NATIONAL UNIVERSITY OF SINGAPORE

SEMESTER 1, 2021/2022

MA2002 Calculus **Tutorial 6** (4^{th} October – 8^{th} October)

TUTORIAL PART I

This part consists of relatively basic questions which cover the course materials. The solutions to these questions will be recorded.

1. For each of the functions

   (a) $^{\diamondsuit} f(x) = 2 + 3x - x^3$ on $\mathbb{R}$, (b) $g(x) = 4x - \tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

   (i) Find the open intervals on which it is increasing and decreasing;

   (ii) Find the coordinates of all its local maximum and minimum points;

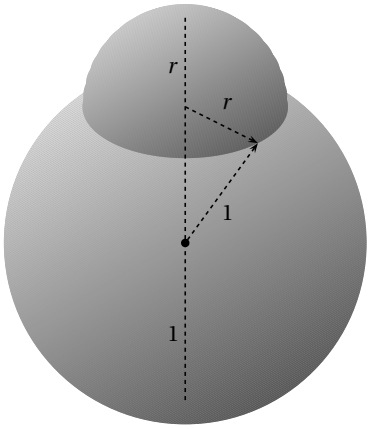
   (iii) Find the open intervals on which it is concave up and concave down;

   (iv) Find the coordinates of all its inflection points;

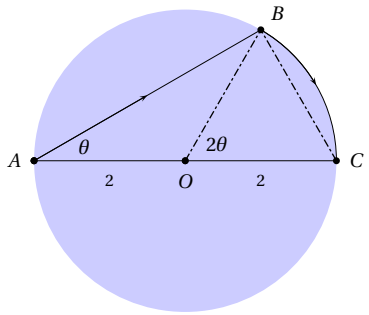
   (v) Use the information from parts (i)–(iv) to sketch its graph.

2. What is the largest possible area for a right triangle whose hypothenuse is 5 cm long?

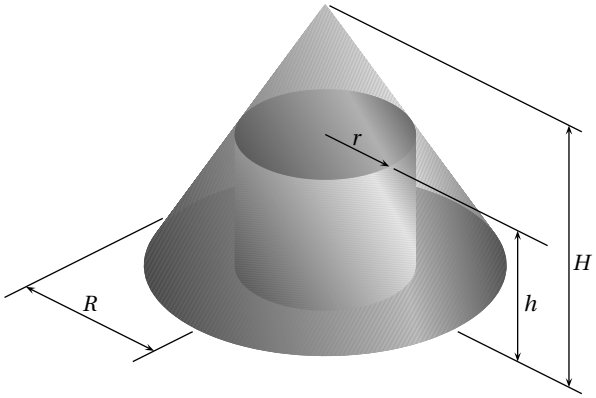
3. A hemisphere bubble is placed on a spherical bubble of radius 1. Find the maximum height of the bubble tower.



4. A woman at a point A on the shore of a circular lake with radius 2 km wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can row a boat at 2 km/h and walk at the rate of 4 km/h. How should she proceed?

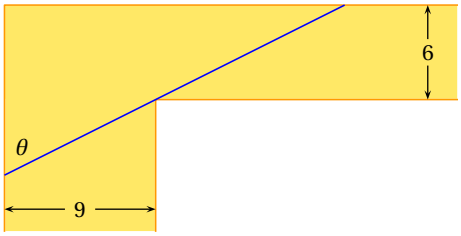


5. A right circular cylinder is inscribed in a cone with height H and base radius R. Find the largest possible volume of such a cylinder.



6. A 216 m^2 rectangular plot (the dimensions to be determined) is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

7. A steel pipe is being carried down a hallway 9 m wide. At the end of the hall there is a right-angled turn into a narrower hallway 6 m wide. What is the approximate length (in feet) of the longest pipe that can be carried horizontally around the corner? Round your answer down to the nearest foot.

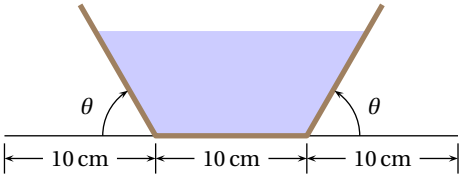


TUTORIAL PART II

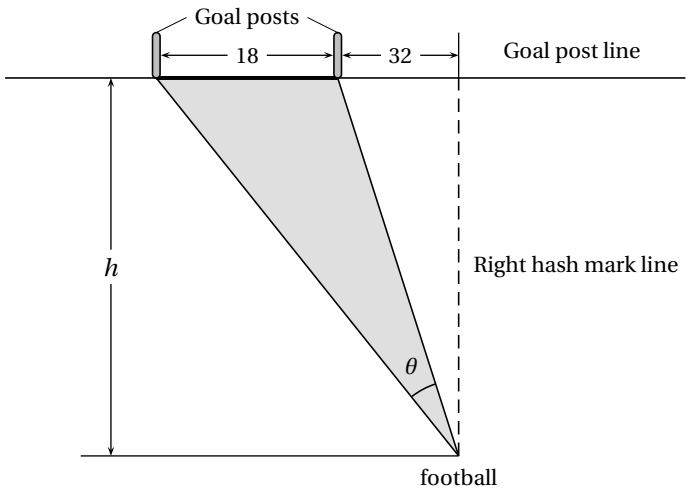
This part consists of relatively difficult questions to promote independent learning and inculcate critical thinking abilities. The solutions will not be recorded. You may attempt them after you have gained a good understanding of the questions in Part I. The complete solution of this part is provided.

1. For the function $f(x) = x + \cos x$ on the interval $[-2\pi, 2\pi]$, find the coordinates of all its local extreme points and inflection points, if any, and then sketch its graph.

2. A rain gutter is to be constructed from a metal sheet of width 30 cm by bending up one third of the sheet on each side through an angle θ. How should θ be chosen so that the gutter will carry the maximum amount of water?



3. An American football player wants to kick a field goal with the ball being on a right hash mark. Assume that the goal posts are 18 ft apart and that the hash mark line is a distance 32 ft from the right goal post. Assume that the field is flat. Find the distance from the goal post line that gives the kicker his largest angle.



Answers to Part I:

1. (a) (i) increasing on $(-1,1)$, decreasing on $(-\infty,-1)$ and on $(1,\infty)$,
   (ii) local maximum at $(1,4)$, local minimum at $(-1,0)$,
   (iii) concave up on $(-\infty,0)$, concave down on $(0,\infty)$,
   (iv) inflection point $(0,2)$.

   (b) (i) increasing on $\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$, decreasing on $\left(-\frac{\pi}{2},-\frac{\pi}{3}\right)$ and on $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$,
   (ii) local maximum at $\left(\frac{\pi}{3},\frac{4\pi}{3}-\sqrt{3}\right)$, local minimum at $\left(-\frac{\pi}{3},-\frac{4\pi}{3}+\sqrt{3}\right)$,
   (iii) concave up on $\left(-\frac{\pi}{2},0\right)$, concave down on $\left(0,\frac{\pi}{2}\right)$,
   (iv) inflection point $(0,0)$.

2. $\frac{25}{4}\,\mathrm{cm}^2$.

3. $1+\sqrt{2}$.

4. Walk to C round the lake directly.

5. $\frac{4}{27}\pi R^2 H$.

6. $18\times 12\,\mathrm{m}$, $72\,\mathrm{m}$.

7. $21\,\mathrm{m}$.