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Securitization of Mortality Risks in Life Annuities

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Abstract

The purpose of this article is to study mortality-based securities, such as mortality bonds and swaps, and to price the proposed mortality securities. We focus on individual annuity data, although some of the modeling techniques could be applied to other lines of annuity or life insurance.

Introduction

The purpose of this article is to study the securitization of mortality risks, especially the longevity risk inherent in a portfolio of annuities or in a pension plan. Life insurance and annuity securitization are now well established. Cummins (2004) has reviewed these securitizations recently. It turns out that none of these securitizations to date has focused on longevity risk. They have involved selling future cash flows, which depend on many risks, such as surrender rates, investment income as well as mortality. The only pure mortality deal is the Swiss Re mortality bond issued in December 2003 (Swiss Re, 2003; MorganStanley, 2003; The Actuary, 2004). Betteto (1999) describes the logic behind the earlier deals as "price efficiency." Cummins (2004) categorizes them as taking advantage of arbitrage opportunities or to invest in new classes of risk that enhance market efficiency. It is interesting that a leader of one of the largest actuarial

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consulting firms forecasts that there will be a flurry of life-based deals in 2004 (Gibson, 2004).

The Swiss Re bond is based on a mortality index of the general population of the United States, United Kingdom, France, Italy, and Switzerland. The term of the bond is 3 years, the price $400 million, and it pays LIBOR plus 135 basis points. If the mortality index exceeds 130 percent of the 2002 level, the principal is reduced. If it goes above 150 percent, the principal is exhausted. MorganStanley's announcement describes this as a 1 in 25 year event (MorganStanley, 2003). He goes on to say that "the appetite for this security from investors was strong." This is the same reaction investors have had to the so-called "catastrophe bonds" based on portfolios of property insurance. In this article, we focus primarily on the other side of the "mortality tail," longevity risk, for which annuity writers (insurers, their reinsurers, and pension plans) have the greatest concern.

Life expectancy throughout the world in recent decades has improved, but that does not necessarily imply that trend can be projected into the future. In addition to uncertainty in mortality forecasts, there are economic and policy changes that make management of longevity risk more important than ever.

According to Mitchell et al. (2001), social security reform and the shift from defined benefit to defined contribution private pension plans should increase demand for individual annuity products in the future. They also find evidence that an individual annuity contract appears to be a more attractive product to consumers today than 10 years ago. As demand for individual annuities increases, insurer's need for risk management of the potential mortality improvements increases as they write new individual annuity business. As Rappaport, Mercer, and Parikh (2002) describe, insurers are keenly interested in understanding the future course of longevity, as well as the protection provided by hedging, asset allocation strategies, reinsurance, and securitization of longevity risk.

In Insurance Securitization, we describe securitization of longevity risk with a mortality bond or a mortality swap. In Pricing the Mortality Risk Bond, we price it using the Wang transform. We illustrate how insurers (or reinsurers or pension plans) can use mortality-based securities to manage longevity risk. In Difficulties in Accurate Mortality Projection we describe the difficulties arising in making mortality projections. We discuss annuity data, including the Individual Annuity Mortality tables and the Group Annuity Experience Mortality (GAEM) reports from Reports of the Transactions of the Society of Actuaries (TSA). The last section is for discussion and conclusions.

INSURANCE SECURITIZATION

We are proposing a new type of mortality bond which is similar to the Swiss Re deal but focused on longevity. The structure is similar to other deals. Generally, the life-based securitizations follow the same structure as the so-called catastrophe-risk bonds. There have been over 30 catastrophe bond transactions reported in the financial press and many papers. Mortality risk bonds are different in several important ways. For example, deviation from mortality forecasts may occur gradually over a long period, as opposed to a sudden property portfolio loss. However, in both, transaction costs are likely to be high relative to reinsurance on a transaction basis.

In both transactions, the insurer (reinsurer or annuity provider) purchases reinsurance from a special purpose company (SPC). The SPC issues bonds to investors. The bond contract and reinsurance convey the risk from the annuity provider to the investors. The SPC invests the reinsurance premium and cash from the sale of the bonds in default-free securities. We will show how this can be set up to allow the SPC to pay the benefits under the terms of the reinsurance with certainty. Now let us be more specific.

Example

As an example of a mortality securitization, consider an insurer[1] that must pay immediate life annuities to ℓ_x annuitants[2] all aged x initially. Set the payment rate at 1,000/year annuitant. Let ℓ_{x+t} denote the number of survivors to year t. The insurer pays 1,000ℓ_{x+t} to its annuitants, which is random, as viewed from time 0. We will define a bond contract to hedge the risk that this portion of the insurer's payments to its annuitants exceeds an agreed upon level.

The insurer buys insurance from its SPC for a premium P at time 0. The insurance contract has a schedule of fixed trigger levels X_t such that the SPC pays the insurer the excess of the actual payments over the trigger. In year t, the insurer pays amount $1000\ell_{x+t}$ to its annuitants. If the payments exceed the trigger for that year, it collects the excess from the SPC, up to a maximum amount. Let us say that the maximum is stated as a multiple of the rate of annuity payments 1000C. Thus in each year $t = 1, 2, \ldots, T$ the insurer collects the benefit B_t from the SPC determined by this formula:

$$B_t = \begin{cases} 1000C & \text{if } \ell_{x+t} > X_t + C \\ 1000(\ell_{x+t} - X_t) & \text{if } X_t < \ell_{x+t} \leq X_t + C \\ 0 & \text{if } \ell_{x+t} \leq X_t. \end{cases} \tag{1}$$

The insurer specifies the annuitant pool in much the same way that mortgage loans are identified in the construction of a mortgage-backed security. The insurer's cash flow to annuitants $1000\ell_{x+t}$ at t is offset by positive cash flow B_t from the insurance:

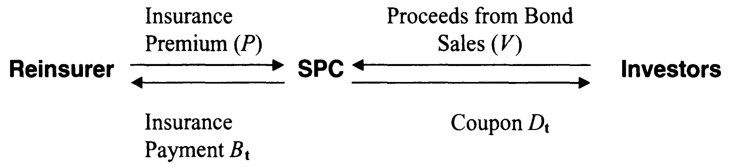
$$\begin{aligned} \text{Insurer's Net Cash Flow} &= 1000\ell_{x+t} - B_t \\ &= \begin{cases} 1000C(\ell_{x+t} - C) & \text{if } \ell_{x+T} > C + X_t \\ 1000X_t & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ 1000\ell_{x+t} & \text{if } \ell_{x+t} \leq X_t. \end{cases} \end{aligned} \tag{2}$$

For this structure, there is no "basis risk" in the reinsurance. Basis risk arises when the hedge is not exactly the same as the reinsurer's risk. The mortality bond covers

[1] The "insurer" could be an annuity writer, an annuity reinsurer, or private pension plan. The counter party could be a life insurer or investor.

[2] The security could be based on a mortality index rather than an actual portfolio. This will avoid the moral hazard problem, but it introduces basis risk.

Figure 1
Mortality Bond Cash Flow Diagram



the same risk, so there is no basis risk. This is in contrast to the Swiss Re deal, which is based on a population index rather than a portfolio of Swiss Re's life insurance polices (or its clients' policies). While there is no basis risk, the contract does not provide full coverage. We will study the distribution of the present value of the excess later.

The Bond Contract

Here is a description of the cash flows between the SPC, the investors, and the insurer as illustrated in Figure 1. SPC payments to the investors:

$$D_t = \begin{cases} 0 & \text{if } \ell_{x+t} > C + X_t \\ 1000C - B_t & \text{if } X_t < \ell_{x+t} \le C + X_t \\ 1000C & \text{if } \ell_{x+t} \le X_t, \end{cases} \tag{3}$$

$$= \begin{cases} 0 & \text{if } \ell_{x+T} > C + X_t \\ 1000(C + X_t - \ell_{x+t}) & \text{if } X_t < \ell_{x+t} \le C + X_t \\ 1000C & \text{if } \ell_{x+t} \le X_t, \end{cases} \tag{4}$$

where D_t is the total coupon paid to investors. The maximum value of ℓ_{x+t} is ℓ_x, attained if nobody dies, but from the perspective of 0, it is a random value between 0 and ℓ_x. Let the market price of the mortality bond be denoted by V. The aggregate cash flow out of the SPC is

$$B_t + D_t = 1000C$$

for each year $t = 1, \ldots, T$ and the principal amount $1000F$ at $t = T$. The SPC will perform on its insurance and bond contract commitments, with probability 1, provided $P + V$ is at least equal to the price W of a default-free fixed-coupon bond with annual coupon $1000C$ and principal $1000F$ valued with the bond market discount factors:

$$P + V \ge W = 1000F d(0, T) + \sum_{k=1}^{T} 1000C d(0, k). \tag{5}$$

The discount factors $d(0, k)$ can be taken from the bond market at the time the insurance is issued. In other words, if the insurance premium and proceeds from sale of the mortality bonds are sufficient, the SPC can buy a "straight bond" and have exactly the required coupon cash flow it needs to meet its obligation to the insurer and the investors. Each year, SPC receives 1000C as the straight bond coupon and then pays D_t to investors and B_t to the insurer. It is always the case that $1000C = D_t + B_t$ is exactly enough to meet its obligations.

