Bayesian Data Analysis, class 2b

Andrew Gelman

Chapter 2. Single-parameter models (part 2)

Discussion of homework due beginning of Class 2b

- Theory problem
- Computing problem
- Applied problem

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Theory problem

- Show that if $y|\theta$ is exponentially distributed with rate θ, then the gamma prior distribution is conjugate ...
- Show that the equivalent prior specification for the mean, $\phi = 1/\theta$, is inverse-gamma. (That is, derive the latter density function.)
- The length of life of a light bulb manufactured by a certain process has an exponential distribution with unknown rate θ. Suppose the prior distribution for θ is a gamma distribution with coefficient of variation 0.5 ... If the coefficient of variation of the distribution of θ is to be reduced to 0.1, how many light bulbs need to be tested?

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Computing problem

- Your task is to estimate the percentatge of the (adult) population in each state (excluding Alaska and Hawaii) who label themselves as "very liberal," replicating the procedure that was used in Section 2.8 to estimate cancer rates ...
- This exercise has four challenges: first, manipulating the data in order to get the totals by state; second, replicating the calculations for estimating the parameters of the prior distribution; third, doing the Bayesian analysis by state; and fourth, making the graphs.

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Applied problem

A (hypothetical) study is performed to estimate the effect of a simple training program on basketball free-throw shooting . . . Let θ be the average improvement in success probability. Give . . .

- A noninformative prior,
- A subjective prior based on your best knowledge, and
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- Equivalent sample size of prior distribution
- Basic single-parameter models (beyond normal location and binomial probability)
- One prior distribution applied to many different cases
- Noninformative priors
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Another sex-ratio example

- The data
- A prior distribution
- Equivalent sample size

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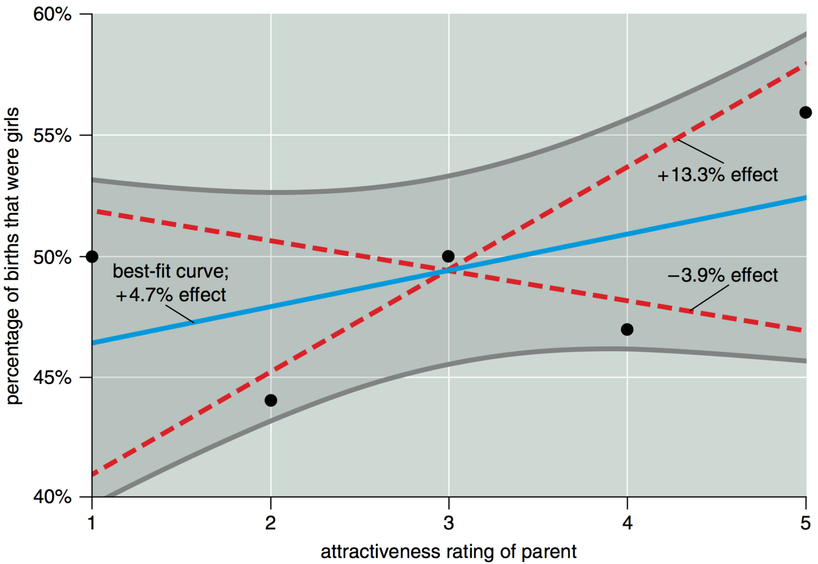
Beauty of parents and sex ratio of children



Pr(girl), comparing beautiful to ugly parents

- Data: difference in Pr(girl) estimated from 3000 respondents
  - 0.08 ± 0.03 (selected comparison)
  - 0.047 ± 0.043 (linear regression)
- Prior distribution: $\mathrm{N}(0, 0.003^2)$
- Equivalent sample size:
- A study with $n = 166{,}000$ people would be weighted equally with the prior

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    Compare sex ratio of prettiest $n/3$ to ugliest $n/3$
    s.e. is $\sqrt{0.5^2/(n/3) + 0.5^2/(n/3)} = 0.5\sqrt{6/n}$
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2.5. Estimating a normal mean with known variance

- Prior $\theta \sim \mathrm{N}(\mu_0, \tau_0^2)$
- Data $\bar{y} \sim \mathrm{N}(\theta, \sigma^2/n)$
- Prior and likelihood both have the form, $e^{A\theta^2+B\theta+C}$
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- Posterior is $\theta|y \sim \mathrm{N}\left(\frac{\frac{1}{\tau_0^2}\mu_0+\frac{n}{\sigma^2}\bar{y}}{\frac{1}{\tau_0^2}+\frac{n}{\sigma^2}}, \frac{1}{\frac{1}{\tau_0^2}+\frac{n}{\sigma^2}}\right)$
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    Instead, do $y_i \sim \text{Poisson}(x_i\theta)$
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2.7. Example: informative prior distribution and multilevel structure for estimating cancer rates

