0} rdrd\theta$$

$$\text{given}\sqrt{r^2+L^2-2rL\cos\theta} < D \quad (2)$$

3.1 Target cell

In the downlink, the interferences come from different TSs.

3.1.1 The first TS

In this part, we consider a target MT in inner region of the reference cell. The transmitting signals in neighboring cells may interfere with the target MT in reference cell when they use the same subcarriers. The SIR of target MT in inner region can be given by

$$SIR_{BS_0-MT} = \frac{P_{BS0}\cdot r^{-\beta}}{\sum_{i=1}^{6} P_{BS1\_c(c=2...6)}\cdot r_i^{-\beta}} = \left(\frac{(D-R_0)\sqrt{L^2+r^2-2rL\cos\theta}}{6(D+R_0)r}\right)^{\beta} \quad (3)$$

Where P_{BS0} is the power from BS_0, and $P_{BS1\_c(c=2...6)}$ is the transmit power of the BS from the neighboring cell. r is the distance between the BS_0 and the MT_0, and r_i is the distance between the neighboring BS and MT. To simplify the calculation, we take the interference to signal ratio (ISR) into account which is the reciprocal of SIR. The ISR would be expressed as

$$ISR_{BS_0-MT} = 1/SIR = \left(\frac{(D-R_0)\sqrt{L^2+r^2-2rL\cos\theta}}{6(D+R_0)r}\right)^{-\beta} \quad (4)$$

It is assumed that all the MTs are uniformly distributed over the area of NRA. The mean of ISR, $E\left[ISR_{BS_0\text{-MT}}\right]$, can be divided into six parts. And $E_i\left[ISR_{BS_0\text{-MT}}\right]$ is the mean of ISR when considering one of the ICIs from six neighboring cells, i is valued as $1,2,\ldots,6$. It is obviously that $E_1[ISR] = E_2[ISR] = \ldots = E_6[ISR]$ according to symmetry and the distribution rule of cells. We focus on the $E_1\left[ISR_{BS_0\text{-MT}}\right]$ when considering the ICI from BS_1, and it can be expressed as

$$E_1\left[ISR_{BS_0-MT}\right] = \frac{\rho_1}{NRA}\cdot \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_{0}^{D-R_0}\left(\frac{(D-R_0)\sqrt{L^2+r^2-2rL\cos\theta}}{6(D+R_0)r}\right)^{-\beta} rdrd\theta \quad (5)$$

Where ρ_1 is the density of accepted calls connected to the BS of the congested cell, and the mean of ISR for subcarrier m, $E\left(ISR_{BS_0\text{-MT}}\right)$ can be expressed as

$$E\left[ISR_{BS_0\text{-MT}}\right] = \sum_{i=1}^{N=6} E_i\left[ISR_{BS_0\text{-MT}}\right] = 6E_1\left[ISR_{BS_0\text{-MT}}\right] \quad (6)$$

Similarly, the variance of ISR, $Var\left[ISR_{BS_0\text{-MS}}\right]$, is expressed as

$$Var\left[ISR_{BS_0\text{-MT}}\right] = \sum_{i-1}^{N=6} Var_i\left[ISR_{BS_0\text{-MT}}\right] = 6Var_1\left[ISR_{BS_0\text{-MT}}\right] \quad (7)$$

Where $Var_1\left[ISR_{BS_0\text{-MT}}\right]$ is the variance of ISR when considering the ICI from BS_1 only, and is given as

$$Var_1\left[ISR_{BS_0-MT}\right] = \frac{\rho_1}{NRA}\cdot \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_{0}^{D-R_0}\left(1/\left(\frac{(D-R_0)\sqrt{L^2+r^2-2rL\cos\theta}}{(D+R_0)r}\right)^{\beta} - E(ISR)\right)^2 rdrd\theta \quad (8)$$