Thus, we see how to set up a longevity risk bond contract for which the longevity risk over T years is passed to the capital market almost completely. Of course, the price of the mortality bond is yet to be addressed. And we need to see how likely it is that some payments will be covered. That is, how good is the insurance coverage? Or from the investor's perspective, how likely is it that they will miss a coupon? At time T, the SPC will have the accumulated value of $P + V - W$ and this is positive with probability 1. This future value belongs to the insurer since it is the sole owner of the SPC. For this article we assume $P + V = W$.

Swaps

The same cash flows, B_t to the insurer and D_t to the bondholders, can be arranged with swap agreements and no principal payment at time T. However, without the principal as collateral, the swap payments are subject to counter-party risk. Assuming there is no counter-party risk, the equivalent swaps contracts are described as follows. Since there is no counter-party risk, the insurer's payment of P at time 0 can be replaced by level annual payments of x where

$$P = x\sum_{k=1}^{T} d(0,k).$$

Then each year, the insurer pays x to the SPC (or a swap originator) and gets a floating benefit $B_t, t = 1, 2, \ldots, T$. There are no other payments. This is fixed for floating swap from the insurer's perspective. So as long as there is no counter-party risk, the insurer can get essentially the same reinsurance benefit from a swap. The swap might be provided by a broker or investment banker.

The same analysis applies to the bondholder's cash flows. In place of paying V for the mortality bond, they can pay a fixed amount y each year in order to receive the same coupons, where

$$y\sum_{k=1}^{T} d(0,k) + 1000F d(0,T) = \sum_{k=1}^{T} E^*[D_t]d(0,k) + 1000F d(0,T).$$

So

$$y\sum_{k=1}^{T} d(0,k) = \sum_{k=1}^{T} E^*[D_t]d(0,k).$$

Then in each year, the SPC gets $x + y$, exactly enough to finance its obligation $B_t + D_t$. The only difference is collateral. If there is no possibility of default on the fixed payments, then SPC will always have just enough cash to meet its floating payment obligations. In this case, swaps can replace the mortality bond. This may save transaction costs. The trade-off is that it introduces default risk.

PRICING THE MORTALITY RISK BONDS

Wang (1996, 2000, 2001) has developed a method of pricing risks that unifies financial and insurance pricing theories. We are going to apply this method to price mortality risk bonds. Let $\Phi(x)$ be the standard normal cumulative distribution function with a probability density function

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

for all x. Wang defines the distortion operator as

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) - \lambda] \tag{6}$$

for $0 < u < 1$ and a parameter λ. Now, given a distribution with cumulative density function $F(t)$, a "distorted" distribution $F^*(t)$ is determined by λ according to the equation

$$F^*(t) = g_\lambda(F)(t) = g_\lambda(F(t)). \tag{7}$$

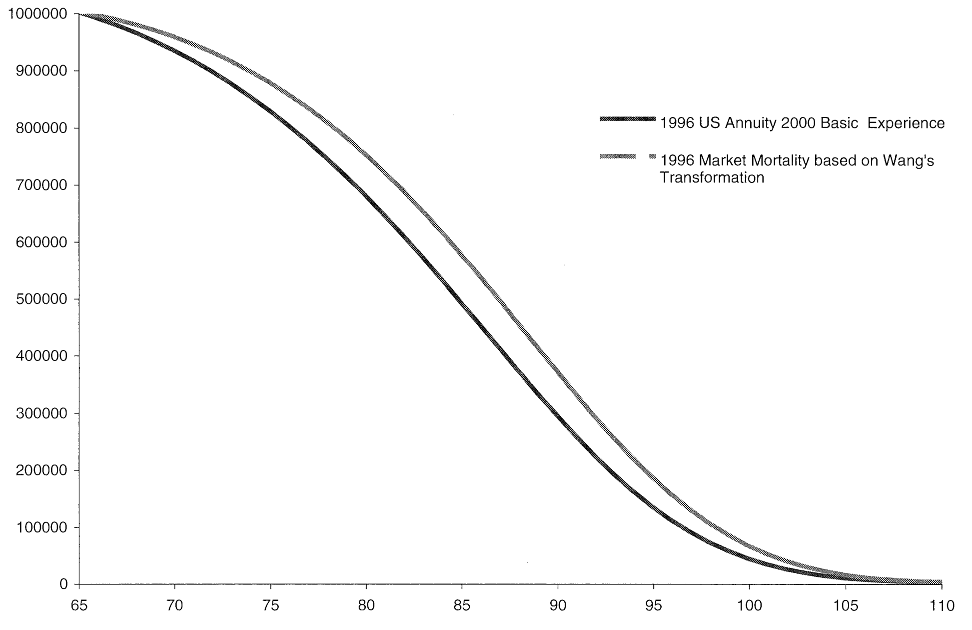
Consider an insurer's liability X over a time horizon $[0, T]$. The value or fair price of the liability is the discounted expected value under the distribution obtained from the distortion operator. Omitting the discount for now, we have the formula for the price:

$$H(X, \lambda) = \mathrm{E}^*(X) = \int x dF^*(x), \tag{8}$$

where $F^*(x) = g_\lambda(F)(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$. The parameter λ is called the market price of risk, reflecting the level of systematic risk. Thus, for an insurer's given liability X with cumulative density function F, the Wang transform will produce a "risk-adjusted" density function F^*. The mean value under F^*, denoted by $E^*[X]$, will define a risk-adjusted "fair-value" of X at time T, which can be further discounted to time zero, using the risk-free rate. Wang's paper describes the utility of this approach. It turns out to be very general and a generalization of well-known techniques in finance and actuarial science. Our idea is to use observed annuity prices to estimate the market price of risk for annuity mortality, then use the same distribution to price mortality bonds.

The Wang transform is based on the idea that the annuity market price takes into account the uncertainty in the mortality table, as well as the uncertainty in the lifetime of an annuitant once the table is given. The market price of risk does not, and need not, reflect the risk in interest rates because we are assuming that mortality and interest

FIGURE 2
The Result of Applying the Wang Transform to the Survival Distribution Based on 1996 IAM Experience for Males (65) and Prices From Best's Review, 1996



rate risks are independent. Moreover, we are assuming that investors accept the same transformed distribution and independence assumption for pricing mortality bonds.

Market Price of Risk

First we estimate the market price of risk λ. We defined our transformed distribution F^* as

$$F^*(t) = g_\lambda(F)(t) = \Phi[\Phi^{-1}({}_tq_{65}) - \lambda]. \tag{9}$$

For the distribution function $F(t) = {}_tq_{65}$, we use the 1996 IAM 2000 Basic Table for a male life age 65 and, separately, for a female life age 65 (Figure 2). Assuming an expense factor equal to 6 percent, we use the August 1996 market quotes of qualified immediate annuities (Kiczek, 1996) and the U.S. Treasury yield curve on August 15, 1996 to get the market price of risk λ by solving the following equations numerically:

$$125.73 = 7.48a_{65}^{(12)} \quad \text{for males,}$$

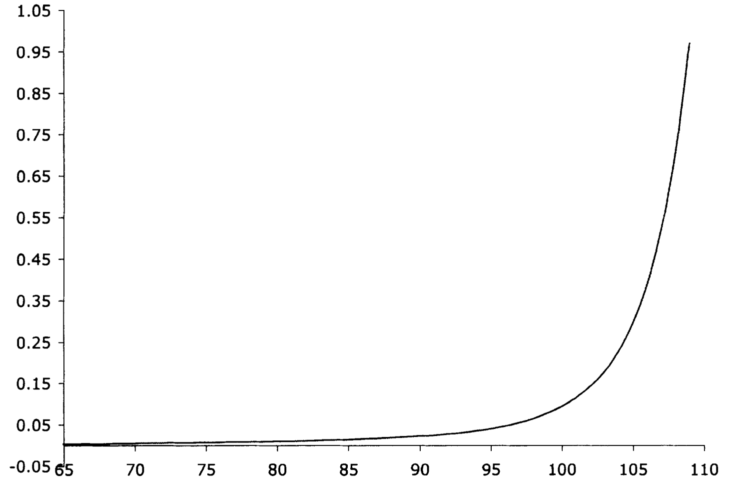
$$135.25 = 6.94a_{65}^{(12)} \quad \text{for females.}$$

