STAT243-PS5

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1 Other students

I discuss some problems with Weijie Xu

2 Question 2

2.1 (a)

First, we look at the format $(-1)^S * 1.d * 2^{e-1023}$. As d is represented as 52 bits and e is represented as 11 bits, we can write the format as

$$(-1)^S * 1.d_1d_2...d_{52} * 2^{e-1023} = (-1)^S * (1 + d_1 * 2^{-1} + d_2 * 2^{-2}... + d_{52} * 2^{-52}) * 2^{e-1023}$$

where $S, d_i \in \{0, 1\}$ and $(e - 1023) \in \{-1023, -1022, ..., 1024, 1025\}$

Then let $2^p - m$ is any one number among $\{1, 2, 3, ..., 2^{53} - 2, 2^{53} - 1\}$ where $m \in \{1, 2, ..., 2^p - 1\}$. Obviously, S=0 and we can find a group of d_i that makes

$$(1 + d_1 * 2^{-1} + d_2 * 2^{-2}... + d_{52} * 2^{-52}) * 2^{p-1} = 2^p - m$$

It is because the range of the left of the equation is $[2^{p-1}, 2^p - 2^{p-53}]$ and the precision is 2^{p-53}

For example

$$2^{53} - 2 = (1 + 1 * 2^{-1}+, ..., +1 * 2^{-50} + 1 * 2^{-51} + 0 * 2^{-52}) * 2^{52}, which\ means\ d_{52} = 0\ and\ other\ d_i = 1$$
$$2^{52} - 3 = (1 + 1 * 2^{-1}+, ..., +0 * 2^{-50} + 1 * 2^{-51} + 0 * 2^{-52}) * 2^{51}, which\ means\ d_{50,52} = 0\ and\ other\ d_i = 1$$

2.2 (b)

If we want to store $2^{53}, 2^{53} + 1, 2^{53} + 2, ...,$ from (a), we can know that p=54. In this way, the precision equals to $2^{p-53} = 2$
It is the same situation when we store $2^{54}, 2^{54} + 1, 2^{54} + 2, ...$ In this situation, p=55. Then the precision equals to $2^{p-53} = 4$

```
options(digits=22)
print(2^53-1)

## [1] 9007199254740991

print(2^53)

## [1] 9007199254740992

print(2^53+1)

## [1] 9007199254740992
```

We can find that $2^{53}+1$ is stored same as 2^{53} in R. Because the precision is 2.

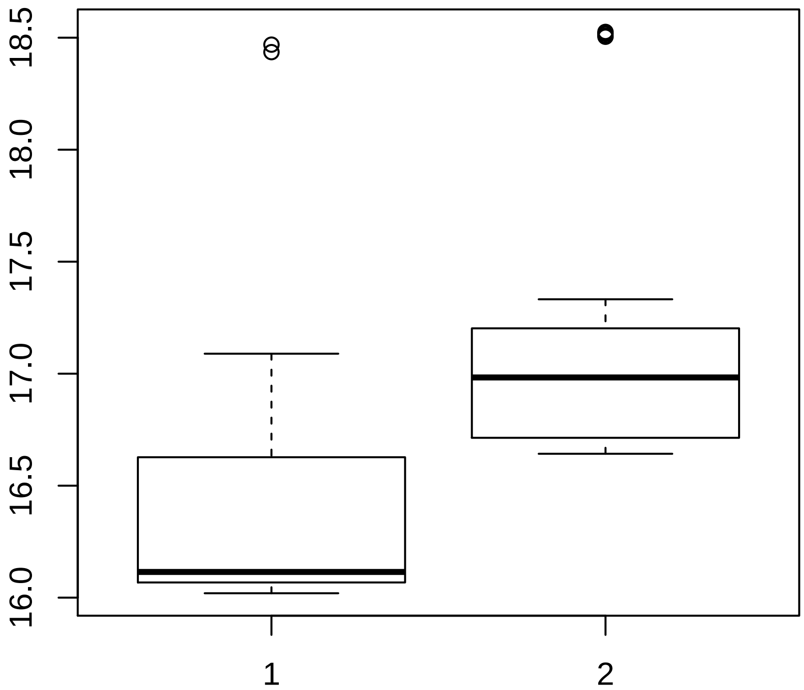
3 Question 3

3.1 (a)

Copy two vectors and it is obviously that copying numeric vector takes more time. It is because that storing each int in R takes 4 bytes while storing each numeric in R takes 8 bytes. In plot, I use log function to the data.

```
library(data.table)
library(microbenchmark)
compare_copytime=function(x,y){
  timex=microbenchmark(copy(x))$time      #get 100 data of time
  timey=microbenchmark(copy(y))$time
  if (mean(timex)>mean(timey)) print("copying the first one takes more time")
  else print('copying the second one cost more time')
  boxplot(log(timex),log(timey))
}
numvec<-rnorm(1e7)
intvec<-as.integer(numvec)
compare_copytime(intvec,numvec)

## [1] "copying the second one cost more time"
```