According to the central limit theorem, the distribution of ISR can be approximated by the Gaussian distribution. It is assumed that the distribution of ISR obeys a Gaussian distribution, so the cumulative distribution function (CDF) of ISR is expressed as follow:

$$P\left(ISR_{BS_0\text{-MT}} \geq ISR^n_{\text{threshold}}\right) = Q\left(\frac{ISR^n_{\text{threshold}} - E\left[ISR_{BS_0\text{-MT}}\right]}{\sqrt{Var\left[ISR_{BS_0\text{-MT}}\right]}}\right) \quad (9)$$

Where $ISR^n_{\text{threshold}}$ represents the threshold when the number of subcarriers required is n. The function $Q(*)$ is the standard normal of CDF, and the function is given by

$$Q(x) = \int_{x}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}dy \quad (10)$$

3.1.1 The second TS

In this section, the interferences mainly come from the RS-MT link. The SIR can be expressed as

$$SIR_{BS_0-MT}=\frac{(D-R_0)^{\beta}\sum_{i=1}^{6}d_i^{\beta}}{(\frac{D}{2}r)^{\beta}} \qquad (11)$$

Where d_i is equal to the distance between the $RS_{i\,(i=1\ldots6)}$ and MT. And r is the distance from the BS_0 to the MT in the target cell. d_i can be expressed as

$$d_i=\sqrt{\left((\frac{L}{2})^2+r^2-2r\left(\frac{L}{2}\right)\cos(\frac{(i-1)\pi}{3}-\theta\right)} \qquad (12)$$

$$given:\sqrt{r^2+L^2-2rL\cos\theta}>D+R_0$$

The ISR would be expressed as

$$ISR_{BS_0-MT}=1/SIR_{BS_0-MT}=\frac{(\frac{D}{2}r)^{\beta}}{(D-R_0)^{\beta}\sum_{i=1}^{6}d_i^{\beta}} \qquad (13)$$

And the $E\left[ISR_{BS_0-MT}\right]$ can be expressed as

$$E\left(ISR_{BS_0-MT}\right)=\frac{\rho_1}{NRA}\cdot\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_{\frac{L\cos\theta-\sqrt{(L\cos\theta)^2-4\cdot(L^2-(D+R_0)^2)}}{2}}^{D-R_0}\frac{(\frac{D}{2}r)^{\beta}}{(D-R_0)^{\beta}\sum_{i=1}^{6}d_i^{\beta}}rdrd\theta \qquad (14)$$

In the same way, the $Var\left[ISR_{BS_0-MT}\right]$ can be expressed as

$$Var(ISR_{BS_0-MT})=\frac{\rho_1}{NRA}\cdot\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_{\frac{L\cos\theta-\sqrt{(L\cos\theta)^2-4\cdot(L^2-(D+R_0)^2)}}{2}}^{D-R_0}\left(\frac{(\frac{D}{2}r)^{\beta}}{(D-R_0)^{\beta}\sum_{i=1}^{6}d_i^{\beta}}-E\left(ISR_{BS_0-MT}\right)\right)^2 rdrd\theta \qquad (15)$$

3.2 The neighbor cell

When the target cell is overload, the MTs in the relay area will communicate with neighbor BS. We will consider the interferences from different TSs.

3.2.1 The first TS

In this part, the interferences mainly come from $BS_{i(i=1..6)}$-MT. The SIR can be expressed as

$$SIR_{BS_i-MT}=\frac{(D+R_0)^{\beta}r^{-\beta}}{[2(D+R_0)^{\beta}+3D^{\beta}+\left(D-R_0\right)^{\beta}](\sqrt{(L)^2+r^2-2rL\cos\theta})^{-\beta}} \qquad (16)$$

L is the distance between target BS and neighbor $BS_{i\,(i=1\ldots6)}$. In the same way, the ISR can be expressed as

$$ISR_{BS_i-MT}=1/SIR_{BS_i-MT}=\frac{[2(D+R_0)^{\beta}+3D^{\beta}+\left(D-R_0\right)^{\beta}](\sqrt{L^2+r^2-2rL\cos\theta})^{-\beta}}{(D+R_0)^{\beta}r^{-\beta}} \qquad (17)$$

