Financial Risk Management

Derivatives

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What are derivatives

A derivative is

- a financial security or contract whose value **derives** from the value of another asset / assets, known as the **underlying** (UL)
- an instrument for **transferring risk** and can therefore be used for
  - **hedging**: alter the exposure to an asset / risk you already have
  - **investment / speculation**: take on an exposure to an asset / risk

Forward

A forward is

- an OTC (over-the-counter) contract in which two counterparties agree, with zero money down, to buy / sell the UCL at a pre-agreed *forward price* at a given *delivery date* in the future

Example

a forward contract to exchange 1m barrels of crude oil in 3 months at a forward price of USD 95/barrel
At the *delivery date*:

- The buyer (Long) delivers: forward price USD 95m
- The seller (Short) delivers: UL 1m barrels of crude oil

Payoff of a forward

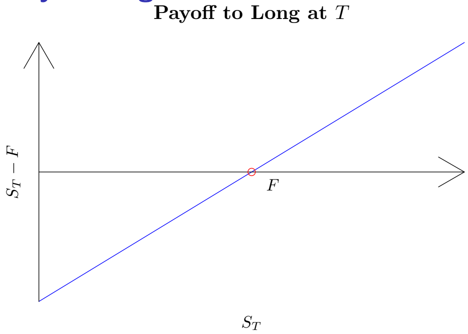
Notations

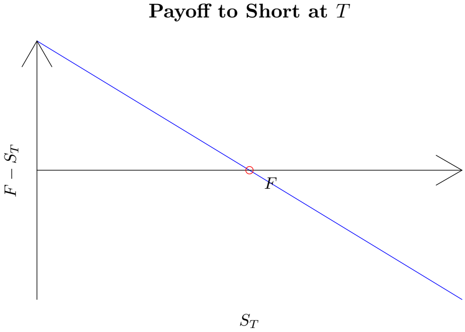
F: forward price

T: delivery date

S_T: the spot price of the underlying on the delivery date

Payoff diagrams





Forwards vs Futures

Futures are **exchange-traded** version of forwards

	Forwards	**Futures**
Buyer-seller interaction	Direct	Via exchange
Default-risk borne by	Individual parties	Exchange
Default controlled by	Collateral	Margin accounts daily “marking to market”
Contract terms	Tailored	Standardized
Unilateral reversal	Difficult	Easy

E-mini S&P 500 Index Futures Contract

Most popular equity index futures contract in the world

- **Contract size**: \$50 $\times$ S&P 500 Index price (0.2 of the standard S&P 500 futures contract)
- **Contract month**: March quarterly expiration cycle (Mar, Jun, Sep, Dec)
- **Trading hours**: CME Globex (essentially around the clock from Sunday evening to late Friday afternoon)
- **Trading termination**: 8.30am on the Settlement Date (3rd Friday of the contract month)
- **Settlement procedure**: Cash settlement based on the Special Opening Quotation on Friday morning of the S&P500 Index
- **Position limits**: 20,000 S&P500 contracts or equivalent net long or short in all contract months combined

Futures contracts - marking to market

- Similar economic effect to forwards, but, due to **marking to market**, gains and losses on futures positions are settled each day
- After **marking to market**, both sides have a zero value position with the new (end of day) futures price.
- The long receives from (pays to) the short any increase (decrease) in the futures price from the previous day

Date	0	1	2	3	$T = 4$
Future price	106	108	104	105	$S_T = 107$
Long receives	0	108-106=2	104-108=-4	105-104=1	107-105=2

- Note that $\sum$(cash flow long receives) $= 1$, equal to the payoff on a forward position where the forward price is the original futures price $S_T - F = 107 - 106 = 1$

Fair forward price

- Consider a stock
  - currently traded at £40
  - does not pay dividends
  - with an expected return of 5% p.a.
  - risk-free rate is 2% p.a.
- How much would you **agree to** today, to pay to buy the stock a year from now?