The market price of risk for males and females, respectively, is shown in Table 1 and Figure 3. The market price of risk is 0.1792 for male annuitants and 0.2312 for female annuitants. Figure 3 shows that the market prices of the annuities are higher than

TABLE 1
The Market Price of Risk, Determined by the 1996 IAM 2000 Basic Table, the U.S. Treasury Constant Maturity Interest Rate Term Structure for August 15, 1996, and Annuity Market Prices From Best's Review (August 1996) Net of Our Assumed Expense Factor 6%. The Payment Rate Is the Dollars Per Month of Life Annuity Per$1,000 of Annuity Premium at the Issue Age. The Market Value Is the Price (Net of Annuity expenses) for $1 Per Month of Life Annuity

	Payment Rate	Market Value	Market Price of Risk
Male (65)	7.48	125.73	0.1792
Female (65)	6.94	135.25	0.2312

FIGURE 3
The Ratio of Standard Deviation to Expected Number of Survivors of an Initial Group of 10,000 Annuitants, Based on the 1994 GAM Female (65) Mortality Distribution



the mortality experience of the 1996 IAM 2000 Basic Table and the market curve lies above the 1996 IAM 2000 Basic mortality experience curve. We think of the 1996 IAM 2000 Basic Table as the actual or physical distribution, which requires a distortion to obtain market prices. That is, a risk premium is required for pricing annuities.

Mortality Bond Strike Levels

A designed portfolio of annuities underlies the mortality bond. The mortality bond contract may set several strike levels X_t. In our example, we set three different improvement levels for male and female (65) immediate annuities which determine the strike levels. We use the Renshaw, Haberman, and Hatzoupoulos (1996) method to predict the force of mortality and discuss this method in Difficulties in Accurate mortality Projection. The improvement levels are determined by the average of 30-year force of mortality improvement forecast for age group 65–74, age group 75–84, and age group 85–94, respectively, based on the 1963, 1973, 1983, and 1996 U.S. individual annuity mortality tables (Table 2).

TABLE 2
Three Different Improvement Levels Determine the Strike Levels

Age Range	Change of Force of Mortality
65–74	−0.0070
75–84	−0.0093
85–94	−0.0103

Including the above improvement factors, the corresponding strike level for each age will be X_t. The number of survivors ℓ_{65+t} is the number of lives attaining age in the survivorship group set in the contract. This means that we set the strike levels X_t as follows:

$$X_t \begin{cases} \ell_x \, {}_t p_x \, e^{0.0070t} & \text{for } t = 1, \ldots, 10 \\ \ell_x \, {}_t p_x \, e^{0.07} e^{0.0093(t-10)} & \text{for } t = 11, \ldots, 20 \\ \ell_x \, {}_t p_x \, e^{0.163} e^{0.0103(t-20)} & \text{for } t = 21, \ldots, 30, \end{cases}$$

where ${}_t P_x$ is the survival probability for the 1996 IAM 2000 Basic table for males or females.

In the annuity market, the price of an immediate annuity is the discounted expected cash flow to a random lifetime of annuitant. The random cash flows are $\{\frac{1}{12} 1000 \ell_{x+t/12} | t = 1, 2, \ldots\}$. The observed price allows us to calculate the market price of risk λ in the Wang transform. The market price of risk λ is obtained from the following equation:

$$12 \ell_x a_{65}^{(12)} = \sum_{t=1/12}^{\infty} \mathrm{E}^*[\ell_{x+t}] d(0, t), \tag{10}$$

where $\ell_x a_{65}^{(12)}$ is the total immediate annuity premium net of the insurer's expenses from a initial number of annuitants ℓ_x and $\mathrm{E}^*[\ell_{x+t}]$ is the transformed expected number of survivors to time t.

In the bond market, we have cash flows $\{D_t\}$ which depend on the same distribution of survivors. We assume that investors accept the same pricing method so that the bond price is

$$V = Fd(0, T) + \sum_{t=1}^{T} \mathrm{E}^*[D_t] d(0, t), \tag{11}$$

where D_t is defined as in (3) and $d(0, t)$ is the discount factor based on the risk-free interest rate term structure at the time the bond is issued. The face amount F is not at risk; it is paid at time T regardless of the number of surviving annuitants. The discount factors are from the U.S. Treasury interest rate term structure on August 15, 1996. The survival distribution in Equation (11) is the distribution derived from the annuity market. It is based on the 1996 U.S. Annuity 2000 Basic Mortality Tables and the Wang transform (9) with $\lambda = 0.1792$ for male annuitants and $\lambda = 0.2312$ for females.

$E^*[D_t]$ is calculated as follows. From (3), we can write the coupon payment as

$$\frac{1}{1000}D_t = \begin{cases} 0 & \text{if } \ell_{x+t} > C + X_t \\ C + X_t - \ell_{x+t} & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ C & \text{if } \ell_{x+t} \leq X_t \end{cases} \tag{12}$$

$$= C - \max(\ell_{x+t} - X_t, 0) + \max(\ell_{x+t} - X_t - C, 0)$$

$$= C - (\ell_{x+t} - X_t)_+ + (\ell_{x+t} - X_t - C)_+. \tag{13}$$

Therefore

$$\frac{1}{1000}\mathrm{E}^*[D_t] = C - \mathrm{E}^*[(\ell_{x+t} - X_t)_+] + \mathrm{E}^*[(\ell_{x+t} - X_t - C)_+].$$

The distribution of ℓ_{x+t} is the distribution of the number of survivors from ℓ_x who survive to age $x + t$, which occurs with probability ${}_tp_x^*$ where ${}_tp_x^*$ is the transformed survival probability. Therefore ℓ_{x+t} has a binomial with parameters ℓ_x and ${}_tp_x^*$. We have a large ℓ_x value so ℓ_{x+t} has approximately a normal distribution with mean $\mathrm{E}^*[\ell_{x+t}] = \mu_t^* = \ell_{xt}p_x^*$ and the variance $\mathrm{Var}^*[\ell_{x+t}] = \sigma_t^{*2} = \ell_{xt}p_x^*(1 - {}_tp_x^*)$.[3] Given a random variable X with $\mathrm{E}[X] < \infty$, integrating by parts shows that

$$\mathrm{E}[(X - k)_+] = \int_k^\infty [1 - F(t)]\, dt,$$

where $F(t) = \Pr(X \leq t)$. For a normal random variable X with mean 0 and variance 1, let $\phi(t) = e^{-u^2/2}/\sqrt{2\pi}$ denote the probability density and $\Phi(t) = \int_{-\infty}^t \phi(u)\, du$ the cumulative density. Then, we have the formula

$$\mathrm{E}[(X - k)_+] = \int_k^\infty [1 - \Phi(t)]\, dt.$$

Using the fact that $\phi'(t) = -t\phi(t)$ and integrating by parts, we can write this integral in terms of $\Phi(t)$ and $\phi(t)$ as

$$\Psi(k) = \int_k^\infty [1 - \Phi(t)]\, dt$$
$$= \phi(k) - k[1 - \Phi(k)].$$

This is a useful form since the functions $\phi(k)$ and $\Phi(k)$ can be calculated with *Excel*. Then we can calculate components of $\mathrm{E}^*(D_t)$:

[3] We are doing the calculation separately for males and females although the notation does not reflect the difference. We can easily adjust this for a mixture of males and females.

$$\begin{aligned}
E^*[(\ell_{x+t} - X_t)_+] &= E^*\left[\left(\ell_{x+t} - \mu_t^* - (X_t - \mu_t^*\right)_+\right] \\
&= \sigma_t^* E^*\left[\left(\frac{\ell_{x+t} - \mu_t^*}{\sigma_t^*} - k_t\right)_+\right] \\
&= \sigma_t^* \Psi(k_t),
\end{aligned}$$

where $k_t = (X_t - \mu_t^*)/\sigma_t^*$. Similarly

$$E^*[(\ell_{x+t} - X_t - C)_+] = \sigma_t^* \Psi\left(k_t + C/\sigma_t^*\right).$$

Finally, we have the formula

$$E^*[D_t] = 1000\left\{C - \sigma_t^*\left[\Psi(k_t) - \Psi\left(k_t + C/\sigma_t^*\right)\right]\right\}. \tag{14}$$

Consider an initial cohort of 10,000 annuitants all of the same sex, $\ell_{65} = 10{,}000$.

