Financial Risk Management

Derivatives

Jiahua (Java) Xu, Ph.D.

2022

What are derivatives

A derivative is

- a financial security or contract whose value **derives** from the value of another asset / assets, known as the **underlying** (UL)
- an instrument for **transferring risk** and can therefore be used for
  - **hedging**: alter the exposure to an asset / risk you already have
  - **investment / speculation**: take on an exposure to an asset / risk

Forward

A forward is

- an OTC (over-the-counter) contract in which two counterparties agree, with zero money down, to buy / sell the UCL at a pre-agreed *forward price* at a given *delivery date* in the future

The contract is an *obligation* of both parties to transact, designed to protect both the buyer and the seller from price fluctuations in the future.

Investment assets and consumption assets

Investment asset: asset normally held for investment purposes

- financial assets: stocks, bonds
- precious metal: gold, silver

consumption asset: asset NOT normally held for investment purposes

- industrial metals: copper, aluminium
- agricultural products: orange juice, pork bellies
- energy products: natural gas, heating oil

Example

a forward contract to exchange 1m barrels of crude oil in 3 months at a forward price of USD 95/barrel

At the *delivery date*:

- The buyer (Long) delivers: forward price USD 95m
- The seller (Short) delivers: UL 1m barrels of crude oil

Payoff of a forward

Notations

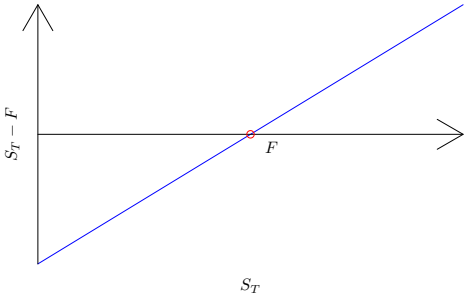
F: forward price

T: delivery date

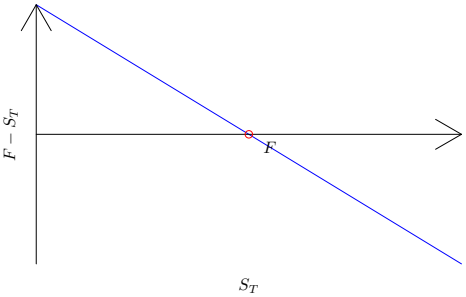
S_T: the spot price of the underlying on the delivery date

Payoff diagrams

Payoff to Long at T



Payoff to Short at T



Fair forward price

- Consider a stock
  - currently traded at £40
  - does not pay dividends
  - with an expected return of 5% p.a.
  - risk-free rate is 2% p.a.
- How much would you **agree to** today, to pay to buy the stock a year from now?

(a) £40
(b) £40.8
(c) £42

Arbitrage-free pricing

Replicate the same cashflow as a long forward contract by buying the stock today using borrowed money and repaying the borrowing with interest at T:

	Today	**Delivery date** T
Long forward	0	$S_T - F$
"Cash and carry" replicating strategy:		
Buy stock today	-40	S_T
Borrow £40 for 1 year at 2%	40	-40.8
Net	0	$S_T - 40.8$

The fair forward price is 40.8; otherwise there is an arbitrage opportunity.

For example, if the actual forward price is quoted at 41.2

		Today	**Delivery date T**
Buy low:	Buy stock today	-40	S_T
	Borrow £40 for 1 year at 2%	40	-40.8
	Net	0	$S_T - 40.8$
Sell high:	Short forward	0	$41.2 - S_T$
Net cash flows		0	£0.4

Holding benefits / costs

- Consider another stock
  - currently traded at £40
  - pays a dividend of £1 in 5 months
  - with an expected return of 5% p.a.
  - risk-free rate is 2% p.a.
- How much would you **agree to** today, to pay to buy the stock a year from now?

$$40 \times (1 + 2\%) - 1 \times (1 + 2\% \times \tfrac{6}{12}) = 39.79$$

For assets that can be traded spot and stored, forwards futures prices are linked to spot prices through the "cost of carry" relationship:

$$F = S \times (1 + r_j)^T - FV(\text{holding benefits}) + FV(\text{holding costs})$$

where

F: forward price

S: current spot price

r_f: risk-free rate

T: maturity of the contract

Holding benefits (costs) are the benefits (costs), typically cashflows, associated with holding the UL that you miss when buying in the future compared to buying now

