Bayesian Data Analysis, class 2a

Andrew Gelman

Chapter 2. Single-parameter models (part 1)

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- The basics:
  - Data model (thus, likelihood)
  - Prior density
  - Posterior density
- Work with analytic conjugate forms
- Likelihood comes before the prior; why?

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2.1. Estimating a probability from binomial data

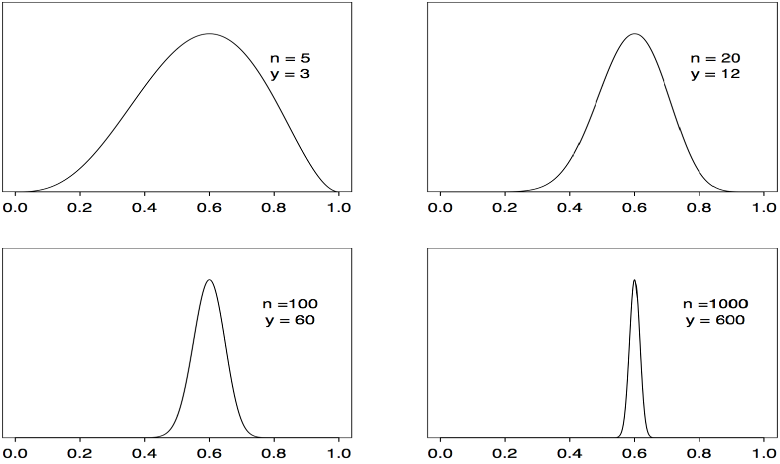

Figure 2.1 *Unnormalized posterior density for binomial parameter θ, based on uniform prior distribution and y successes out of n trials. Curves displayed for several values of n and y.*

2.1. Estimating a probability from binomial data

- Bayes' original example
- $\frac{y}{n}$, $\frac{y+1}{n+2}$, and other alternatives
- Consider some examples

- Dependence of θ and n

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2.2. Posterior as compromise between data and prior information

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- When does it happen that posterior variance is *larger* than the prior variance?
  - Bad luck
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- Research problem: when do models have "warning lights"?

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2.3. Summarizing posterior inference

- Simulation draws
- Simulations + analytics
- Posterior intervals

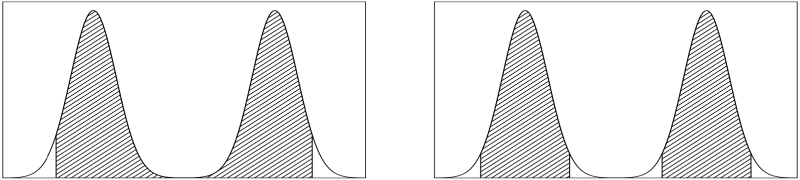

Figure 2.2 *Hypothetical posterior density for which the 95% central interval and 95% highest posterior density region dramatically differ: (a) central posterior interval, (b) highest posterior density region.*

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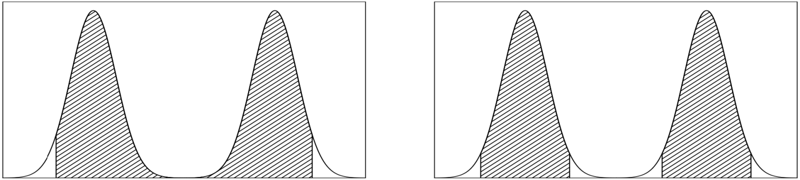

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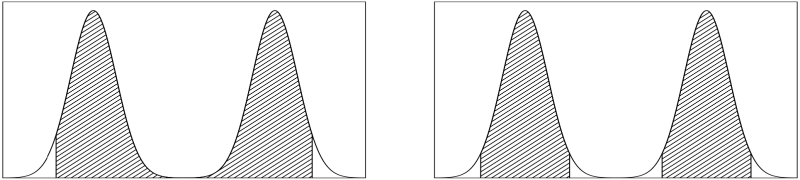

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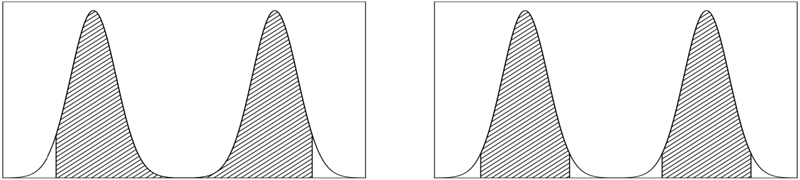