Table 3 shows prices for mortality bonds and reinsurance for a group of 10,000 male and female annuitants, respectively, with \$1,000 annual payout per person, with the strike levels defined as above, the annual aggregate cash flow out of the SPC \$700,000 (=1000C) and a 7 percent coupon rate for both straight bond and mortality bond. The price of the mortality bond on male (65) immediate annuitants is \$998.85 per \$1000 of face value. Similarly, the bond price for the female (65) immediate annuitants is \$995.57 per \$ 1000. With the above setup, the reinsurance price is \$11,493 for male (65) and \$44,337 for female (65). It gives the insurer 30-year protection. If the number of survivors exceeds the strike level X_t in year t, the SPC will pay the insurer the excess

TABLE 3

The Survival Distribution Underlying the 1996 Immediate Annuity Market Based on the 1996 U.S. Annuity 2000 Basic Mortality Table, the Wang transform, the Average Immediate Annuity Market Quotes in August 1996 and the U.S. Treasury Interest Rates on August 15, 1996

	Male (65)	Female (65)
Market price of risk (λ)	0.1792	0.2312
Number of annuitants	10,000	10,000
Annuity annual payout per person	1,000	1,000
Total premium from annuitants	99,650,768	107,232,089
Improvement level age 65–74	−0.0070	−0.0070
Improvement level age 75–84	−0.0093	−0.0093
Improvement level age 85–94	−0.0103	−0.0103
Face value of straight bond	10,000,000	10,000,000
Face value of mortality bond	10,000,000	10,000,00
Coupon rate of straight bond and mortality bond	0.07	0.07
Annual aggregate cash flow out of SPC (1000C)	700,000	700,000
Straight bond price	10,000,000	10,000,000
Mortality bond price	9,988,507	9,955,663
Reinsurance premium	11,493	44,337

(B_t) up to \$700,000 and the total coupon the investors will get that year is max[0, 700,000 – B_t]. Compared with the total immediate annuity premium the insurer collects from the annuitants (\$99,650,768 for male (65) and \$107,232,089 for female (65)), the reinsurance premium the insurer pays the SPC is only a negligible proportion of the total annuity premium (0.012% for male and 0.041% for female).

HOW GOOD IS THE HEDGE?

We point out that, given the distribution of survivors, there is very little variance in the cash flows. For example, given the survivor function ${}_tp_x$ of ℓ_{x+t}, we can describe ℓ_{x+t} as a binomial distribution. It is the number of successes in $N = \ell_x$ trials with the probability of a success on a given trial of ${}_tp_x$. The distribution of ℓ_{x+t} is approximately normal with parameters $E[\ell_{x+t}] = N_tp_x$ and $\mathrm{Var}[\ell_{x+t}] = N_tp_x(1 - {}_tp_x)$. The coefficient of variation is the ratio of σ_t/μ_t. The graph of the coefficient of variation of the number of survivors for an initial group of 10,000 annuitants, based on the 1994 GAM female (65) survival distribution is shown in Figure 3. Note that for a bond of duration 30 -years, the coefficient of variation rises to a maximum of about 1 percent, so there is little risk, *given the table*. The risk arises from uncertainty in the table. In calculating the bond value, we have to evaluate the expected value $E(\ell_{x+t})$ carefully. It is not enough to estimate a mortality table and then estimate the expected value. That approach would ignore the uncertainty in the table.

In order to illustrate this further, suppose that the possible tables are labeled with a random variable θ. The conditional distribution $\ell_{x+t} \mid \theta$ depends on θ. The unconditional moments are

$$\begin{aligned} \mathrm{E}[\ell_{x+t}] &= \mathrm{E}[\mathrm{E}[\ell_{x+t} \mid \theta]] = N\mathrm{E}[\mathrm{E}[{}_tp_x \mid \theta]] \\ \mathrm{Var}[\ell_{x+t}] &= \mathrm{E}[\mathrm{Var}[\ell_{x+t} \mid \theta]] + \mathrm{Var}[\mathrm{E}[\ell_{x+t} \mid \theta]]. \end{aligned} \tag{15}$$

Even if, as in Figure 3, there is very little variance in $\mathrm{E}[\ell_{x+t} \mid \theta]$ for all θ and the range of $t \leq 30$, there is still variance due to table uncertainty (the first term). We have little experience to guide us in estimating the terms $\mathrm{E}[\mathrm{E}[{}_tp_x \mid \theta]]$ and $\mathrm{E}[\mathrm{Var}[{}_tp_x \mid \theta]]$. Of course, this uncertainty occurs in all kinds of mortality calculation, not just mortality bonds.

We use the simulation to examine the impact of mortality shocks which shift the mortality tables to the insurer and the investors. With the setup shown in Table 3, we assume that the uncertainty v_t at time t in the mortality table follows a normal distribution with mean 0 and variance 1. The distribution of mortality shocks ε_t at time t is a beta distribution with parameters a and b. The mortality improvement shock ε_t is expressed as a percentage of the force of mortality μ_{x+t}, so it ranges from 0 to 1, that is, $0 < \epsilon_t < 1$ with probability 1.

Before performing the simulation, we first examine the mean and standard deviation of the annual percentage mortality improvement based on the U.S. 1963, 1973, 1983, and 1996 IAM tables for the males aged from (65) to (94). We conclude that its mean μ_m is equal to 0.0122 and the standard deviation σ_m is 0.0099. In the following simulation, we assume the coefficient of variation, CV, in different shock scenarios is constant, that is,

$$\mathrm{CV} = \sigma_m/\mu_m = 0.0099/0.0122 = 0.8139.$$

Without the shock, the survival probability for an age (x) at year t with the market expectation is $p^*_{x+t} = \exp(-\mu^*_{x+t})$. With the shock, the new survival probability p'_{x+t} can be expressed as

$$p'_{x+t} = (e^{-\mu^*_{x+t}})^{1-\varepsilon_t} = \left(p^*_{x+t}\right)^{1-\varepsilon_t}.$$

The random number of survivors ℓ'_{x+t+1} at time $t + 1$ is conditional on last period's survival number ℓ'_{x+t}, the shock parameters ε_t, and the mortality table random parameters v_t:

$$\ell'_{x+t+1} = \ell_{x+t}p'_{x+t} + v_t\sqrt{\ell'_{x+t}p'_{x+t}(1 - p'_{x+t})}.$$

Table 4 presents the results of simulations of the number of survivors ℓ_{85} at time $t = 20$, the present value of annuity payments and the present value of cash flows to bondholders. Each simulation includes a shock improvement to market mortality, modeled by multiplying the force of mortality by a factor $1 - \epsilon_t$ in each year. With a small mortality improvement shock $\mathrm{E}[\varepsilon_t] = 0.01$ (Table 4), that is, $a = 1.49$ and $b = 147.51$, the present value of total annuity payments increase from 99,650,768 without shock to 101,081,752 on average. In this scenario, investors will lose 3.43 percent [= (9,988,507 −9,646,354)/9,988,507] of their expected total payments. When there is a big shock $\mathrm{E}[\varepsilon_t] = 0.5$, the present value of total annuity payments will increase by 12.21 percent and the investors will lose 37.61 percent of their total expected payments on average. The impact of different mortality shock is illustrated in Figure 4. The mortality bond coupons are reduced as the SPC pays reinsurance benefits to the insurer. This hedges the insurer's risk that the number of survivors exceeds the market's expected value.

The mortality bond price and reinsurance premium are very sensitive to an insurer's expense rate. With a given annuity market quote and a given strike level, the net annuity premium increases with a decrease in the expense factor and thus the market price of risk λ increases. This implies that the market predicts a higher future survival rate ${}_tp^*_x$ and anticipates that the number of survivors is more likely to exceed the given strike level X_t. The mortality bond price goes down because the investors are more likely to lose higher proportion of their coupons and the reinsurance premium correspondingly goes up. The results for an increase in the expense rate are just on the opposite (see Table 5).

Longevity risk could easily extend over 50 years or more.[4] Most long-term bonds mature within 30 years. It is conceivable that a reinsurer can issue a very long-term bond (through the SPC), essentially default free except for mortality risk, which would appeal to investors. This would increase the reinsurer's capacity to issue long-term contracts to its client companies.

[4] The authors thank Thomas P. Edwards and his colleagues for pointing this out after reading the first draft of this article.

