

NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2021/2022

MA2002 Calculus

Homework Assignment 1

IMPORTANT:

- (i) Write your name and student number on every page of your answer scripts.
- (ii) Scan your scripts as a single PDF document. Other formats are not acceptable.
- (iii) Rename your PDF document by student number. For example, A1234567X.pdf.
- (iv) Log in to LumiNUS, and upload your PDF document to one of the subfolders in Files — Student Submission — Homework Assignment 1, depending on the first letter of your name on your student card.
- (v) Submit by 13th September 2021 (Thursday) 23:59.

1. Let $f(x) = \frac{1}{1-x^2}$ and $g(x) = \sqrt{x+1}$. [10]

- (i) Write down the domains of f and g .
- (ii) Find $f \circ g$ and $g \circ f$ and their domains.

2. For each of the following, evaluate the limit. (l'Hôpital's rule is **not** allowed.) [15]

(a) $\lim_{x \rightarrow -2} \frac{x^4 + 3x^3 + x^2 + 4}{x^4 + 4x^3 + 3x^2 - 4x - 4}$.

(b) $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 7} - \sqrt{x^3 + 3}}{\sqrt{x+1} - \sqrt{2x-1}}$.

3. Prove the following using only the precise definition of limit. (Any theorem on limit is **not** allowed.) [15]

(a) $\lim_{x \rightarrow -\sqrt{2}} (x^2 - \sqrt{2}x) = 4$.

(b) $\lim_{x \rightarrow 1} \frac{x}{x^2 + 1} = \frac{1}{2}$.

4. Find the values of real constants a and b such that $\lim_{x \rightarrow \infty} (\sqrt{ax^2 + 1} - \sqrt{x^2 + bx}) = 2021$. Justify your answers. [10]

5. Using only the precise definitions of limit and infinite limit, prove that the following two statements are equivalent: [10]

(i) $\lim_{x \rightarrow a} f(x) = \infty$;

(ii) $\lim_{x \rightarrow a} \frac{1}{f(x)} = 0$ and $f(x) > 0$ for all x in an open interval containing a except at a .