(a) £40
(b) £40.8
(c) £42

Arbitrage-free pricing

Replicate the same cashflow as a long forward contract by buying the stock today using borrowed money and repaying the borrowing with interest at T:

	Today	**Delivery date T**
Long forward	0	$S_T - F$
"Cash and carry" replicating strategy:		
Buy stock today	-40	S_T
Borrow £40 for 1 year at 2%	40	-40.8
Net	0	$S_T - 40.8$

The fair forward price is 40.8; otherwise there is an arbitrage opportunity.

For example, if the actual forward price is quoted at 41.2

		Today	**Delivery date** T
Buy low:	Buy stock today	-40	S_T
	Borrow £40 for 1 year at 2%	40	-40.8
	Net	0	$S_T - 40.8$
Sell high:	Short forward	0	$41.2 - S_T$
Net cash flows		0	£0.4

Cost of carry relationship

- Consider another stock
  - currently traded at £40
  - pays a dividend of £1 in 5 months
  - with an expected return of 5% p.a.
  - risk-free rate is 2% p.a.
- How much would you **agree to** today, to pay to buy the stock a year from now?

$$40 \times (1 + 2\%) - 1 \times (1 + 2\% \times \frac{6}{12}) = 39.79$$

For assets that can be traded spot and stored, forwards futures prices are linked to spot prices through the "cost of carry" relationship:

$$F = S \times (1 + r_j)^T - FV(\text{holding benefits}) + FV(\text{holding costs})$$

where

F: forward price

S: current spot price

r_f: risk-free rate

T: maturity of the contract

Holding benefits (costs) are the benefits (costs), typically cashflows, associated with holding the UL that you miss when buying in the future compared to buying now

FV: future value, i.e. compounded to T at risk-free rate

Forward and future – applications

- Contracts for difference (CFDs)
- Profitable alpha (futures overlay) strategies
- Commodities investing

Contracts for difference (CFDs)

A CFD is a contract between a buyer and a seller, stipulating that:

- if the price of the UL increases, the seller pays the buyer the increase
- if the price falls, the buyer pays the seller the decrease

Cashflows	Today	T
to CFD buyer (long)	0	$S_T - S$
to CFD seller (short)	0	$S - S_T$

In addition:

- the buyer pays the seller daily interst on the initial value of the UL
- the seller pays the buyer any dividends / coupons on the underlying
- margin requirements for end user

CFDs vs forwards & futures

Cashflows	Today	From today to T	T
to CFD buyer (long)	0	interest on S	$S_T - S$
to CFD seller (short)	0	0	$S - S_T$

Is the forward price the expected spot price?

- is $F = \mathbb{E}[S_T]$
- Simplest setting: ignoring holding costs / benefits, using simple compounding at an annual risk-free rate r_f, for a 1-year forward

$$F = S \times (1 + r_f)$$

- According to standard finance theories, the (risky) UL should earn a risk premium π

$\mathbb{E}[S_T] = S \times (1 + r_f + \pi)$

therefore $F \neq \mathbb{E}[S_T]$

- Actual payoff on a long forward: $S_T - F$
- The **expected** payoff on a long forward is

$$\mathbb{E}[S_T - F] = S \times (1 + r_f + \pi) - S \times (1 + r_f) = S \times \pi$$

- The expected return on the long forward (as a percentage of the *current price of the underlying*) is the risk premium
- By going long (short) forwards / futures you assume (lay off) the risk premium on the UL

Portable alpha strategies

- According to the CAPM, the risk premium on a stokc or portfolio is:

$\mathbb{E}(R_i) - R_f = \beta[\mathbb{E}(R_m) - R_f]$ where $\beta = \frac{\sigma_{R_i,R_m}}{\sigma^2_{R_m}}$

where:

R_i, R_m: the returns on the portfolio and the "market", respectively

R_f: the risk-free rate of return

$\mathbb{E}[]$: expected value

σ_{R_i,R_m}: the covariance between R_i and R_m

$\sigma^2_{R_m}$: the variance of R_m

- The expected excess return on a stock is compensation for taking on (non-diversifiable) "market" risk

Thank you!

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References I