TABLE 4
Simulation Results for Mortality Shocks of 1%, 5%, 10%, 25%, and 50% Mortality Improvements in Excess of Market Expectation (10,000 Trials)

		Present Value		Percentage Change	
	l_{85}	Annuity Payments	Coupons and Principal	Annuity Payments (%)	Coupons and Principal (%)
Shock parameters:		$a = 1.49, b = 147.51, \mathrm{E}[\varepsilon_t] = 0.01, \sigma[\varepsilon_t] = 0.0081$			
Mean	5,882	101,081,752	9,646,354	1.44	−3.43
Maximum	6,061	102,034,832	9,733,716	2.39	−2.55
95th percentile	5,934	101,356,632	9,724,704	1.71	−2.64
5th percentile	5,854	100,930,352	9,494,340	1.28	−4.95
Minimum	5,850	100,910,696	9,022,497	1.26	−9.67
Standard deviation	26	138,312	76,634		
Shock parameters:		$a = 1.38, b = 26.30, \mathrm{E}[\varepsilon_t] = 0.05, \sigma[\varepsilon_t] = 0.0407$			
Mean	6,014	101,784,128	9,166,880	2.14	−8.23
Maximum	6,906	106,551,160	9,733,260	6.92	−2.56
95th percentile	6,282	103,214,240	9,691,183	3.58	−2.98
5th percentile	5,867	100,998,600	7,989,176	1.35	−20.02
Minimum	5,850	100,911,688	6,277,006	1.27	−37.16
Standard deviation	135	720,514	538,599		
Shock parameters:		$a = 1.26, b = 11.37, \mathrm{E}[\varepsilon_t] = 0.10, \sigma[\varepsilon_t] = 0.0814$			
Mean	6,187	102,709,848	8,521,295	3.07	−14.69
Maximum	7,933	112,101,464	9,733,680	12.49	−2.55
95th percentile	6,754	105,734,760	9,653,101	6.11	−3.36
5th percentile	5,881	101,073,520	6,599,827	1.43	−33.93
Minimum	5,850	100,910,768	4,901,722	1.26	−50.93
Standard deviation	285	1,522,121	995,280		
Shock parameters:		$a = 0.88, b = 2.65, \mathrm{E}[\varepsilon_t] = 0.25, \sigma[\varepsilon_t] = 0.2035$			
Mean	6,753	105,752,992	7,192,883	6.12	−27.99
Maximum	9,954	123,485,544	9,733,768	23.92	−2.55
95th percentile	8,372	114,514,088	9,624,507	14.92	−3.64
5th percentile	5,891	101,128,584	4,562,061	1.48	−54.33
Minimum	5,850	100,910,584	3,818,042	1.26	−61.78
Standard deviation	790	4,266,107	1,668,137		
Shock parameters:		$a = 0.25, b = 0.25, \mathrm{E}[\varepsilon_t] = 0.50, \sigma[\varepsilon_t] = 0.4070$			
Mean	7,847	111,815,112	6,232,152	12.21	−37.61
Maximum	10,005	123,776,936	9,733,797	24.21	−2.55
95th percentile	10,003	123,768,536	9,733,007	24.20	−2.56
5th percentile	5,850	100,912,208	3,808,961	1.27	−61.87
Minimum	5,850	100,910,520	3,808,223	1.26	−61.87
Standard deviation	1,696	9,326,669	2,436,069		

FIGURE 4
The Change in Expected Present Values of Annuity Payments (Solid Line) and Bondholder Payments (Broken Line) Are Shown as a Function of the Mortality Shocks E[ε_t]. The Numerical Values Are Shown in Table 4

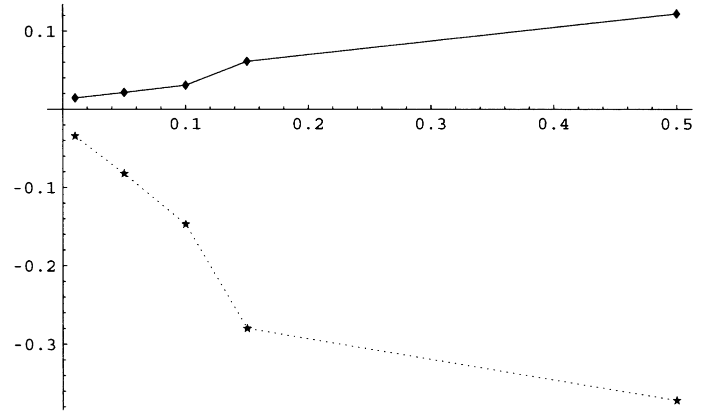


TABLE 5
The Sensitivity of Mortality Bond Price and Reinsurance Price to the Change of an Insurer's Expense Rate

	Male		Female	
Expense Factor (%)	Mortality Bond Price	Reinsurance Price	Mortality Bond Price	Reinsurance Price
4	9,316,726	683,274	9,279,932	720,068
6	9,988,507	11,493	9,955,663	44,337
8	10,000,000	0	10,000,000	0

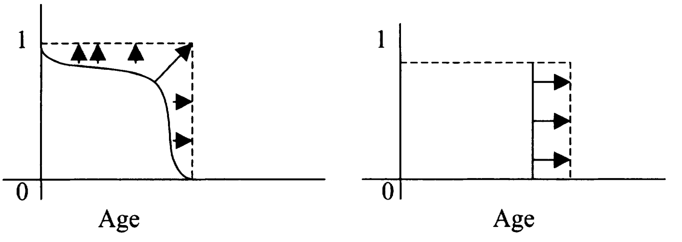
Reinsurers may find annuity securitization to be an efficient means of increasing capacity despite transaction costs simply because reinsurers must hold more capital to write the same risk. With greater capacity, better contracting terms (longer terms, e.g.), and potentially lower cost (more efficient use of capital), securitization may be a feasible tool for reinsurer to hedge its mortality risks.

For investors, the risk of losing a large proportion of annual coupon is relatively low (e.g., in our setup), even if for a big mortality improvement shock. The mortality bond may be a good candidate for the investors to diversify their investment portfolio.

DIFFICULTIES IN ACCURATE MORTALITY PROJECTION

General and insured population mortality have improved remarkably over the last several decades. For example, the force of mortality for male aged (65) decreases from 0.0222 based on the U.S. 1963 IAM Table to 0.0111 based on the U.S. 1996 IAM Table. At old age, probabilities of death are decreasing, increasing the need for living benefits. The calculation of expected present values (needed in pricing and reserving) requires

FIGURE 5
Two Views of Mortality Improvement, Rectangularization on the Left and Steady Progress on the Right



an appropriate mortality projection in order to avoid underestimation or overestimation of future costs which will jeopardize an insurer's profit or its market share.

Rogers (2002) shows that mortality operates within a complex framework and is influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions, and health behaviors. Not all of these factors improve with time. For example, for biological variables, recent declines in mortality rates were not distributed evenly over the disease categories of underlying and multiple causes of death. According to Stallard (2002), successes against the top three killers (heart diseases, cerebrovascular diseases, and malignant neoplasms) did not translate into successes against many of the lower ranked diseases. Moreover, Olshansky (2004) points out, a projected "quantum leap" in mortality depends on new biomedical technologies, administered to enough people to have an impact on the population. This may be difficult to do, even if these were a technological breakthrough.

Different Opinions in Mortality Trend

Improvement. Buettner (2002) concludes that there are today two alternative views about the future improvement of mortality at older ages: compression versus expansion (sometimes also called rectangularization vs. steady progress), illustrated in Figure 5. Mortality compression occurs when age-specific mortality declines over a widening range of adult ages, but meets natural limits for very advanced ages. As a result, the survivor curve would approach a rectangle, and mortality across countries may indeed converge to similar patterns.

In the case of steady progress, there are no natural limits to further reductions in mortality at higher ages. The age at which natural limits set in does not exist. Consequently, all age groups, especially higher age groups, would continue to experience declining mortality. The Human Genome Project is producing a rapidly expanding base of knowledge about life processes at their most fundamental level. Some experts have predicted that the genes for the aging process will be identified and drugs to retard the aging process will be developed in the not distant future. It is worth noting that genetic technology, including the mapping of the human genome, has developed much faster than forecasts. Anti-aging drugs may be available sooner than anyone forecasts.

Life Table Entropy. Life table entropy refers to a phenomenon that further improvement of already high life expectancies may become increasingly more difficult. The gains in survival a century ago were greater than they have been more recently. For instance, Rogers (2002) shows that the survival gains achieved between 1900 and 1920 are large compared to the modest gains realized between 1980 and 1999. Hayflick (2002) suggests that,

> . . .Those who predict enormous gains in life expectation in the future based only on mathematically sound predictions of life table data but ignore the biological facts that underlie longevity determination and aging do so at their own peril and the peril of those who make health policy for the future of this country.

