Bayesian Data Analysis, class 5b

Andrew Gelman

Chapter 6: Model checking (part 1)

Discussion of homework due beginning of Class 5b

- Theory problem
- Computing problem
- Applied problem

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Theory problem

- $2J$ parameters $\theta_1, \ldots, \theta_{2J}$ clustered into two groups, with exactly half being drawn from a $N(1,1)$ distribution, and the other half being drawn from a $N(-1,1)$ distribution, but we have not observed which parameters come from which distribution
- Are $\theta_1, \ldots, \theta_{2J}$ exchangeable under this prior distribution?
- Show that this distribution cannot be written as a mixture of independent and identically distributed components.
- de Finetti's theorem

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Computing problem

- Simulation of a stochastic process

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Applied problem

- Stochastic learning model with 2 or 3 parameters
- Weakly informative prior
- Simulate data from fake dogs

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6. Model checking (part 1)

- Comparing estimates and predictions to substantive knowledge
- Comparing predictions to observed data
- Graphical and numerical tests
- P-values and u-values

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6.1. The place of model checking in applied Bayesian statistics

- With great power comes great responsibility
- Sensitivity analysis
- All models are false
- Real Bayes vs. super-Bayes

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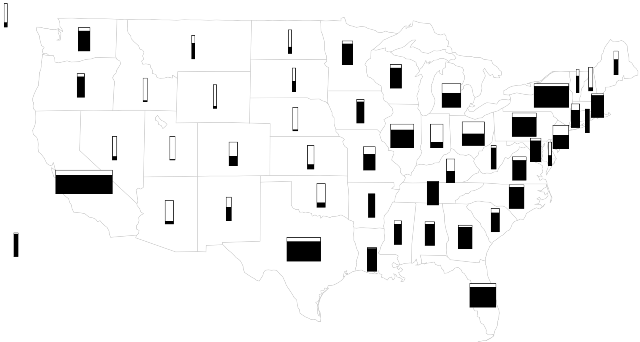

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- We're talking about prior information!

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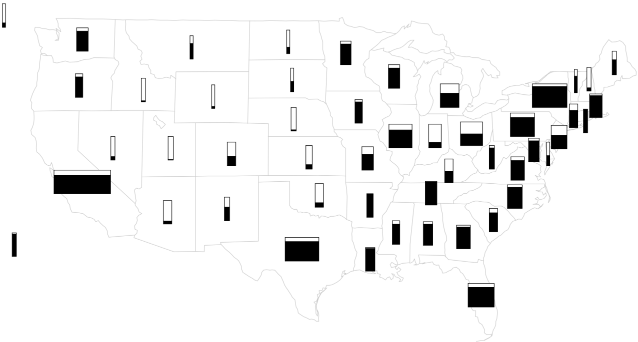

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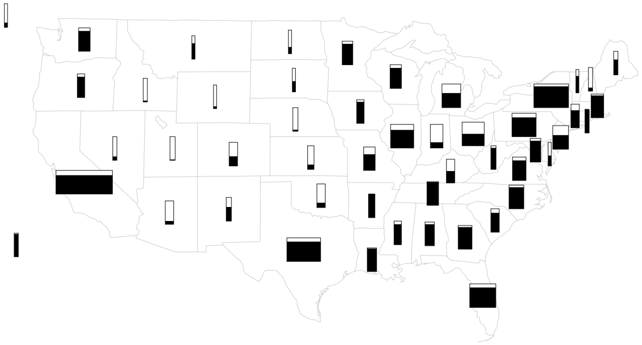

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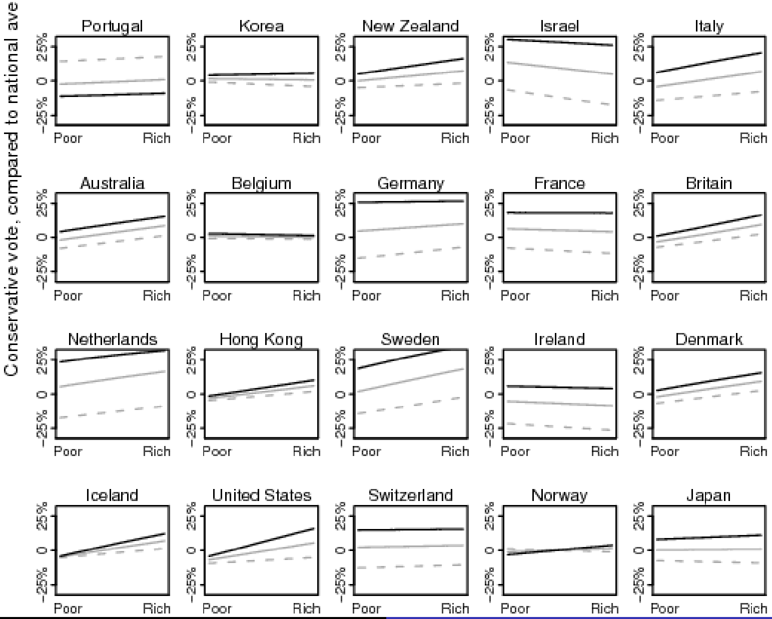
What's the matter with Portugal?



