Bayesian Data Analysis, class 5a

Andrew Gelman

Chapter 5: Hierarchical models (part 2)

Discussion of homework due beginning of Class 4b

- Theory problem
- Computing problem
- Applied problem

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Theory problem

- Normal approximation to the posterior distribution from Cauchy data
- Second derivative, plotting the normal density

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Computing problem

- Poisson regression: check that posterior inferences are consistent with true parameter values

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Applied problem

- Basketball shooting again: θ_i is improvement in success probability for person i
- Prior distribution for mean and standard deviation of θ_i in the population
- Sidestepping causal questions

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5. Hierarchical models (part 2)

- The 8 schools example
- Meta-analysis
- Weakly informative priors for hierarchical variance parameters

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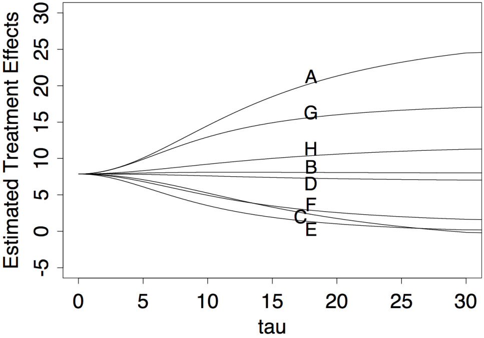
Estimates conditional on the group-level variance

School	Estimated treatment effect, y_j	Standard error of effect estimate, σ_j
A	28	15
B	8	10
C	-3	16
D	7	11
E	-1	9
F	1	11
G	18	10
H	12	18



Posterior uncertainties

School	Estimated treatment effect, y_j	Standard error of effect estimate, σ_j
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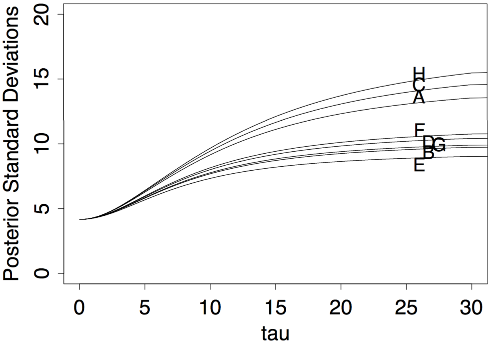

Inference for the group-level variance

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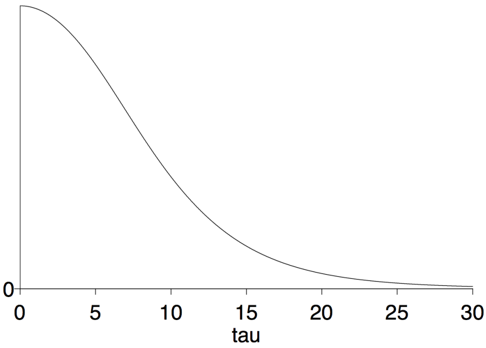
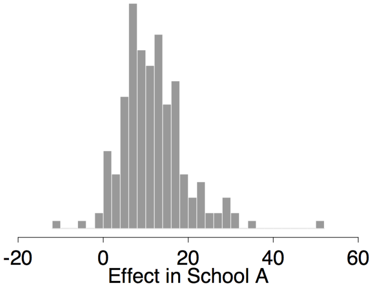


A couple of qois



How do these differ?

Some lingering questions

- What if the model were applied not to 8 schools but to 8 unrelated objects?
- Inference for μ

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5.6. Hierarchical modeling applied to a meta-analysis

- Transform $\frac{y_2}{n_2} - \frac{y_1}{n_1}$ to log-odds with approximately normal errors
- Apply 8-schools model
- Three levels of inference:
- Write the notation for each inference

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  - Average effect
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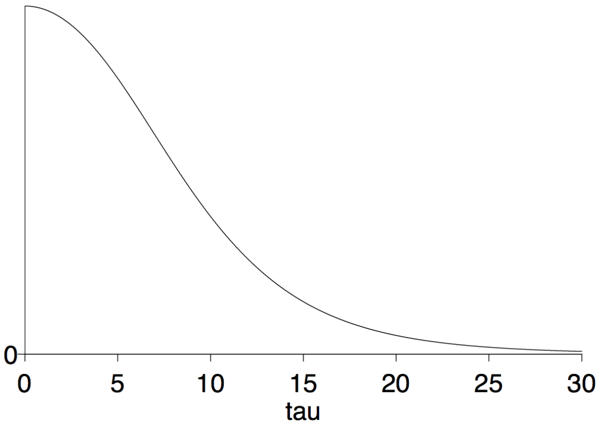
5.7. Weakly informative priors for hierarchical variance parameters