And the $E\left[ISR_{BS_0-MT}\right]$ can be expressed as

$$E\left(ISR_{BS_i-MT}\right)=\frac{6}{\pi\cdot D^2}\cdot\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_{0}^{D}\frac{[2(D+R_0)^{\beta}+3D^{\beta}+\left(D-R_0\right)^{\beta}]}{(D+R_0)^{\beta}r^{-\beta}}\cdot(\sqrt{L^2+r^2-2rL\cos\theta})^{-\beta}rdrd\theta \qquad (18)$$

In the same way, the $Var\left[ISR_{BS_i-MT}\right]$ can be expressed as

$$Var(ISR_{BS_i-MT})=\frac{6}{\pi\cdot D^2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\int_{0}^{D}([\frac{2(D+R_0)^{\beta}+3D^{\beta}+(D-R_0)^{\beta}}{(D+R_0)^{\beta}r^{-\beta}}\cdot\left(\sqrt{L^2+r^2-2rL\cos\theta}\right)^{-\beta}]-E\left(ISR\right))^2 rdrd\theta \qquad (19)$$

3.2.2 The second TS

In this part, the interferences mainly come from six cells that can be expressed as

$$I_1=S_R\times(D-R_0)^{\beta}\times(\sqrt{r^2+L^2-2rL\cos\theta})^{-\beta}$$

$$I_2=S_R\times(\frac{D}{2})^{\beta}\times\left(\sqrt{r^2+(\frac{3L}{2})^2-2r\left(\frac{3L}{2}\right)\cos(\frac{\pi}{6}-\theta}\right)^{-\beta}$$

$$I_3=S_R\times(\frac{D}{2})^{\beta}\times\left(\sqrt{r^2+(\frac{3L}{2})^2-2r\left(\frac{3L}{2}\right)\cos(\frac{\pi}{6}-\theta}\right)^{-\beta}$$

$$I_4=S_R\times D^{\beta}\times\left(\sqrt{r^2+L^2-2rL\cos(\pi-\theta}\right)^{-\beta}$$

$$I_5=S_R\times D^{\beta}\times\left(\sqrt{r^2+L^2-2rL\cos(\frac{2\pi}{3}-\theta}\right)^{-\beta}$$

$$I_6 = S_R \times D^{\beta} \times \left(\sqrt{r^2 + L^2 - 2rL\cos(\frac{2\pi}{3} + \theta}\right)^{-\beta}$$

$$given: \sqrt{r^2 + L^2 - 2rL\cos(\theta)} < D \quad (20)$$

Where $BS_{i\text{-}j}$ represents j^{th} cell located at the i^{th} layer.

I_j is interference from $BS_{i\text{-}j}$.

The receiving power can be expressed as

$$P_{receive\_power} = S_R \cdot \left(\frac{D}{2}\right)^{\beta} \cdot \left(\sqrt{r^2 + (\frac{L}{2})^2 - 2r\left(\frac{L}{2}\right)\cos(\theta}\right)^{-\beta} \quad (21)$$

Where S_R is the lowest power for each MT, and the

SIR_{RS_i-MT} can be expressed as

$$SIR_{RS_i-MT} = \frac{P_{receive\_power}}{\sum_{i=1}^{6} S_i} = \frac{(\frac{D}{2})^{\beta} \cdot \left(\sqrt{r^2 + (\frac{L}{2})^2 - 2r\left(\frac{L}{2}\right)\cos(\theta}\right)^{-\beta}}{\sum_{i=1}^{6} I_i} \quad (22)$$

The ISR would be expressed as

$$ISR_{RS_i-MT} = 1/SIR_{RS_i-MT} = \frac{\sum_{i=1}^{6} I_i}{(\frac{D}{2})^{\beta} \cdot \left(\sqrt{r^2 + (\frac{L}{2})^2 - 2r\left(\frac{L}{2}\right)\cos(\theta}\right)^{-\beta}} \quad (23)$$