Deterioration. Although general population mortality has improved over time, the improvement may be overstated. Substantial mortality improvements often come after periods of mortality deterioration. For example, between 1970 and 1975, males aged 30–35 saw annual mortality improvement of over 2 percent, but this may be an adjustment to the 1.5 percent annual mortality decline that occurred during the previous 5-year period. Moreover, there is still a chance for a resurgence of infectious diseases. Deaths due to influenza could increase with the introduction of new influenza strains or with the shortages of the influenza vaccine. Rogers (2002) argues that although HIV is now controlled, it is not eradicated and could expand, or variants of HIV could develop that could increase mortality. Drug-resistant infectious diseases such as tuberculosis could increase. Goss, Wade, and Bell (1998) find that age-adjusted annual death rates for ages 85 and over in the United States actually deteriorated by 0.72 percent per year for males and by 0.52 percent for females during the observation period 1990–1994.

There is no agreement among experts on the future of mortality. Steady improvement is the trend, but changes in either direction are feasible.

Technical Difficulties in Mortality Projections

Quality of Data. Good quality complete data are a prerequisite for a reliable mortality projection. However, in reality it is not easy to obtain data for research. For example, although detailed data on old-age mortality are collected in most countries of the developed world, they are not so commonly available for developing countries. Buettner (2002) claims that even in developed countries, the quality of age reporting deteriorates among the very old.

The Society of Actuaries' series of studies of life annuity experience is of limited value for several reasons. First, it is not timely. Second, it is appropriate only for the products the policy holders owned (whole life, term life, or annuities, e.g.). So, it cannot be used directly to assess mortality for new products or similar products issued on a new basis (e.g., underwriting annuities for select mortality).

Thulin, Caron, and Jankunis (2002) note that complexity of annuity products nowadays often makes mortality projection difficult. Sometimes, an insurer has to introduce new entries with different mortality assumptions into the insured pool. For instance, trends in the marketplace are blurring traditional distinctions in the following two key areas:

(1) Work site products sold on an individual basis increasingly show features traditionally associated with group products.

(2) Group products sold on the basis of individual election in the workplace (voluntary products) with minimal participation requirements compete directly with individual products.

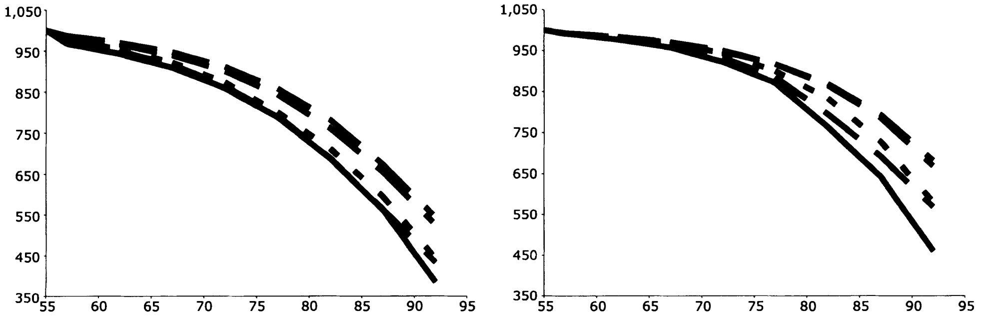
They severely limit insurers' ability to underwrite to discern mortality differentials. New sources of underwriting information are becoming a way of life for insurers, as pressure on costs and hastened issue pressure create an underwriting environment with less documentation and information. One solution is making more data available to researchers and making it available sooner.

The Society of Actuaries publishes tables and mortality reports from time to time. The individual annuity mortality (IAM) tables are intended for estimation of insurance company liabilities and these tables are based on actual insurance industry experience. We use the projected IAM tables to determine the strike levels for our annuity mortality bond. The Society also published periodic group annuity mortality reports of actual experience. While the reports do not contain complete mortality tables, they are not adjusted and not as conservative as the IAM tables. Moreover, the experience reports were made more frequently than the IAM tables were constructed. In this section, we use the group annuity mortality tables for the illustration and prediction of future mortality trends although the same skill can be applied on the individual annuity mortality tables.

The *GAM Experience Reports* on Annuities (1952, 1962, 1975, 1983, 1984, 1987, 1990, 1994, 1996) describe the mortality improvement from 1951 to 1992. The Reports give the number of deaths observed among a cohort of annuitants in 5-year age groups observed for 1 year. The observations of deaths and exposures are summarized in the Appendix to this article. The *Reports* provide data, but do not construct mortality tables. We show graphs of this experience in Figure 6. For male and female data, the survival curves generally rise with the observation period. The change between 1981 and 1991 for females is an exception since there is some deterioration at the later ages. That is, the lowest rates at each age are for the 1951 observations, the next to lowest are for 1961, and so on. The trend in improvement is increasing on average, with the largest increase occurring between 1971 and 1981 for males and females.

FIGURE 6

Number of Survivors of an Initial Cohort of 1,000 Male (left) and Female Lives at Age 55, Based on the Society of Actuaries *TSA Reports* for 1951, 1961, 1971, 1981, and 1991 on Group Annuity Experience, Without Adjustments



Projection Models. Recent changes in mortality challenge mortality projection models. The competitive nature of the insurance market means that an insurer cannot raise its price at will. A sound projection model is crucial.

However, the revealed weakness and problems of poor fitting may arise because most projection models do not capture the dynamics of mortality that is changing in a dramatic and fundamental way.

Renshaw, Haberman, and Hatzoupoulos (1996) suggest a generalized linear model which showed mortality declining over time with the rates of decline not being necessarily uniform across the age range. It incorporates both the age variation in morality and the underlying trends in the mortality rates. The advantage of this model is that the predictions of future forces of mortality come directly from the model formula. We adopt this model for investigating the performance of mortality derivatives based on a portfolio of life annuities.

During a certain period, the force of mortality, $\mu(x, t)$, at age x, in calendar year t, is modeled using the following formula:

$$\begin{aligned}\mu(x,t) &= \exp\left[\beta_0 + \sum_{j=1}^{s} \beta_j L_j(x') + \sum_{i=1}^{r} \alpha_i t'^i + \sum_{i=1}^{r}\sum_{j=1}^{s} \gamma_{i,j} L_j(x') t'^i\right] \\ &= \exp\left\{\sum_{j=0}^{s} \beta_j L_j(x')\right\} \exp\left\{\sum_{i=1}^{r} \left(\alpha_i + \sum_{j=1}^{s} \gamma_{ij} L_j(x')\right) t'^i\right\}, \end{aligned} \tag{16}$$

where

$$t' = \frac{t - 1971.5}{20.5} \quad \text{and} \quad x' = \frac{x - 74.5}{17.5}.$$

Sithole, Haberman, and Varrall (2000) use the same model. They note that first factor in (16) is the equivalent of a Gompertz–Makeham graduation term. The second multiplicative term is an adjustment term to predict an age-specific trend. The γ_{ij} terms may be pre-set to 0. The age and time variables are rescaled to x' and t' so that both are mapped onto the interval $[-1, +1]$ after transforming ages and calendar years. $L_j(x)$ is the Legendre polynomial defined below:

$$\begin{aligned} L_0(x) &= 1 \\ L_1(x) &= x \\ L_2(x) &= (3x^2 - 1)/2 \\ L_3(x) &= (5x^3 - 3x)/2 \\ &\vdots \\ (n+1)L_{n+1}(x) &= (2n+1)xL_n(x) - nL_{n-1}(x), \end{aligned}$$

where n is a positive integer and $-1 \leq x \leq 1$.

TABLE 6
Group Annuities, 6-Parameter Log-Link Model. All of the Coefficients Are Significant at the 1% Level

	Male		Female	
Coefficient	Value	Standard Error	Value	Standard Error
β_0	−2.7744	0.0087	−3.3375	0.0111
β_1	1.3991	0.0139	1.7028	0.0179
β_2	0.1053	0.0114	0.1543	0.0146
β_3	−0.1073	0.0127	−0.0872	0.0163
α_1	−0.2719	0.0116	−0.2660	0.0149
$\gamma_{1,1}$	0.0839	0.0178	−0.1294	0.0228
Adjusted R^2		0.9944		0.9930
Sum of squared errors		0.0701		0.0899

The data are the actual group annuity mortality experience for calendar years $t = 1951$, 1961, 1971, 1981, ..., 1992. Since the GAM Experience Reports are 5-year age group results, we assume that the ratio of the total number of deaths in each group over the total number of exposures in that group (the average death rate in that group) represents the death rate of the middle-point age of that group. We use the middle-point age as our observation in the regression. The experience was analyzed for the middle-point age ranges $x = 57$ to 92 years for both male and female, giving a total of 120 data cells for males and 120 for females.