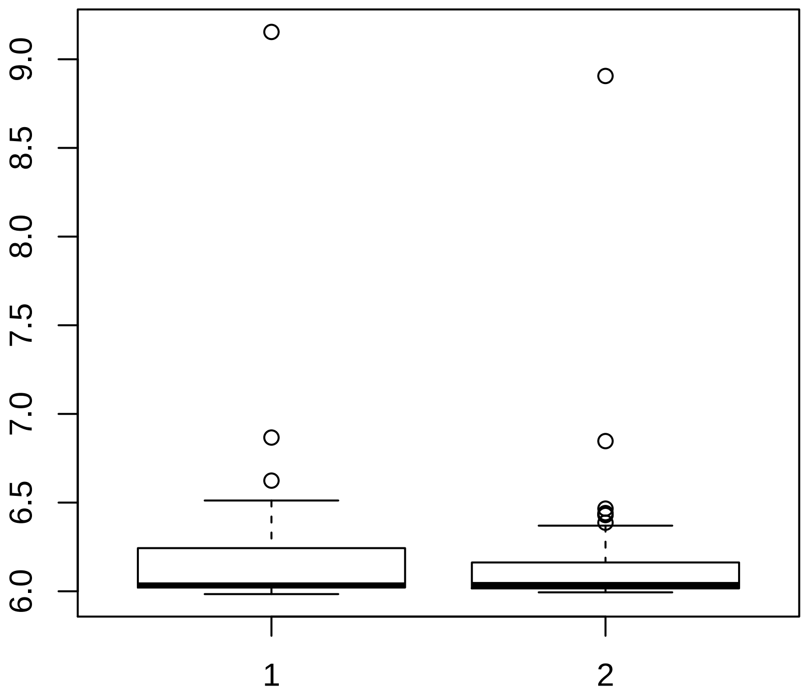


3.2 (b)

I find that the time used to take a subset of these two vectors is nearly same.

```
compare_subtime=function(x,y){
  timex=microbenchmark(xsub<-x[1:5*1e6])$time        #get 100 data of time
  timey=microbenchmark(ysub<-y[1:5*1e6])$time
  if (mean(timex)>mean(timey)) print("taking a subset of the first one takes more time")
  else print('taking a subset of the second one cost more time')
  boxplot(log(timex),log(timey))
}
compare_subtime(intvec,numvec)

## [1] "taking a subset of the first one takes more time"
```



4 Question4

4.1 (a)

Breaking Y into n individual column-wise computations sometimes may not speed up a lot. Because if there is one task needs much more time than any other one, it still takes much time even though we breaking Y into n individual columns. In this way, it can not speed up a lot while it costs much more communication.

Therefore sometimes it is better to break up Y into p blocks of m = n/p columns rather than into n individual column-wise computations

4.2 (b)

1. In approach one, each worker deals with a $\frac{n}{p} \times n$ matrix , plus the memory used to store matrix X, Y. Then the total memory used in a single moment is $\frac{n}{p} \times n \times p + 2n^2 = 3n^2$

In approach two, each worker deals with a $\frac{n}{p} \times \frac{n}{p}$ matrix , plus the memory used to store matrix X, Y. Then the total memory used in a single moment is $\frac{n}{p} \times \frac{n}{p} \times p + 2n^2 = 2n^2 + \frac{n^2}{p}$

Therefore, the second approach use less memories.

2. In approach one, each worker deals with a matrix, so the total number needed to be passed to workers is p

In approach two, each worker deals with p matrix, so the total number needed to be passed to workers is $p \times p = p^2$

Therefore, the first approach is better for minimizing the communication.

5 Question5

From the question2, we know numerics are written as the following format:

$$(-1)^S * 1.d_1 d_2 ... d_{52} * 2^{e-1023} = (-1)^S * (1 + d_1 * 2^{-1} + d_2 * 2^{-2} ... + d_{52} * 2^{-52}) * 2^{e-1023}$$

I only discuss the situation the number is smaller than 1, without loss of generality.
It is obviously that only $n = \sum_{i=1}^{52} a_i * 2^{-i}$can be accurately stored.
And other numbers are stored as $m = \sum_{i=1}^{52} a_i * 2^{-i}$, where a_i minimizes $\|m - \sum_{i=1}^{52} a_i * 2^{-i}\|$
Then we take example of 0.2+0.3. If $0.2 = \sum_{i=1}^{52} a_i * 2^{-i}$, a_i are fixed there, it shows that a_i minimizes

$$\|0.2 - \sum_{i=1}^{52} a_i * 2^{-i}\|$$

And

$$\|0.2 - \sum_{i=1}^{52} a_i * 2^{-i}\| = \|0.3 - (2^{-1} - \sum_{i=1}^{52} a_i * 2^{-i})\|$$

So 0.3 is store as $2^{-1} - \sum_{i=1}^{52} a_i * 2^{-i}$ in R. Then 0.2+0.3=0.5.
However, if the result can not be written as binary format, like 0.1+0.2=0.3. Then we can not guarantee that it is true.

```
0.2+0.3==0.5

## [1] TRUE

0.1+0.4==0.5

## [1] TRUE

0.1+0.2==0.3

## [1] FALSE
```