And the $E\left[ISR_{RS_i-MT}\right]$ can be expressed as

$$E\left(ISR_{RS_i-MT}\right) = \frac{\rho_2}{RA} \cdot \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \int_{D}^{\frac{2L\cos\theta + \sqrt{(2L\cos\theta)^2 - 4\left(L^2 - D^2\right)}}{2}} \frac{\sum_{i=1}^{6} I_i}{(\frac{D}{2})^{\beta}} \cdot \frac{r}{\left(\sqrt{r^2 + (\frac{L}{2})^2 - 2r\left(\frac{L}{2}\right)\cos(\theta}\right)^{-\beta}} dr d\theta \quad (24)$$

Where ρ_2 is the density of calls inside the RA.

IV. SUBCARRIERS REQUIRED ANALYSIS

In this section, we will calculate the mean value of subcarriers required in related cells. It is divided into three parts including the number of subcarriers required at the NRA and RA, and RSs of the reference cell. For each link of OFDMA two-hop cellular system, a group of subcarriers is allocated to incoming users to meet their requirements for data rate. In this work, we assumed that all users have the same data rate R_{req}. Due to the different users have different SIR, they will require different number of subcarriers. The R_{req} is expressed as

$$\begin{aligned} R_{req} &= W \sum_{m=1}^{n} \log_2\left(1 + SIR_m\right) \\ &= W \sum_{m=1}^{n} \log_2\left(1 + \frac{1}{ISR_m}\right) \end{aligned} \quad (25)$$

Where W represents the bandwidth of each subcarrier, and n represents the number of subcarriers required for an incoming user. ISR_m represents the ISR experienced by a call when using the m^{th} subcarrier on downlink transmission. To avoid blocking, the number of subcarriers required per MT needs to be higher than n. Due to the neglect of frequency dependent fast fading, the ISR on each subcarrier is same, and then, n can be calculated by

$$n = \frac{R_{req}}{W \times \log_2\left(1 + \frac{1}{ISR}\right)} \quad (26)$$

In order to calculate the probability of the required subcarriers, it is necessary to find the location where the user needs the maximum number of subcarriers, and then calculate the maximum value. It can be seen that the maximum number of subcarriers are required by a user when the distance between transmitter and receiver is the farthest because the receiver suffers from the most severe ICI from six neighboring cells. Then, W can be expressed as

$$n_{max} = \frac{R_{req}}{W \times \log_2\left(1 + \frac{1}{ISR_{max}}\right)} \quad (27)$$

Where ISR_{max} represents the maximum ISR the receiver probably experienced. On the other hand, we can derive ISR from following equation when the number of subcarriers required is determined.

$$ISR_n = \frac{1}{2^{\frac{R_{req}}{W \times n}} - 1} \quad (28)$$

ISR_n represents the value of ISR when the number of subcarriers needed is N. Besides that, we introduced a new parameter, $Q(n) = P(n_{\text{req}} = n)$, which means the probability of an incoming call requiring number of subcarriers n to meet its data rate. $P(n_{\text{req}} = n)$ can be given by

$$P(n_{\text{req}} = n) = P(n-1 < n_{\text{req}} \le n) \tag{29}$$

Due to $1 < n \le n_{max}$, so $P(n_{\text{req}} = n)$ represent the probability when $n-1 < n_{\text{req}} \le n$ and n_{req} is an integer. According to the equation (28), when the number of subcarriers required is lower than n, the value of ISR must be under β that is the threshold of ISR when n subcarriers required. So, the equation (29) can be rewritten as

$$P(n_{\text{req}} = n) = P\left(ISR_{\text{threshold}}^{n-1} < ISR \le ISR_{\text{threshold}}^{n}\right) \tag{30}$$