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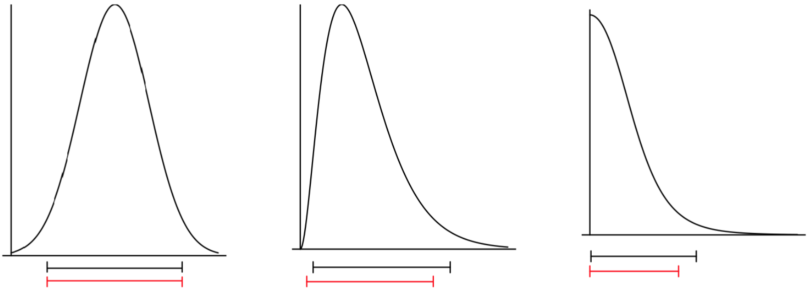
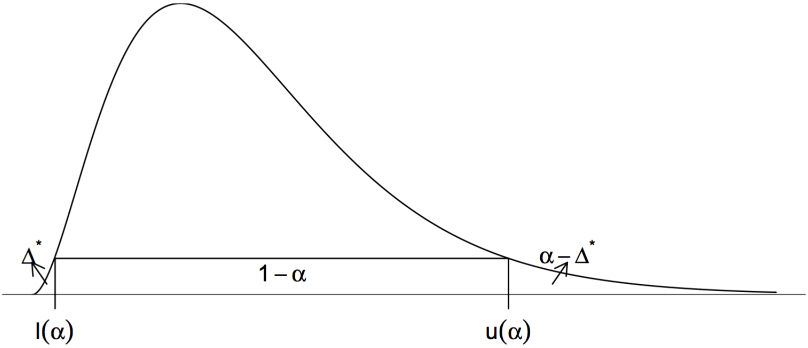
Central and shortest intervals



Shortest posterior intervals



Posterior intervals: choices and goals

- Central intervals
- Highest posterior density intervals
- Nested or non-nested?
- Asymmetry and multiple modes
- Multidimensional contours
- The purpose of interval estimation

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Constructing shortest posterior intervals from simulations

- Empirical Spin
- Smoothing
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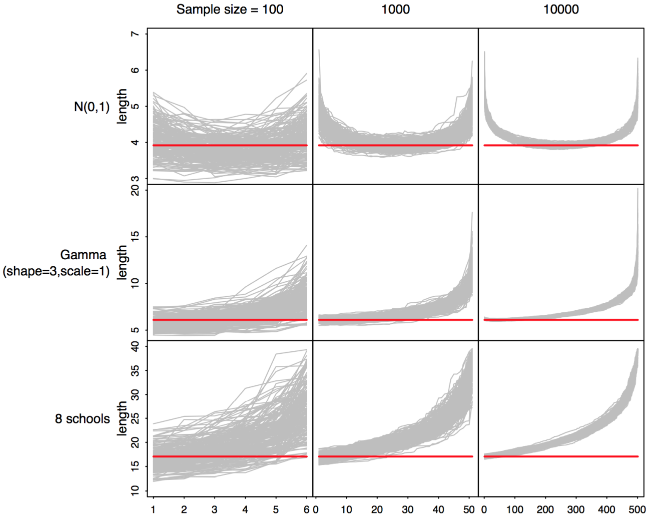
Sampling variability of the empirical shortest posterior interval



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- Interpretations
  - Population
  - State of knowledge
  - Software defaults (statistician in a box)
- Binomial model:
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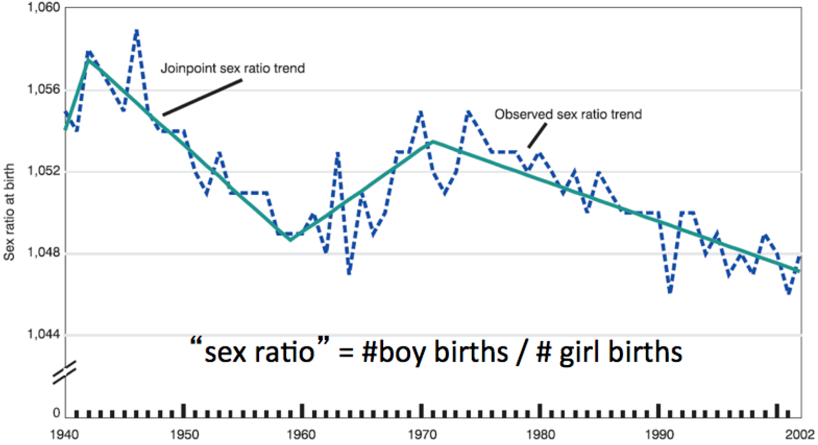
Probability of a girl birth