- A puzzling pattern in a map
- Raw data y_j/n_j and Bayes estimate $\frac{\alpha+y_j}{\beta+n_j}$
- Estimating the prior distribution from data

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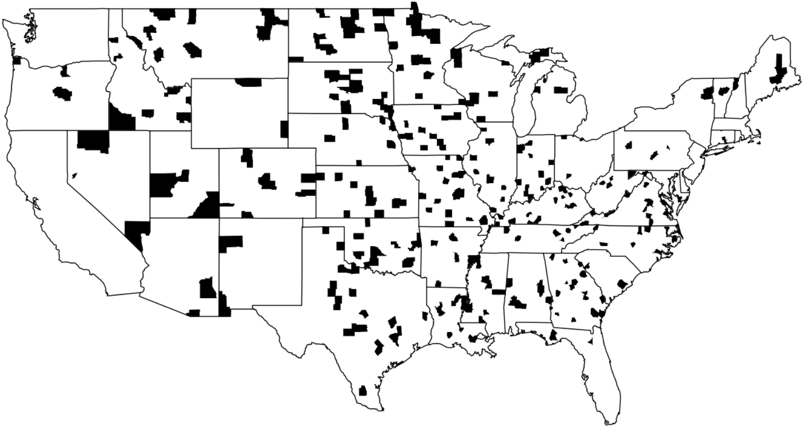
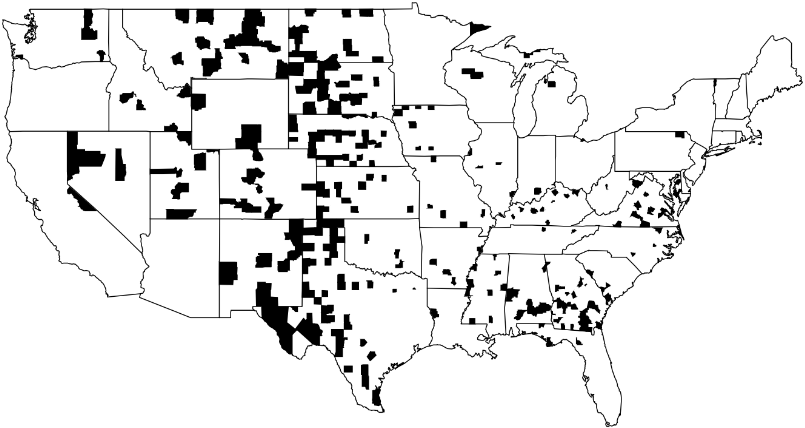
A puzzling pattern in a map

Highest kidney cancer death rates



But wait!

Lowest kidney cancer death rates



All at once

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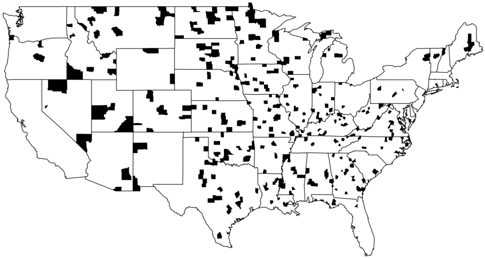