And $Q(n)$ can be expressed as

$$Q(n) = \begin{cases} P\left(ISR \ge ISR_{\text{threshold}}^{n_{\max}}\right), n = 0 \\ 1 - P\left(ISR \ge ISR_{\text{threshold}}^{1}\right), n = 1 \\ P\left(ISR \ge ISR_{\text{threshold}}^{n-1}\right) \\ \quad - P\left(ISR \ge ISR_{\text{threshold}}^{n}\right), \\ \quad 1 < n \le n_{max} \end{cases} .$$

$$\text{and} \sum_{n=0}^{n_{max}} Q(n) = 1 \tag{31}$$

When $n = 0$, the user is blocked and the probability is equal to $P\left(ISR \ge ISR_{\text{threshold}}^{n_{max}}\right)$. When $n = 1$, the probability is $1 - P\left(ISR \ge ISR_{\text{threshold}}^{1}\right)$. The average number in one hop of required subcarriers for a user in reference cell,$N_{\text{avg\_peruser}}$, is given by

$$N_{\text{avg\_peruser}} = \sum_{n=0}^{n_{\max}} n \times Q(n) \tag{32}$$

From equation (32), we can obtain the average subcarriers required for the three links denoted as $N_{\text{avg\_BS-MT}}$, $N_{\text{avg\_BS-RS}}$ $N_{\text{avg\_RS-MT}}$ respectively. $N_{\text{avg\_BS-MT}}$ represents the required subcarriers from link BS_0-MT_0 that located at the NRA. $N_{\text{avg\_BS-RS}}$ and $N_{\text{avg\_RS-MT}}$ represent the required subcarriers of link BS_i-RS_i and RS_i-MT_S for MTs that are located in the RA.

V. Optimization Problem

In this paper, we assume that all first layer neighboring cells have the same number of calls. This means that all neighboring BSs can support the same number of relayed calls and experience the same interference. That is to say, there are only two decision variables, ρ_1 and ρ_2. Moreover, the constraints based on the interference at the six neighboring BSs become similar and are reduced to one condition. The values of ρ_1 and ρ_2 have to be non-negative. We will force that they cannot be zero so that each disc can have some calls accepted. If one of the densities is zero at some point, some discs will have no calls accepted, which means that some users will never be able to make a call unless they change their position. Hence, all values have to be greater than zero.

The target of the optimization problem is to maximize the total number of calls inside the congested cell $N_{\text{avg\_cell}}$. The total number of calls is calculated by multiplying the density of calls by the area where these calls reside. Consequently, the objective function of the optimization problem can then be given by

$$\text{Max } N_{\text{avg\_cell}} = 6\rho_1 \cdot NRA + 6\rho_2 \cdot RA \tag{33}$$

Subject to

(1) $N_{\text{avg\_BS-MT}} \cdot NRA \le N_{total}$

(2) $\left(N_{\text{avg\_BS-RS}} + N_{\text{avg\_RS-MT}}\right) \cdot RA + R_{load\_ratio} \cdot N_{total} \le N_{total}$

(3) $0 < \rho_1 < 1, 0 < \rho_2 < 1$

Where R_{load_ratio} is the load ratio for neighboring cell. Solving this optimization problem yields the maximum number of calls that can be supported inside a congested cell under the given R_{load_ratio}. The optimization problem can be solved using the linear programming package in MATLAB.

VI. Simulation Results

In this paper, the simulation results are based on the analytical model that has been discussed above and calculated by using Monte-Carlo method. We consider that the subcarriers allocation scheme is random. The number of subcarriers needed is only to satisfy the data rate requirement R_{req}. If the subcarriers are not enough, the call will be blocked. The average number of subcarriers needed by all users in the reference cell is related to the path loss exponent, the annulus width and the load rate. It is proved in the following simulations.

