**NATIONAL UNIVERSITY OF SINGAPORE**

SEMESTER 1, 2021/2022

**MA2002 Calculus** **Tutorial 10** ($1^{st}$ November – $5^{th}$ November)

TUTORIAL PART I

This part consists of relatively basic questions which cover the course materials. The solutions to these questions will be recorded.

1. Find the area of the region enclosed by the curve $y = x^4$ and the line $y = 8x$.

2. (i) Compute the area of the ellipse
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$
where $a > 0$, $b > 0$.

   (ii) Compute the volume of the ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$
where $a > 0$, $b > 0$, $c > 0$.

3. For each of the following, find the volume of the solid generated by revolving the region enclosed by the given curves/lines about the specified line.
   (a) $y = x^3$, $y = 0$ and $x = 2$; about the $x$-axis.
   (b) $x = \sqrt{2\sin 2y}$ $(0 \le y \le \pi/2)$ and $x = 0$; about the $y$-axis.
   (c) $y = 4 - x^2$ and $y = 2 - x$; about the $x$-axis.
   (d) $y = x^2$, the $x$-axis and $x = 1$; about $x = -1$.
   (e) $y = \sin x$ $(0 \le x \le \pi)$ and $y = 0$; about the $y$-axis.
   (f) $x = (y-3)^2$ and $x = 4$; about $y = 1$.

4. Find the lengths of the following curves.
   (a) $y = \sqrt{2 - x^2}$ from $x = 0$ to $x = 1$.
   (b) $y = \ln(\cos x)$ from $x = 0$ to $x = \pi/3$.
   (c) $x = \dfrac{y^3}{6} + \dfrac{1}{2y}$ from $y = 2$ to $y = 3$.

5. Find the areas of the surfaces generated by revolving the following curves about the indicated axes.

   (a) $y = \sqrt{2x - x^2},\ 1/2 \le x \le 3/2;$ about the $x$-axis.

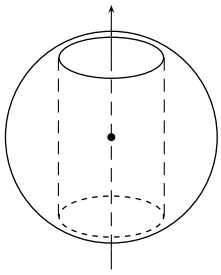
   (b) $x = 2\sqrt{4 - y},\ 0 \le y \le 15/4;$ about the $y$-axis.

   (c) $x^{2/3} + y^{2/3} = 1$ in the first and second quadrants; about the $x$-axis.

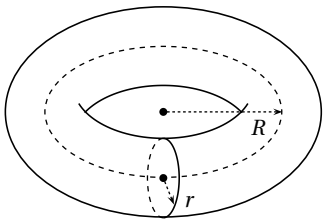
## TUTORIAL PART II

This part consists of relatively difficult questions to promote independent learning and inculcate critical thinking abilities. The solutions will not be recorded. You may attempt them after you have gained a good understanding of the questions in Part I. The complete solution of this part is provided.

1. Consider a sphere of radius $R$ which has a hole of radius $r$ ($r < R$) bored through its center. Find the volume of the resulting solid.

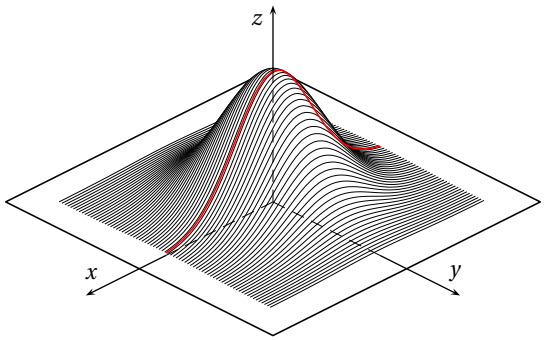


2. Find the volume and surface area of a torus with radii $r$ and $R$, where $r < R$, as shown below.




3. (i) Using the cylindrical shell method, find the volume of the solid bounded above by the surface $z = e^{-(x^2+y^2)}$ and bounded below by the $xy$-plane.

   (ii) Express the volume in part (i) by another method. Hence, evaluate the improper integral $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\,dx$.




**Answers to Part I:**

1. $\frac{48}{5}$.

2. (i) $\pi ab$, (ii) $\frac{4\pi abc}{3}$.

3. (a) $\frac{128\pi}{7}$, (b) $2\pi$, (c) $\frac{108\pi}{5}$, (d) $\frac{7\pi}{6}$, (e) $2\pi^2$, (f) $\frac{128\pi}{3}$.

4. (a) $\frac{\sqrt{2}\pi}{4}$, (b) $\ln(2+\sqrt{3})$, (c) $\frac{13}{4}$.

5. (a) $2\pi$, (b) $\frac{35\sqrt{5}\pi}{3}$, (c) $\frac{12\pi}{5}$.