In fitting the Equation (16), we found that when the parameter $\gamma_{1,2}$ is excluded from the formula (for male and female), all of the remaining six parameters in the model are statistically significant. Although the six-parameter model which excludes the quadratic coefficient in age effects from the trend adjustment term was next fitted to the data, the revised models seem to be appropriate for making predictions of future forces of mortality

$$\mu(x,t) = \exp[\beta_0 + \beta_1 L_1(x') + \beta_2 L_2(x') + \beta_3 L_3(x') + \alpha_1 t' + \gamma_{11} L_1(x')t'].$$

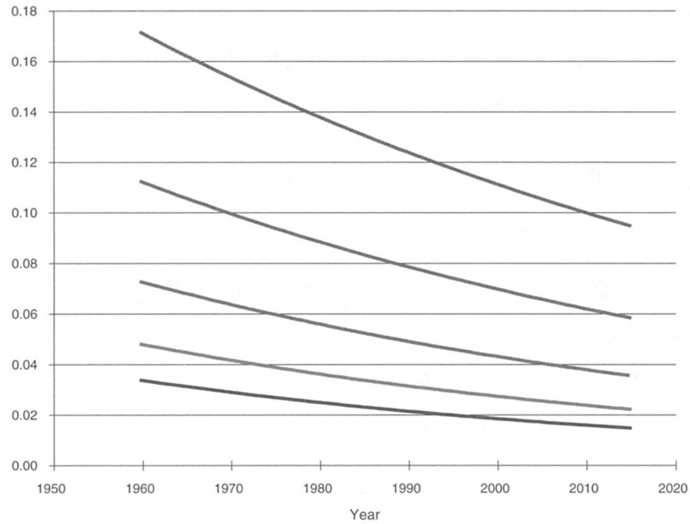
Details of the revised fit are given in Table 6.

Figure 7 shows the male group annuity predicted forces of mortality based on the 6-parameter model given by (17). All of the predicted forces of mortality progress smoothly with respect to both age and time, and the model naturally predicts a reduction in the rate of improvement in mortality at the old ages. There are errors in the estimate which should tell us how confident we can be in projecting mortality into the future, assuming the dynamics of mortality improvement continues as it has in the observation period. This is potentially dangerous. As we have pointed out earlier, there is a good bit of controversy with regard to the dynamics of mortality improvement.

We also note that these results are based on group annuity experience. Individual annuity experience may be very different. For example, anti-selection should be a much more important issue. As the market for individual immediate annuities develops, insurers will have to adjust their estimates to reflect the change in the market mortality. They may have to apply underwriting techniques and control for moral hazard and anti-selection when they issue annuities, just as they now do for life insurance.

FIGURE 7

Male Group Annuity Mortality Predicted Forces of Mortality Based on 6-Parameter Log-Link Model and *TSA* Reports 1951–1992. The Top Curve Is the Force of Mortality for Age 85, the One Just Below It Is for Age 80, Then 75, 70, and the Bottom Curve Is for Age 65. The Greatest Improvement (Steepest Slope) Is at Age 85



Since individual annuity mortality tables are more likely to capture the information asymmetry, we use the projection based on individual annuity mortality tables to determine different mortality-improvement levels for different age groups specified in the mortality bond contract.

DISCUSSION AND CONCLUSIONS

Financial innovation has led to the creation of new classes of securities that provide opportunities for insurers to manage their underwriting and to price risks more efficiently. Cummins and Lewis (2002) establish that risk expansion helps to explain the development of catastrophic risk bonds and options in the 1990s. A similar expansion is needed to manage longevity risk. There is a growing demand for a long-term hedge against improving annuity mortality. We have shown how innovation in swaps and bond contracts can provide new securities which can provide the hedge insurers need.

There is a trend of privatizing social security systems with insurers taking more longevity risk. Moreover, the trend to defined contribution corporate pension plans is increasing the potential market for immediate annuities.

This is an opportunity and also a challenge to insurers. Insurers will need increased capacity to take on longevity risk, and securities markets can provide it. This will allow annuity insurers to share this "big cake." Securitization of mortality risks has long duration, high capacity, and possibly low cost. Demand for new securities arises when new risks appear and when existing risks become more significant in magnitude. And we now have the technology to securitize the mortality risks based on modern financial models. Securitization in the annuity and life insurance markets has been relatively rare, but we have argued that this may change. We explored the securitization of mortality risks showing how it can help solve the difficulties in managing annuity mortality risk.

APPENDIX: SUMMARY OF DATA

We collected the data from the Society of Actuaries Transactions Reports for each of the years for which there was data. We used reports for calendar years published for the years 1951, 1961, 1971, and each year from 1981 to 1992. The last report is based on 1992 experience. We understand that the Society of Actuaries is reviving its experience studies.

Group Annuity Experience 1951, 1961, 1971, and 1981

	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
			1951			
55–59	335.70	11.00	1174.25	10.00	1509.95	21.00
60–64	12102.34	308.00	3847.76	57.00	15950.10	365.00
65–69	39871.68	1413.00	4602.89	91.00	44474.57	1504.00
70–74	17218.98	958.00	1737.57	63.00	18956.55	1021.00
75–79	5873.40	484.00	666.00	37.00	6539.40	521.00
80–84	1774.33	226.00	209.00	26.00	1983.33	252.00
85–89	374.08	68.00	51.25	8.00	425.33	76.00
90–94	47.42	15.00	7.00	2.00	54.42	17.00
			1961			
55–59	1,371.88	36.00	2,454.63	18.00	3,826.51	54.00
60–64	23,718.46	605.00	9,902.34	116.00	33,620.80	721.00
65–69	96,620.43	3,371.00	19,390.30	333.00	116,010.73	3,704.00
70–74	60,560.45	3,371.00	10,594.01	349.00	71,154.46	3,720.00
75–79	26,772.96	2,275.00	3,901.58	195.00	30,674.54	2,470.00
80–84	7,701.84	1,002.00	1,057.17	109.00	8,759.01	1,111.00
85–89	1,717.08	310.00	275.00	35.00	1,992.08	345.00
90–94	254.42	59.00	39.00	7.00	293.42	66.00
			1971			
55–59	3,611.23	85.00	3,574.90	26.00	7,186.13	111.00
60–64	33,806.66	791.00	18,521.74	177.00	52,328.40	968.00
65–69	120,227.85	4,022.00	41,802.04	595.00	162,029.89	4,617.00
70–74	93,795.47	4,955.00	28,542.94	746.00	122,338.41	5,701.00
75–79	63,066.93	5,269.00	16,284.46	747.00	79,351.39	6,016.00
80–84	28,166.41	3,113.00	6,815.79	510.00	34,982.20	3,623.00
85–89	8,022.23	1,315.00	1,699.37	213.00	9,721.60	1,528.00
90–94	1,328.05	338.00	251.95	51.00	1,580.00	389.00
			1981			
55–59	26,599.21	440.00	11,124.59	99.00	37,723.80	539.00
60–64	82,756.29	1,568.00	32,978.18	347.00	115,734.47	1,915.00
65–69	185,232.93	4,924.00	73,727.06	1,003.00	258,959.99	5,927.00
70–74	157,276.45	6,571.00	68,210.94	1,397.00	225,487.39	7,968.00
75–79	97,763.34	6,189.00	42,614.73	1,347.00	140,378.07	7,536.00
80–84	48,755.90	4,727.00	20,588.86	1,093.00	69,344.76	5,820.00
85–89	19,601.58	2,719.00	7,936.75	681.00	27,538.33	3,400.00
90–94	4,980.49	990.00	2,087.62	294.00	7,068.11	1,284.00