Fig. 4. shows the changing curve of $N_{\mathrm{avg\_cell}}$ in the congestion cell when the width of annulus R_0 increases, and the optimized result of $N_{\mathrm{avg\_cell}}$ is compared with the case when no inter-cell relay is done and the case when inter-cell relay is performed but without optimization. From this figure, it is observed that in the case of optimization, the total number of calls $N_{\mathrm{avg\_cell}}$ increases as R_0 becomes larger. This is because when the width of annulus increases, the subcarriers requested for a user will decrease. When the neighboring cells are light-load, it is observed that the number of supported calls $N_{\mathrm{avg\_cell}}$ can be up to five times the case if inter-cell relay is not allowed. It can also be seen that in the case of optimization, the number of supported calls $N_{\mathrm{avg\_cell}}$ can be 1.5 times than those of the case of non-optimized. We assume that the density of congested cell is non-limited, therefore when the width of annulus is very large, the trend of changing curve is smooth. This is because the SIR will be stable as the width of annulus R_0 become larger.

Fig. 5. shows the relationship of the total number of calls $N_{\mathrm{avg\_cell}}$ in a congested cell with path loss β and width of annulus R_0 when the the load ratio of the neighboring cell R_{load_ratio} is 15%. It is seen that $N_{\mathrm{avg\_cell}}$ in the congested cell will increase with the β. This is because as β increases, it means that the SIR will increase and fewer subcarriers are needed to satisfy the signal quality.

Fig. 6. shows the total number of calls $N_{\mathrm{avg\_cell}}$ in a congested cell given a certain load ratio R_{load_ratio} for the neighboring cell when the width of annulus R_0 is 100m. It can be seen that the total number of calls decreases as R_{load_ratio} increases. When the load ratio R_{load_ratio} reaches 80%, the total number of calls by using inter-cell relay with optimization will close to the case of no inter-cell relay. Therefore when R_{load_ratio} is high, this method of inter-cell relay is not fit for solving the congestion problem. We can find that the variation in the load of other neighboring cells has higher effect on the capacity of the congested cell when optimization is performed. This is because that as the load ratio R_{load_ratio} increases, the number of subcarriers given for RA will decreases.

Table I. *Parameters for numerical analysis.*

Parameters for Numerical Analysis	Value
Radius of Cells R_{BS} (meters)	500
Transmit Power for BS P_{transmit} (dBm)	30
Subcarrier Bawidth W (KHz)	30
Path Loss Exponent β	2, 3 , 4
Data Rate Requirement R_{req} (Kbps)	64
Total subcarriers for a cell N_{total}	200
Load ratio of the neighboring cell R_{load_ratio}	15%

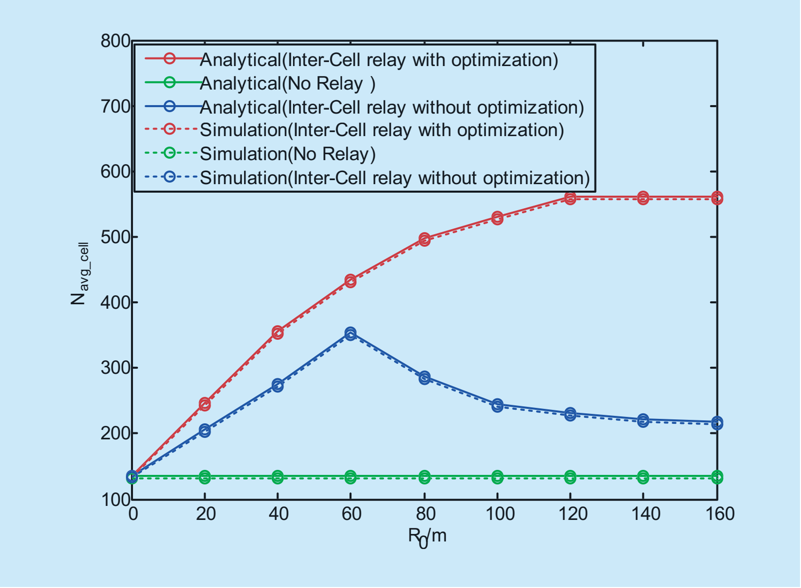

Fig. 4. *Comparison of the number of supported calls inside congested cell under three different cases.*