Result of the uniform prior distribution, $p(\tau) \propto 1$, for $\tau \in (0, \infty)$:



Prior distributions for the hierarchical variance parameter

- What is a good "weakly informative prior"?
  - $\log \tau \sim \text{Uniform}(-\infty, \infty)$
  - $\tau \sim \text{Uniform}(0, \infty)$
  - $\tau \sim \text{Inverse-gamma}(0.001, 0.001)$
  - $\tau \sim \text{Cauchy}^{+}(0, A)$
- Polson and Scott (2011):
  "The half-Cauchy occupies a sensible 'middle ground' ... it performs very well near the origin, but does not lead to drastic compromises in other parts of the parameter space."

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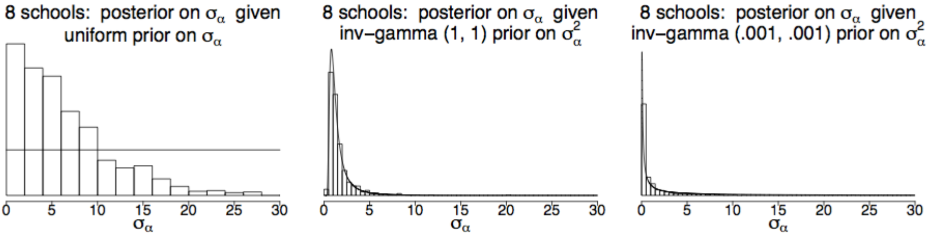

- Inv-gamma prior cuts off at 0

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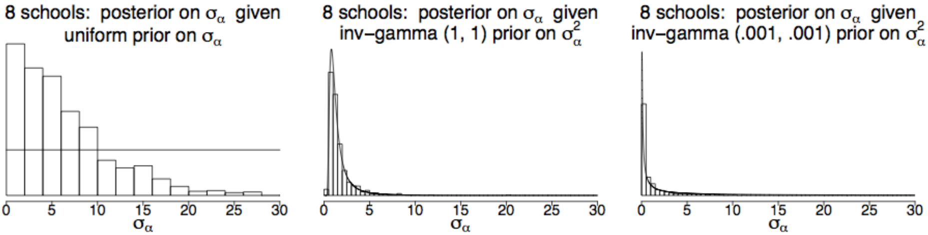

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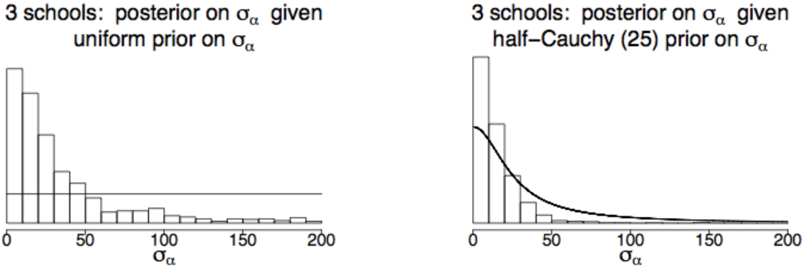
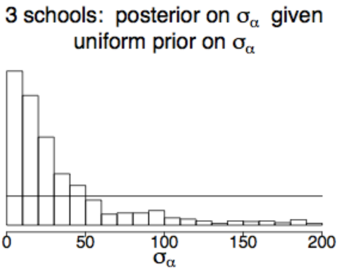
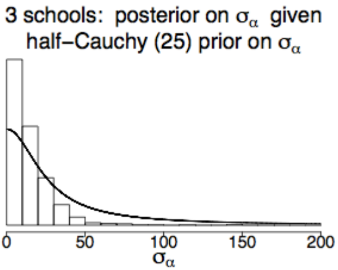


- Uniform prior doesn't cut off the long tail

Problems with uniform prior





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Priors and distributions for multilevel models

- Variance parameters indistinguishable from 0
- 8 schools
- 3 schools
- 1 or 2 schools??

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Summary of Chapter 5

- Estimate hyperparameters from data
- Partial pooling
- Exchangeability and going beyond
- Difficulties when #groups is small

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