Group Annuity Experience 1982–1985

Attained Age	Male Exposure	Male Deaths	Female Exposure	Female Deaths	Total Exposure	Total Deaths
			1982			
55–59	28,631.53	453.00	11,754.62	92.00	40,386.15	545.00
60–64	89,455.43	1,753.00	35,433.49	336.00	124,888.92	2,089.00
65–69	192,308.39	5,097.00	75,640.56	985.00	267,948.95	6,082.00
70–74	162,420.78	6,740.00	72,661.69	1,354.00	235,082.47	8,094.00
75–79	103,419.33	6,465.00	48,058.37	1,540.00	151,477.70	8,005.00
80–84	52,549.11	4,861.00	23,671.10	1,231.00	76,220.21	6,092.00
85–89	21,392.48	2,989.00	9,443.51	832.00	30,835.99	3,821.00
90–94	5,716.77	1,082.00	2,526.42	322.00	8,243.19	1,404.00
			1983			
55–59	33,163.65	510.00	13,783.18	117.00	46,946.83	627.00
60–64	98,632.53	1,868.00	41,665.68	435.00	140,298.21	2,303.00
65–69	195,074.64	5,153.00	79,663.64	1,103.00	274,738.28	6,256.00
70–74	170,348.65	6,995.00	72,621.93	1,511.00	242,970.58	8,506.00
75–79	107,213.60	6,964.00	48,482.16	1,613.00	155,695.76	8,577.00
80–84	57,936.04	5,399.00	24,237.52	1,388.00	82,173.56	6,787.00
85–89	22,035.27	3,111.00	9,528.77	895.00	31,564.04	4,006.00
90–94	6,136.86	1,218.00	2,725.40	373.00	8,862.26	1,591.00
			1984			
55–59	40,574.69	580.00	16,305.25	132.00	56,879.94	712.00
60–64	119,381.14	2,212.00	48,941.94	448.00	168,323.08	2,660.00
65–69	221,883.84	5,695.00	91,062.97	1,241.00	312,946.81	6,936.00
70–74	200,590.93	8,196.00	86,304.56	1,870.00	286,895.49	10,066.00
75–79	129,357.81	8,141.00	60,361.35	2,106.00	189,719.16	10,247.00
80–84	67,297.97	6,288.00	31,781.28	1,771.00	99,079.25	8,059.00
85–89	26,575.80	3,766.00	12,400.26	1,211.00	38,976.06	4,977.00
90–94	7,743.72	1,574.00	3,681.76	573.00	11,425.48	2,147.00
			1985			
55–59	43,299.71	656.00	17,016.15	146.00	60,315.86	802.00
60–64	123,040.09	2,386.00	50,603.92	565.00	173,644.01	2,951.00
65–69	223,999.93	6,226.00	93,571.37	1,368.00	317,571.30	7,594.00
70–74	207,718.42	9,000.00	90,306.94	2,050.00	298,025.36	11,050.00
75–79	137,102.94	9,186.00	65,194.85	2,426.00	202,297.79	11,612.00
80–84	71,953.72	7,141.00	35,412.31	2,137.00	107,366.03	9,278.00
85–89	28,655.87	4,287.00	14,095.45	1,437.00	42,751.32	5,724.00
90–94	8,411.94	1,812.00	4,179.97	671.00	12,591.91	2,483.00

Group Annuity Experience 1986–1989

Attained Age	Male Exposure	Male Deaths	Female Exposure	Female Deaths	Total Exposure	Total Deaths
			1986			
55–59	44,010.72	627.00	16,677.86	112.00	60,688.58	739.00
60–64	122,620.42	2,163.00	50,381.10	476.00	173,001.52	2,639.00
65–69	227,995.35	5,699.41	95,512.26	1,261.00	323,507.61	6,960.41
70–74	216,055.50	8,098.29	93,727.78	1,966.00	309,783.28	10,064.29
75–79	146,182.97	8,610.00	68,834.32	2,324.00	215,017.29	10,934.00
80–84	78,070.67	7,153.00	38,836.55	2,108.00	116,907.22	9,261.00
85–89	31,484.42	4,005.00	15,650.49	1,406.00	47,134.91	5,411.00
90–94	9,097.10	1,678.00	4,672.65	690.00	13,769.75	2,368.00

(continued)

(Continued)

	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
			1987			
55–59	47,303.94	598.00	17,781.62	134.00	65,085.56	732.00
60–64	129,028.29	2,138.00	53,226.99	533.00	182,255.28	2,671.00
65–69	238,848.85	5,773.00	101,240.19	1,356.00	340,089.04	7,129.00
70–74	223,665.17	8,714.00	98,442.35	2,054.00	322,107.52	10,768.00
75–79	157,461.29	9,443.00	74,752.64	2,525.00	232,213.93	11,968.00
80–84	83,820.45	7,671.00	43,600.05	2,452.00	127,420.50	10,123.00
85–89	34,094.97	4,590.00	18,036.28	1,677.00	52,131.25	6,267.00
90–94	9,836.78	1,921.00	5,395.54	825.00	15,232.32	2,746.00
			1988			
55–59	49,424.32	683.00	18,162.87	141.00	67,587.19	824.00
60–64	132,778.58	2,252.00	53,788.54	513.00	186,567.12	2,765.00
65–69	235,874.82	5,587.00	102,022.53	1,295.00	337,897.35	6,882.00
70–74	221,164.05	8,388.00	99,853.21	2,116.00	321,017.26	10,504.00
75–79	162,202.31	9,530.00	78,542.78	2,630.00	240,745.09	12,160.00
80–84	88,225.65	8,012.00	47,418.51	2,583.00	135,644.16	10,595.00
85–89	35,929.54	4,707.00	20,142.57	1,879.00	56,072.11	6,586.00
90–94	10,484.98	2,002.00	5,926.74	845.00	16,411.72	2,847.00
			1989			
55–59	45,167.60	580.00	19,788.90	138.00	64,956.50	718.00
60–64	120,348.84	2,008.00	53,312.98	488.00	173,661.82	2,496.00
65–69	201,223.57	4,827.00	94,345.49	1,235.00	295,569.06	6,062.00
70–74	180,723.00	6,748.00	88,016.87	1,829.00	268,739.87	8,577.00
75–79	134,297.88	7,852.00	70,107.48	2,357.00	204,405.36	10,209.00
80–84	72,524.22	6,606.00	41,921.07	2,353.00	114,445.29	8,959.00
85–89	29,672.14	3,992.00	18,031.93	1,628.00	47,704.07	5,620.00
90–94	8,245.34	1,704.00	5,114.09	820.00	13,359.43	2,524.00

Group Annuity Experience 1990–1992

	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
			1990			
55–59	53,375.95	686.00	24,851.00	174.00	78,226.95	860.00
60–64	146,190.29	2,333.00	67,235.53	596.00	213,425.82	2,929.00
65–69	258,735.98	5,949.00	122,669.86	1,562.00	381,405.84	7,511.00
70–74	238,694.07	8,911.00	116,031.28	2,327.00	354,725.35	11,238.00
75–79	189,088.76	11,105.00	95,064.28	3,186.00	284,153.04	14,291.00
80–84	109,583.14	9,912.00	62,967.19	3,520.00	172,550.33	13,432.00
85–89	48,022.47	6,572.00	30,700.37	2,778.00	78,722.84	9,350.00
90–94	14,672.14	2,842.00	10,005.89	1,445.00	24,678.03	4,287.00
			1991			
55–59	50,731.54	661.00	22,245.01	158.00	72,976.55	819.00
60–64	137,582.08	2,383.00	60,722.23	543.00	198,304.31	2,926.00
65–69	240,820.91	5,774.00	114,994.74	1,557.00	355,815.65	7,331.00
70–74	230,909.08	8,685.00	115,825.34	2,433.00	346,734.42	11,118.00
75–79	188,317.23	10,961.00	96,727.27	3,360.00	285,044.50	14,321.00
80–84	112,587.59	10,048.00	66,245.62	3,791.00	178,833.21	13,839.00
85–89	48,883.89	6,713.00	33,022.70	2,996.00	81,906.59	9,709.00
90–94	15,033.98	2,901.00	10,909.55	1,624.00	25,943.53	4,525.00

(continued)

(Continued)

Attained Age	Male		Female		Total	
	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
			1992			
55–59	47,790.52	689.00	20,925.44	156.00	68,715.96	845.00
60–64	122,033.83	2,143.00	55,616.52	466.00	177,650.35	2,609.00
65–69	216,153.60	5,124.00	107,068.38	1,429.00	323,221.98	6,553.00
70–74	212,415.17	7,526.00	111,099.67	2,260.00	323,514.84	9,786.00
75–79	173,061.53	9,440.00	91,863.84	3,044.00	264,925.37	12,484.00
80–84	106,152.91	9,177.00	63,719.81	3,349.00	169,872.72	12,526.00
85–89	47,214.93	6,190.00	33,278.32	2,984.00	80,493.25	9,174.00
90–94	15,059.41	2,859.00	11,268.86	1,634.00	26,328.27	4,493.00

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