- In general population, Pr(girl birth) = 0.485



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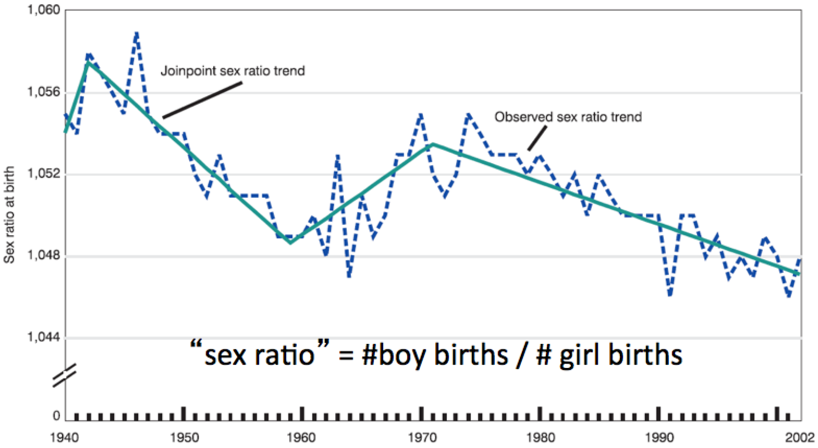
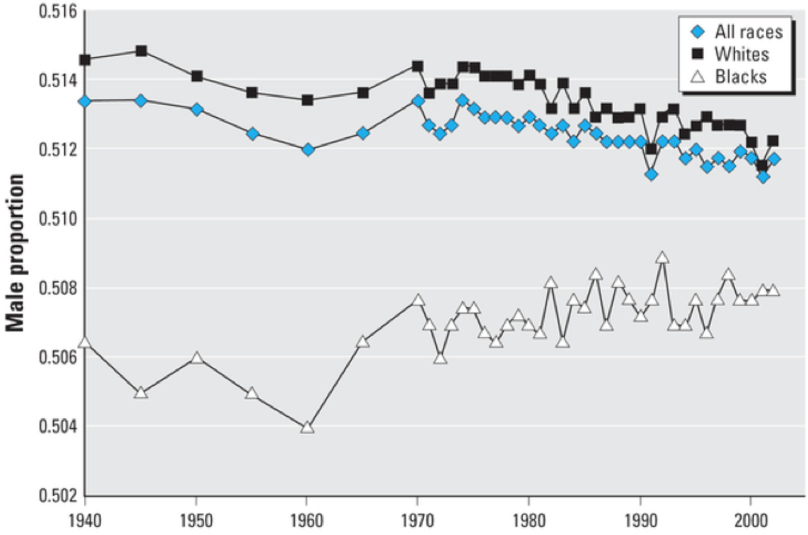


Variation!



All races
Whites
Blacks
Male proportion
0.516
0.514
0.512
0.510
0.508
0.506
0.504
0.502
1940
1950
1960
1970
1980
1990
2000

Example: probability of a girl birth given placenta previia

- Data: 437 girls out of 980 births, $y/n = 437/980 = 0.446$
- Consider different Beta(α, β) priors

$\frac{\alpha}{\alpha+\beta}$	$\alpha + \beta$	Posterior median of θ	95% posterior interval for θ
0.500	2	0.446	$[0.415, 0.477]$
0.485	2	0.446	$[0.415, 0.477]$
0.485	5	0.446	$[0.415, 0.477]$
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0.485	20	0.447	$[0.416, 0.478]$
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Constructing a prior distribution

- $p =$ probability of a girl birth given placenta previa
- Suppose we believe p should be between 0.47 and 0.50
  - Beta(α, β) prior with mean 0.485 and sd 0.01
    Need $\frac{\alpha}{\alpha+\beta} = 0.485$ and $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = 0.01^2$
    Solve to get $\alpha + \beta = 2500$ (actually, 2496.75) and $\alpha = 1212.5$, $\beta = 1287.5$
- Combine with $n = 980$ data points

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