FV: future value, i.e. compounded to T at risk-free rate

Futures

Futures contract: fungible, standardized contract for delivery of a specific commodity at a specific delivery or maturity date for an agreed-upon price (the **futures price**), to be paid at contract maturity

Futures market: market for trading **futures contracts** wherein buyers and sellers in a centralized futures exchange (wherein some flexibility is sacrificed for **liquidity**)

The **futures exchange** establishes features of the contract:

- **size** of the contract: mass, volume, number of units
- acceptable **grade** of the commodity
- contract **delivery dates**
- nature of **settlement**: cash, warehouse receipts

The trader with the **long position** (the buyer) commits to purchasing the commodity on the delivery date.

The trader with the **short position** (the seller) commits to delivering the commodity on the delivery date.

Forwards vs Futures

Futures are **exchange-traded** version of forwards

	Forwards	**Futures**
Buyer-seller interaction	Direct	Via exchange
Default-risk borne by	Individual parties	Clearinghouse
Default controlled by	Collateral	Margin accounts daily "marking to market"
Contract terms	Tailored	Standardized
Unilateral reversal	Difficult	Easy

E-mini S&P500 Index Futures Contract

Most popular equity index futures contract in the world

- **Contract size**: \$50 $\times$ S&P500 Index price (0.2 of the standard S&P500 futures contract which has a multiplier of \$250)
- **Contract month**: March quarterly expiration cycle (Mar, Jun, Sep, Dec)
- **Trading hours**: CME Globex (essentially around the clock from Sunday evening to late Friday afternoon)
- **Trading termination**: 8.30am on the Settlement Date (3rd Friday of the contract month)
- **Settlement procedure**: Cash settlement based on the Special Opening Quotation on Friday morning of the S&P500 Index
- **Position limits**: 20,000 S&P500 contracts or equivalent net long or short in all contract months combined

Futures contracts - marking to market

- Similar economic effect to forwards, but, due to **marking to market**, gains and losses on futures positions are settled each day
- After **marking to market**, both sides have a zero value position with the new (end of day) futures price.
- The long receives from (pays to) the short any increase (decrease) in the futures price from the previous day

Date	0	1	2	3	$T = 4$
Future price	106	108	104	105	$S_T = 107$
Long receives	0	108-106=2	104-108=-4	105-104=1	107-105=2

- Note that $\sum$(cash flow long receives) $= 1$, equal to the payoff on a forward position where the forward price is the original futures price $S_T - F = 107 - 106 = 1$
- **Convergence property**: at maturity, the futures price and spot price must converge; $F_T = P_T$

The clearinghouse

Clearinghouse: designated intermediary between a buyer and a seller in a financial market that validates and finalizes the transaction, ensuring that both the buyer and the seller honor their contractual obligations.

- Traders on both sides face the **clearinghouse** rather than each other.
- The **clearinghouse** bears the risk of non-performance by any trader.
- Contracts are therefore **fungible**: traders can reverse a position by entering the countervailing position with the clearinghouse.

Margin and open interest

Margin: a good-faith deposit to guarantee contract performance

Marking to market: daily settling (realizing) of gains and losses with the clearinghouse

notespecialtaxtreatment

Maintenance margin: minimum quantity that a trader must hold in reserve with the clearinghouse

- Margin safeguards the position of the clearinghouse.
- If the mark-to-market value of the trader's account falls below the maintenance margin, the trader receives a **margin call**.
- A **margin call** requires the trader to replenish the margin account to the maintenance margin. Otherwise, the position is reduced to a size commensurate with the remaining funds (the trader is **bought in** by the exchange).

Open interest: the number of contracts outstanding

Hedging and speculation

Hedgers use futures to insulate themselves (**hedge**) against price movements in the underlying asset.

- Example: both airplane manufacturers and bauxite miners might seek to hedge their exposure to the price of aluminium.

Speculators use futures to profit from movements in futures prices.

