# Bayesian Data Analysis, class 3a

Andrew Gelman

Chapter 3: Introduction to multiparameter models (part 1)

# Discussion of homework due beginning of Class 2b

- Theory problem
- Computing problem
- Applied problem

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- Show that if $y|\theta$ is exponentially distributed with rate $\theta$, then the gamma prior distribution is conjugate ...
- Show that the equivalent prior specification for the mean, $\phi = 1/\theta$, is inverse-gamma. (That is, derive the latter density function.)
- The length of life of a light bulb manufactured by a certain process has an exponential distribution with unknown rate $\theta$. Suppose the prior distribution for $\theta$ is a gamma distribution with coefficient of variation 0.5 ... If the coefficient of variation of the distribution of $\theta$ is to be reduced to 0.1, how many light bulbs need to be tested?

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# Computing problem

- Your task is to estimate the percentatge of the (adult) population in each state (excluding Alaska and Hawaii) who label themselves as "very liberal," replicating the procedure that was used in Section 2.8 to estimate cancer rates . . .
- This exercise has four challenges: first, manipulating the data in order to get the totals by state; second, replicating the calculations for estimating the parameters of the prior distribution; third, doing the Bayesian analysis by state; and fourth, making the graphs.

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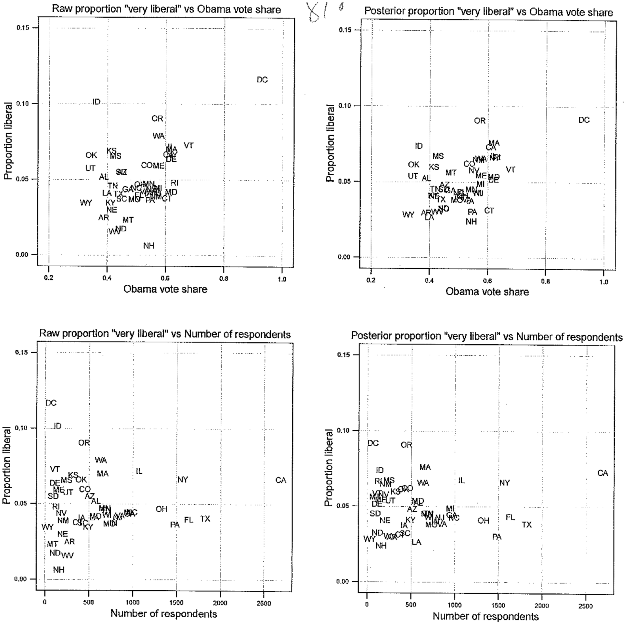
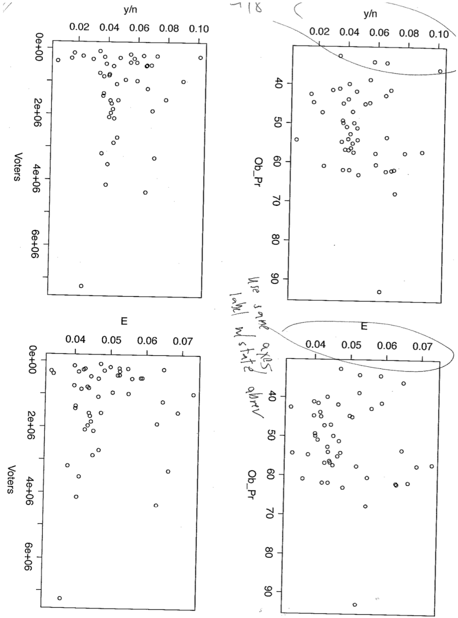
# Good graphs (but could be even better!)



# Bad graphs (but that's how you learn!)



# Applied problem

A (hypothetical) study is performed to estimate the effect of a simple training program on basketball free-throw shooting . . . Let $\theta$ be the average improvement in success probability. Give . . .

- A noninformative prior,
- A subjective prior based on your best knowledge, and
- A weakly informative prior.

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# One student's answer

- A noninformative prior:

  > "$U(-1, 1)$ *because the change in probabilities can't be less than* $-1$ *or more than 1"*

- A subjective prior based on your best knowledge:

  > "*Beta*$(2, 5)$*: most of the probability lies below* $\theta = 0.5$ *(mean = 0.29, mode = 0.20), and the probability of improving by more than 0.8 is essentially zero"*

- A weakly informative prior:

  > *"Assuming that taking 50 practice shots a day for a month does not worsen success probability, we can choose* $U(0, 1)$*"*

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- Important algebra and geometry
- Basic computation
- Similarities and differences compared to classical statistics

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- Factorization, $p(\theta_1|y)p(\theta_2|\theta_1, y)$
- What is a nuisance parameter?

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- Simple scenario of independence:
  - $y_1 \sim \text{dist}(\theta_1)$, prior $p(\theta_1)$
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- $y_1, \ldots, y_n \sim \mathrm{N}(\mu, \sigma^2)$
- Try $p(\mu, \sigma^2) \propto \sigma^{-2}$
  - Equivalent to $p(\mu, \sigma) \propto \sigma^{-1}$
    Equivalent to $p(\mu, \log \sigma) \propto 1$
- Unnormalized joint posterior density:
  $p(\mu, \sigma^2|y) \propto (2\pi)^{-n/2}\sigma^{-n-2}\exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^n (y_i-\mu)^2\right)$
- Integrate out $\mu$: $\sigma^2|y \sim \text{Inv-}\chi^2(n-1, s^2)$
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- Prior distribution $p(\mu, \sigma^2) \propto \sigma^{-2}$
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- $(\bar{y}, s^2, n)$ are *sufficient statistics*
- What happens if we change the prior distribution?

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- $y_1, \ldots, y_n \sim \mathrm{N}(\mu, \sigma^2)$
- Conjugate family of priors:
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    $\mu|\sigma^2 \sim$ Normal with variance proportional to $\sigma^2$
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# Joint prior distributions

- Dependent prior on $\mu, \sigma$: does this make sense?
- Examples of multi-parameter models
  - Normal model: location $\mu$, scale $\sigma$
    Student-$t$ model: d.f. $\nu$, location $\mu$, scale $\sigma$
    Two location parameters, $\mu_1, \mu_2$
    Baseline and treatment effect, $\mu, \theta$
    Regression coefficients: $\beta_0, \beta_1, \beta_2, \ldots$
    Main effects and interactions

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# 3.4. Multinomial model for categorical data

- $y_1, \ldots, y_k \sim \text{Multinomial}(n; \theta_1, \ldots, \theta_k)$
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# 3.5. Multivariate normal model with known variance

- Important algebra

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# 3.6. Multivariate normal with unknown mean and variance

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