NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2021/2022

MA2002 Calculus **Tutorial 7** (11^{th} October – 15^{th} October)

TUTORIAL PART I

This part consists of relatively basic questions which cover the course materials. The solutions to these questions will be recorded.

1. Find the following limits.

(a) $\displaystyle\lim_{t\to -3} \frac{t^3-4t+15}{t^2-t-12}$,

(b) $\displaystyle\lim_{x\to 0} \frac{8x^2}{\cos x - 1}$,

(c) $\displaystyle\lim_{t\to 0} \frac{t(1-\cos t)}{t-\sin t}$,

(d) $\displaystyle\lim_{x\to \infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}}$,

(e) $\displaystyle\lim_{x\to (\pi/2)^-} \frac{\sec x}{\tan x}$,

(f) $\displaystyle\lim_{x\to 1} \frac{\sqrt{3-x}-\sqrt{1+x}}{x^2+x-2}$,

(g) $\displaystyle\lim_{x\to a} \frac{x^m-a^m}{x^n-a^n} \quad (a\neq 0,\ m,n\in\mathbb{Z}^+)$,

(h) $\displaystyle\lim_{x\to \pi/2} (\sec x - \tan x)$.

2. Let $a>0$. Use Riemann sums to compute $\displaystyle\int_0^a x^3\,dx$. You may use the following formula

$$\sum_{i=1}^{n} i^3 = 1^3+2^3+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

3. Suppose that h is continuous and that $\displaystyle\int_{-1}^{1} h(r)\,dr = 0$ and $\displaystyle\int_{-1}^{3} h(r)\,dr = 6$. Find $\displaystyle\int_{3}^{1} h(r)\,dr$.

4. Find $\dfrac{dy}{dx}$ of the following functions.

(a) $\displaystyle y=\int_0^{x^2} \cos(t^{1/3})\,dt$,

(b) $\displaystyle y=\int_\pi^{\sqrt{x}} \sin t\,dt$,

(c) $\displaystyle y=\int_{\tan x}^{0} \frac{dt}{(1+t^2)^2}$.

5. Let f be a continuous function on $\mathbb{R}$. Suppose that g and h are differentiable. Find a formula for the derivative of
$$F(x) = \int_{g(x)}^{h(x)} f(t)\,dt.$$
Hence evaluate the derivative of $F(x) = \displaystyle\int_{\cos x}^{5x} \cos(t^2)\,dt$.

6. Let f be a continuous function on $\mathbb{R}$. Define
$$F(x) = \int_a^x f(t)(x-t)\,dt.$$
Evaluate $F''(x)$.

TUTORIAL PART II

This part consists of relatively difficult questions to promote independent learning and inculcate critical thinking abilities. The solutions will not be recorded. You may attempt them after you have gained a good understanding of the questions in Part I. The complete solution of this part is provided.

1. Find the following limits.

   (a) $\displaystyle\lim_{x\to 0} \frac{\tan x - x - x^3/3}{\sin^5 x}$, (b) $\displaystyle\lim_{x\to\infty} \left(\sqrt{x^2+x+1} - \sqrt{x^2-x}\right)$.

2. Let $0 < a < b$. Use Riemann sums to compute $\displaystyle\int_a^b x^{-2}\,dx$.

3. Let f be a continuous function on $\mathbb{R}$. Suppose that f is periodic with period T ($T > 0$), i.e., $f(x+T) = f(x)$ for every $x \in \mathbb{R}$. Prove that
$$\int_a^{a+T} f(x)\,dx = \int_0^T f(x)\,dx \quad \text{for any } a \in \mathbb{R}.$$

Answers to Part I:

1. (a) $-\dfrac{23}{7}$, (b) -16, (c) 3, (d) 3, (e) 1, (f) $-\dfrac{\sqrt{2}}{6}$, (g) $\dfrac{m}{n}\,a^{m-n}$, (h) 0.

2. $\dfrac{a^4}{4}$.

3. -6.

4. (a) $2x\cos(x^{2/3})$, (b) $\dfrac{\sin\sqrt{x}}{2\sqrt{x}}$, (c) $-\cos^2 x$.

5. $h'(x)f(h(x)) - g'(x)f(g(x))$, $\quad 5\cos(25x^2) + \sin x \cos(\cos^2 x)$.

6. $f(x)$.