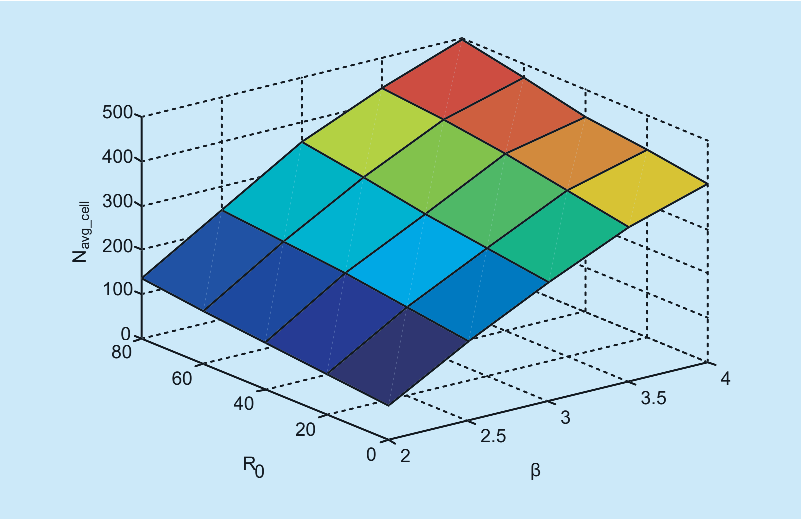

Fig. 5. *The relationship of the number of supported calls with different path loss exponent and width of annulus.*

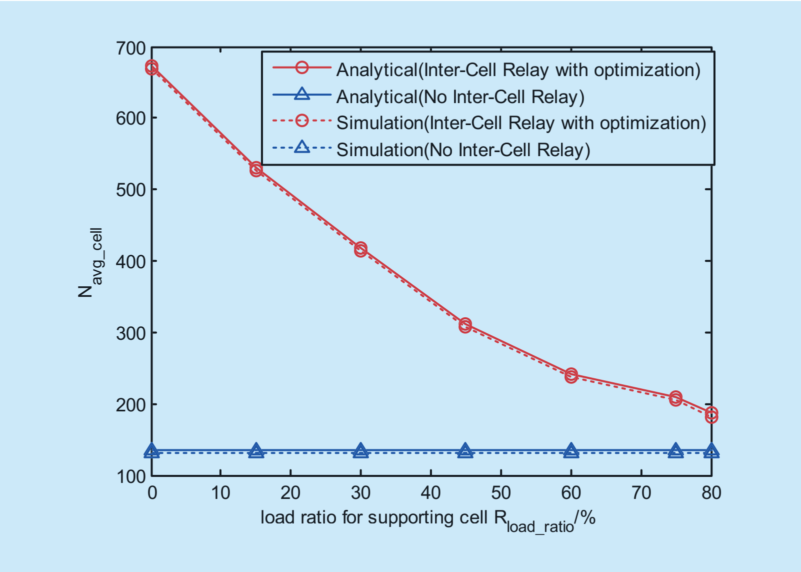

Fig. 6. *The relationship between the number of supported calls and load ratio.*

VII. CONCLUSION

In this paper, we propose a inter-cell relay method to solve the congestion problem and optimize the maximum number of calls in congested cell by using the variation of densities because it is easier for readers to follow. From the simulation result, we can find that this method is more suitable when the neighboring cells are light-load, and the case of optimization can achieve better performance than others. Moreover, two-hop relay is adopted in this papar, but if more than two-hop realy is used, the entire area of the congested cell will become the relay area, and more calls will be forwarded to the neighboring cell. But it will lead to more complex signal interference scenarios. In addition, the next generation of wireless networks will be wireless heterogeneous networks, including macrocell, microcell, picocell, femtocell and so on. Therefore, our next reserching work is to consider the mutil-hop inter-cell relay to solve congestion problem in wireless heterogeneous networks.

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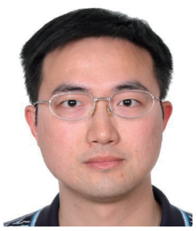

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