6.3. Posterior predictive checking

- Compare observed data to replications simulated from the model
- Replication and p-values
- Several examples

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Replications and p-values

- $\theta, y, y^{\mathrm{rep}}$
- Classical p-value is $p\text{-value}(y|\theta) = \Pr\left(T(y^{\mathrm{rep}}) \geq T(y) | \theta, y\right)$
- Bayesian posterior p-value is $\Pr\left(T(y^{\mathrm{rep}}) \geq T(y) | y\right) = \int p\text{-value}(y|\theta) p(\theta|y) d\theta$
- Compute using simulation

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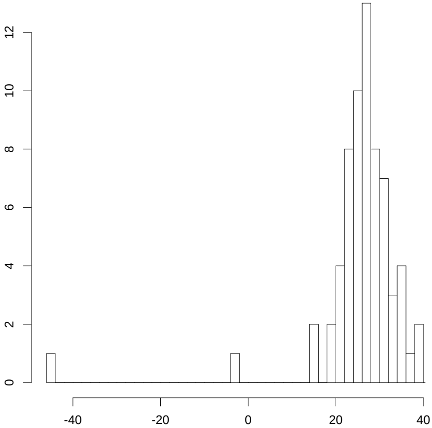
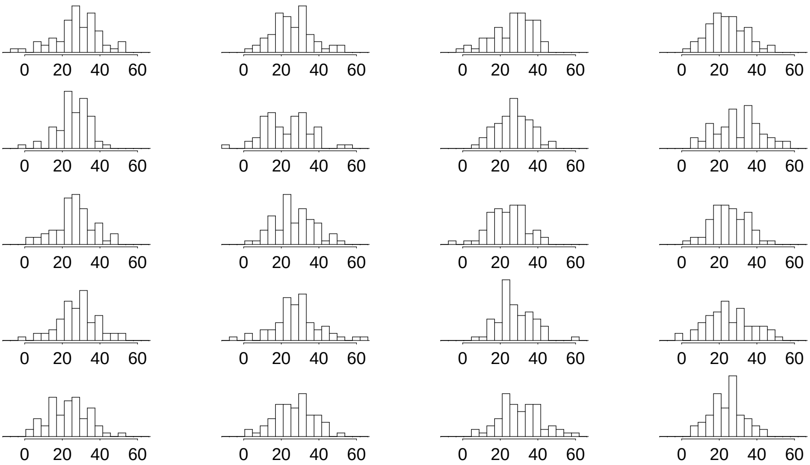
A dataset

A normal distribution is fit to the following data:

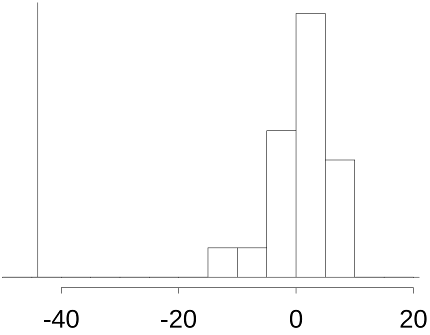


20 replications under the fitted model



Comparison using a numerical test statistic

$T(y) = \min_i y_i$:



Numerical posterior predictive checks

- Limitations of the likelihood principle
- Multiple comparisons
- Interpreting *p*-values

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Connections to classical testing

- p-values and u-values
- The purpose of a model check
- Going beyond pivotal test statistics
- Two examples

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Example where the posterior predictive p-value is stuck near 0.5 and this makes sense

- Data $y \sim \mathrm{N}(\theta, 1)$
- Prior $\theta \sim \mathrm{N}(0, A^2)$ with $A = 100$
- Test statistic $T(y) = y$ (the sample mean)
- To compute posterior predictive check:

- If model is correct, test will essentially never "reject"

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  - Compare to data:

$$p\text{-value} = \Phi\left(\frac{y - \mathrm{E}(y^{\mathrm{rep}}|y)}{\mathrm{sd}(y^{\mathrm{rep}}|y)}\right) = \Phi(-y/14{,}000)$$

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Example where the posterior predictive p-value is stuck near 0.5 and this seems bad

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- Prior $\theta \sim \mathrm{N}(0, A^2)$ with $A = 100$
- Test statistic $T(y) =$ sample skewness of the 1000 observations
- Now suppose that all you've observed is $y_1, \ldots, y_{10}$
- How to understand this result?

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  - $T(y)$ is a random variable (with 99% of the observations missing)
  - $\Pr(T(y^{\rm rep}) > T(y)) \approx 0.5$ for just about any $y_1, \ldots, y_{10}$
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