

**NATIONAL UNIVERSITY OF SINGAPORE**

SEMESTER 1, 2021/2022

**MA2002 Calculus**

**Homework Assignment 2**

**IMPORTANT:**

- (i) Write your name and student number on every page of your answer scripts.
- (ii) Scan your scripts as a single PDF document. Other formats are not acceptable.
- (iii) Rename your PDF document by student number. For example, A1234567X.pdf.
- (iv) Log in to LumiNUS, and upload your PDF document to one of the subfolders in Files — Student Submission — Homework Assignment 2, depending on the first letter of your name on your student card.
- (v) Submit by 27<sup>th</sup> September 2021 (Thursday) 23:59.
- (vi) l'Hôpital's rule is not allowed.

1. Find the values of constants  $a$  and  $b$  such that  $f$  is continuous on  $\mathbb{R}$ , where [10]

$$f(x) = \begin{cases} \frac{\sin(\sin \pi x)}{x+1} & \text{if } x < -1, \\ ax+b & \text{if } -1 \leq x \leq 2, \\ \frac{\sqrt{x+3}-\sqrt{2x+1}}{\sqrt{3x-1}-\sqrt{4x-3}} & \text{if } x > 2. \end{cases}$$

2. Use Intermediate Value Theorem to prove that the following equation has at least four real roots. [8]

$$(x-1)(x-3)(x-5)(x-7)(x-9) = (x-2)(x-4)(x-6)(x-8)(x-10).$$

3. Use only the definite of derivative, find [10]

$$\frac{d}{dx} \sqrt{x^2 + \sqrt{x}}, \quad x > 0.$$

4. Let  $a > 0$  and  $b > 0$  be constants. Suppose that the parabolas [8]

$$C_1 : y^2 = 4a(a-x) \quad \text{and} \quad y^2 = 4b(b+x)$$

intersect at a point  $P$ . Prove that the tangent line to  $C_1$  at  $P$  and the tangent line to  $C_2$  at  $P$  are perpendicular.

5. Find the values of constants  $a, b, c$  such that  $f$  is differentiable on  $\mathbb{R}$ , where [10]

$$f(x) = \begin{cases} \frac{a - \cos x}{x} & \text{if } x < 0, \\ bx + c & \text{if } x \geq 0. \end{cases}$$

6. Find the absolute maximum and minimum values of the following functions. [14]

(a)  $f(x) = \sqrt[5]{x+1}(x-2)^2$  on  $[-2, 3]$ .

(b)  $g(x) = x - \cos 2x - 2 \sin x + 2 \cos x$  on  $[-\pi, \pi]$ .