NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2021/2022

MA2002 Calculus **Homework Assignment 4**

IMPORTANT:

(i) Write your name and student number on every page of your answer scripts.

(ii) Scan your scripts as a single PDF document. Other formats are not acceptable.

(iii) Rename your PDF document by student number. For example, A1234567X.pdf.

(iv) Log in to LumiNUS, and upload your PDF document to one of the subfolders in Files — Student Submission — Homework Assignment 4.

(v) Submit by 8th November 2021 (Monday) 23:59.

1. Use Riemann sums to evaluate the following definite integrals. [8]

   (a) $\displaystyle\int_1^3 2^x\,dx.$

   (b) $\displaystyle\int_0^6 \cos 3x\,dx.$ [*Hint*: $\displaystyle\sum_{i=0}^{n}\cos(i\theta) = \frac{1}{2} + \frac{\sin(\frac{2n+1}{2}\theta)}{2\sin\frac{\theta}{2}}$].

2. Express the following as definite integrals and evaluate the limits. [10]

   (a) $\displaystyle\lim_{n\to\infty}\left(\frac{1}{n^5}\sum_{i=1}^{n} i^3\sqrt{n^2-i^2}\right).$

   (b) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\frac{n^3}{16n^4-i^4}.$

3. Find the following limits. [8]

   (a) $\displaystyle\lim_{x\to 0}\left(\frac{\cos x}{\cos 3x}\right)^{\csc^2 x}.$

   (b) $\displaystyle\lim_{x\to\infty}\left(\frac{x^2+\sqrt{x}+1}{x^2-\sqrt{x}+1}\right)^{x^{3/2}}.$

4. Find the following definite integrals. [8]

   (a) $\displaystyle\int_1^3 \frac{dx}{(x+3)\sqrt{x^2+2x-3}}.$ [*Hint*: Trigonometric substitution.]

   (b) $\displaystyle\int_0^1 \sqrt{1-x^2}\,\sin^{-1} x\,dx.$ [*Hint*: Integration by parts.]

5. Let m, n be nonnegative integers and define $I(m,n) = \int_0^1 x^m (\ln x)^n \, dx$. Find a relation between $I(m,n)$ and $I(m,n-1)$ $(n \geq 1)$ and find a general formula of $I(m,n)$. [6]

   [*Hint*: This is an improper integral.]

6. Let f be a continuous function on $[a,b]$. If $\int_a^b [f(x)]^2 \, dx = 0$, prove that $f(x) = 0$ for all $x \in [a,b]$. [5]

   [*Hint*: Prove by contradiction. The precise definition of limit may be necessary.]

7. Let f and g be continuous increasing functions on $[0,1]$. Prove that [5]

$$\int_0^1 f(x)\, dx \int_0^1 g(x)\, dx \leq \int_0^1 f(x) g(x)\, dx.$$

   [*Hint*: Use mean value theorem for definite integral.]