- Speculators take **long** position if they expect an increase in price and a **short** position if they expect a decrease in price.
- Usually, **transaction costs** in future markets are considerably smaller than in markets for the UL.
- Speculators also gain the advantage of **leverage** because margin requirements are generally much less than the value of the UL.

Forward and future – applications

- Contracts for difference (CFDs)
- Profitable alpha (futures overlay) strategies
- Commodities investing

Contracts for difference (CFDs)

A CFD is a contract between a buyer and a seller, stipulating that:

- if the price of the UL increases, the seller pays the buyer the increase
- if the price falls, the buyer pays the seller the decrease

Cashflows	Today	T
to CFD buyer (long)	0	$S_T - S$
to CFD seller (short)	0	$S - S_T$

In addition:

- the buyer pays the seller daily interest on the initial value of the UL
- the seller pays the buyer any dividends / coupons on the underlying
- margin requirements for end user

CFDs vs forwards & futures

Cashflows	Today	From today to T	T
to CFD buyer (long)	0	interest on S	$S_T - S$
to CFD seller (short)	0	0	$S - S_T$

Is the forward price the expected spot price?

- is $F = \mathbb{E}[S_T]$
- Simplest setting: ignoring holding costs / benefits, using simple compounding at an annual risk-free rate r_f, for a 1-year forward

$$F = S \times (1 + r_f)$$

- According to standard finance theories, the (risky) UL should earn a risk premium π

$\mathbb{E}[S_T] = S \times (1 + r_f + \pi)$

therefore $F \neq \mathbb{E}[S_T]$

- Actual payoff on a long forward: $S_T - F$
- The **expected** payoff on a long forward is

$$\mathbb{E}[S_T - F] = S \times (1 + r_f + \pi) - S \times (1 + r_f) = S \times \pi$$

- The expected return on the long forward (as a percentage of the *current price of the underlying*) is the risk premium
- By going long (short) forwards / futures you assume (lay off) the risk premium on the UL

Portable alpha strategies

- According to the CAPM, the risk premium on a stock or portfolio is:

$\mathbb{E}(R_i) - R_f = \beta[\mathbb{E}(R_m) - R_f]$ where $\beta = \frac{\sigma_{R_i,R_m}}{\sigma^2_{R_m}}$

where:

R_i, R_m: the returns on the portfolio and the "market", respectively

R_f: the risk-free rate of return

$\mathbb{E}[]$: expected value

σ_{R_i,R_m}: the covariance between R_i and R_m

$\sigma^2_{R_m}$: the variance of R_m

- The expected excess return on a stock is compensation for taking on (non-diversifiable) "market" risk

If you run the regression:

$$R_i - R_f = \alpha + \beta(R_m - R_f) + \epsilon$$

and securities are efficiently priced, alpha should not be statistically significantly different from zero

- For actively managed portfolios, if alpha is positive this is typically interpreted as evidence of stock-picking ability

Use futures overlays to create a position where you earn beta in one asset category and alpha in another:

- You have £1m to invest and a target beta of 1 wrt the S&P500
- You believe there are no alpha opportunities in S&P500 stocks but you have identified a market-neutral (zero-beta) hedge fund that you believe will generate positive alpha

To capture the S&P beta return as well as the hedge fund alpha:

- invest £1m in the "market-neutral" hedge fund
- go long £1m 1-yr S&P500 futures

Commodity forwards / futures pricing

- The cost of carry relationship

$$F = S + \text{intest cost} - \text{holding benefit} + \text{holding costs}$$

- For commodities, this becomes

$$F = S + \text{intest cost} - \text{convenience yield} + \text{storage costs}$$

Convenience yield

- The convenience yield (CY) is a holding benefit
- Unlike holding benefits on financial assets (eg dividends or coupons), CY is NOT a cashflow ensured if you are long the underlying
- The CY captures the "intangible" benefits of holding the underlying spot for those who consume it / use it in production

Investing in commodities

Commodities as an asset class

- low (negative) correlations with stocks and bonds
- hedge against inflation ("real return" asset class)
- exposure to emerging markets growth ("commodities super cycle")
- strong performance (relative to equities) during 2000-2007

Interest rate futures: convexity adjustment

For **interest rate futures** longer than about two years, factors related to settlement influence the value of the contracts:

- Traders will tend to invest the proceeds from **daily settlement** at prevailing rates, so futures contracts for which there is a mark-to-market gain from an increase in rates (or loss from a decrease in rates) will outperform similar forward contracts
- **Eurobond** futures are settled at the beginning of the period to which the rate applies, calculated as the present value of what the settlement would be if it were made at the end of the period, and the adverse impact of delaying settlement on holders of forward contracts is greater when there is a gain

So:

$$\text{forward rate} = \text{future rate} - c$$

where $c > 0$ is the convexity adjustment

Interest rate futures: estimating forward interest rates

interest rate futures can be used to bootstrap zero curves (e.g. SONIA):

$$R_F = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

$$R_2 = \frac{R_F(T_2 - T_1) + R_1 T_1}{T_2}$$

Interest rate futures: duration-based hedging

If we assume that the change in forward yield Δy is the same for all maturities, then:

$$\Delta P = -PD_P \Delta y$$

$$\Delta V_F = -V_F D_F \Delta y$$

$$N^* = \frac{PD_P}{V_F D_F}$$

V_F: contract price for one interest rate futures contrast

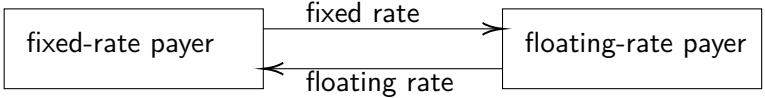
D_F: duration of underlying asset at maturity of the futures contract

P: forward value of the portfolio being hedged at maturity of the hedge (often assumed to be the same as the initial value of the portfolio)

Swap

Interest rate swap

Arrangement, applied to some notional principal, wherein interest at a predetermined fixed rate is exchanged for interest at a floating reference rate is exchanged for interest at a flating interest rate, with one or more regular exchanges being made for an agreed period of time



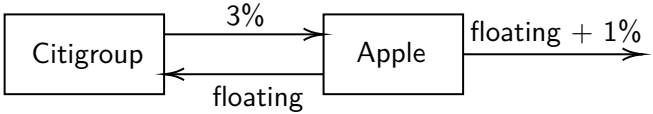
Overnight indexed swap

Interest rate swap wherein a fixed rate of interest (the OIS rate) is exchanged for a reference rate of interest calculated from a realized overnight rate (e.g. SONIA, SOFR).

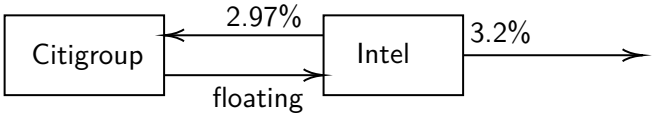
- If there is only one exchange, the OIS rate is a **risk-free zero rate** equivalent to the UL overnight rate.
- Otherwise, the OIS rates define a risk-free bond worth par.
- An OIS rate can be contrasted with a LIBOR swap, wherein the LIBOR rate for a period is known as the start of the period, so the floating rate of the first exchange is known.

Interest rate swaps: transferring liabilities

Apple uses a swap to convert **floating**-rate borrowings to **fixed**-rate borrowings:

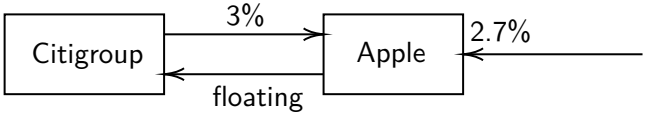


Intel uses a swap to convert **fixed**-rate borrowings to **floating**-rate borrowings:

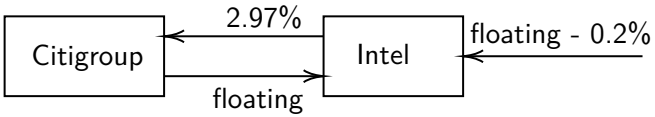


Interest rate swaps: transferring assets

Apple converts a **fixed**-rate investment to **floating**-rate investment:



Intel converts a **floating**-rate investment to **fixed**-rate investment:

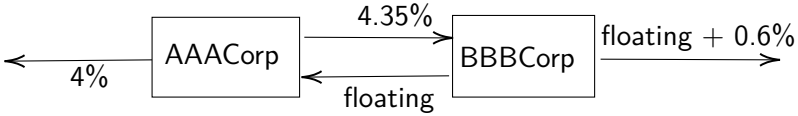


Comparative advantage

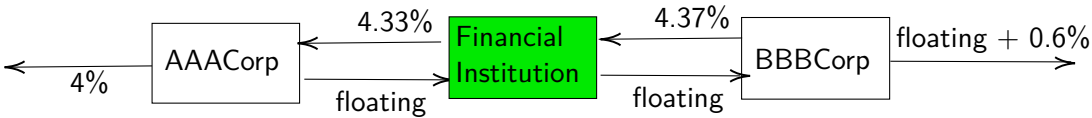
Why use swaps? One reason might be **comparative advantage**: a company might have a relative advantage to borrowing in either fixed-rate markets or floating-rate markets.

	fixed rate	floating rate
AAACorp	4.0%	floating - 0.1%
BBBCorp	5.2%	floating + 0.6%

Here, AAACorp and BBBCorp might seek to collaborate:



In practice, the swap might be **brokered** by a financial institution:



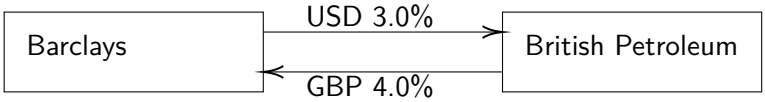
Why has comparative advantage not been arbitraged away?

The **maturities** of contracts available via fixed-rate financing are generally different than those available via floating-rate financing:

- Fixed-rate contracts are often longer than floating-rate contracts
- The spread over the reference rate can effectively be adjusted by floating-rate lenders
- Fixed-rate lenders often lack this option

Currency swaps

fixed-for-fixed currency swap: arrangement wherein principal and interest payments in one currency are exchanged for principal and interest payments in another currency

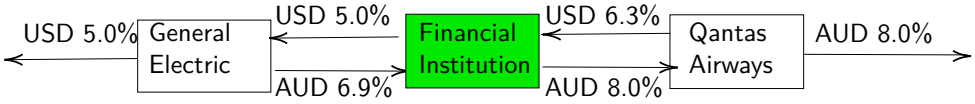


Variations:

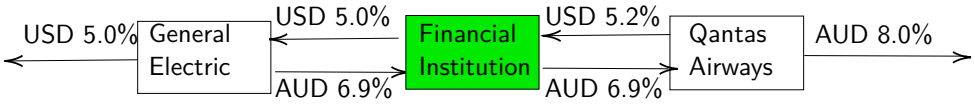
- fixed-for-floating currency swap
- floating-for-floating currency swap
- quanto (or diff swap): arrangement wherein a rate observed in one currency is applied to a principal amount in another currency

Currency swaps: example with comparative advantage

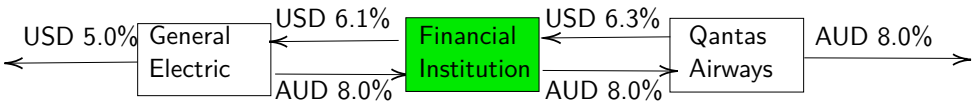
Suppose that General Electric has a **comparative advantage** to borrowing in USD and Qantas Airways has a **comparative advantage** to borrowing in AUD. Financial institution could reduce both of their costs by taking on FX risk:



It might be more cost-effective for Quantas Airways to bear some FX risk:



Or it might be more cost-effective for General Electric to bear some FX risk:



Other swaps

Equity swap: agreement to exchange the total return (dividends plus gains) of an equity index for a fixed for floating rate of interest.

Credit default swap: agreement that generates a payment if a particular company (the reference entity) defaults

- the **protection buyer** pays the **CDS spread** (and insurance premium) over the life of the contract
- in the event of default, the **protection seller** pays an amount that would restore the value of the hypothetical portfolio of the bonds of the reference entity to the value of its principal

Options:

- entendable swap: one party can extend the swap arrangement
- puttable swap: one party can terminate the swap arrangement early
- swaption: option on a swap

Thank you!

Contact

Jiahua (Java) Xu

UCL Centre for Blockchain Technologies

66-72 Gower Street

jiahua.xu@ucl.ac.uk

References I