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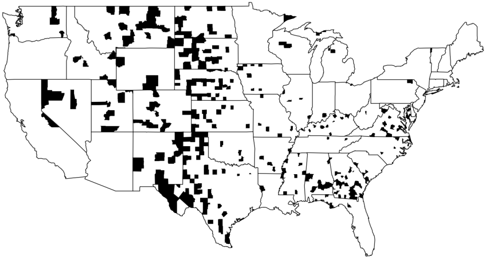
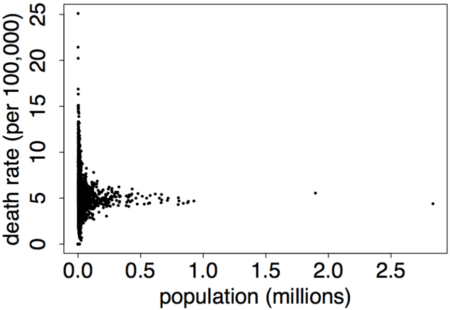
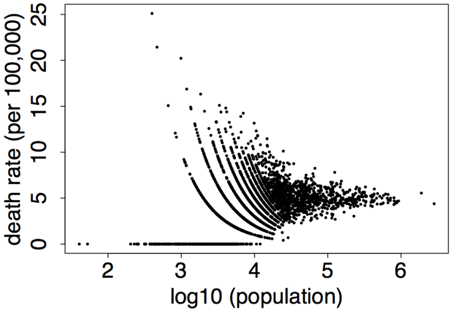
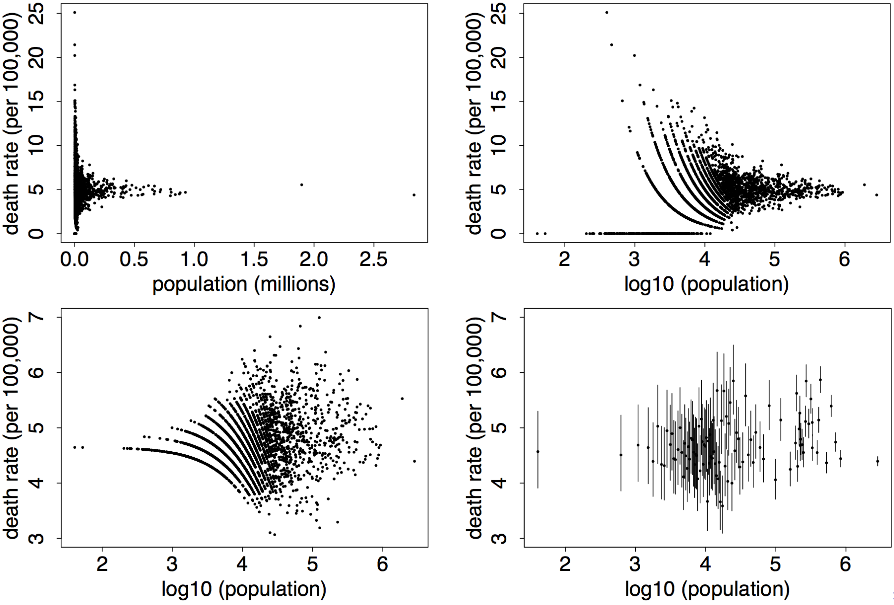


Cancer rates and sample size





Raw data and Bayes estimates



The parable of the two exams

- Option 1: 100 questions
- Option 2: 1 question, score is 0 or 100
- Which exam would you take to maximize the chance of getting the job?

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Estimating the prior distribution from data

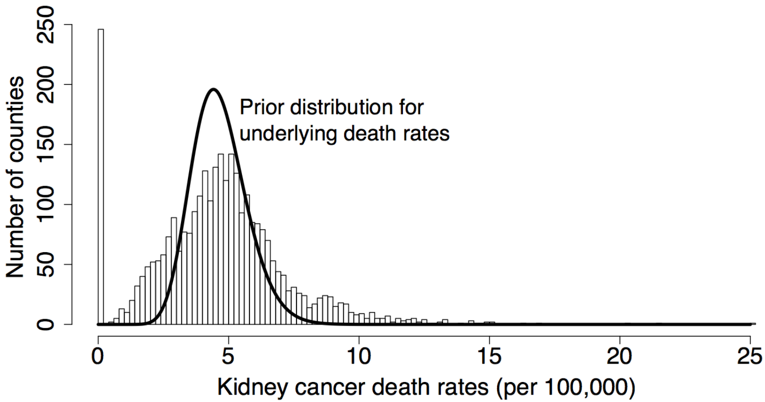

Figure 2.10 *Empirical distribution of the age-adjusted kidney cancer death rates,* $y_j/(10n_j)$*, for the 3071 counties in the U.S., along with the* $Gamma(20, 430000)$ *prior distribution for the underlying cancer rates* θ_j*.*

Connecting cancer rate example to Bayesian themes

- Model $y_j \sim \text{Poisson}(10 n_j \theta_j)$
    - What are the assumptions here?
- Flat Gamma$(0, 0)$ prior gives posterior mean $\frac{y_j}{n_j}$
- Informative Gamma $(20, 430{,}000)$ prior gives posterior mean $\frac{20+y_j}{430{,}000+n_j}$
- Equivalent sample size (in each county) of 20, prior mean (in each county) of $\frac{20}{430{,}000} = 4.65 \times 10^{-5}$
- One prior, many different datasets
- Data-based prior distribution

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- Do parameter estimates look reasonable?
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- For county j, observe y_j deaths out of $10n_j$ person-years
- Imagine predicting second half from first half:
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    y_{j2} deaths out of $5n_j$ person-years
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- Unnormalized densities
- Uniform prior distributions on different scales
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- Posterior as weighted average of prior and data
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