Set Theory

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August 2022

Intro to Set Theory

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- A branch of mathematics
- Collects objects into *sets* and studies the properties

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Sets

A **set** is a collection of objects

$$S = \{s_1, s_2, s_3, ...s_n\}$$

The objects can be anything

- Can think of it as a group whose members have something in common that we care about
- We usually use variables or units of observation

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Elements in a set (or not)

We can denote an object as being in a set:

$$s_4 \in S$$

Or we can show an object is *not* in a set:

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$$M \subset S$$

M is a proper subset of S if and only if all elements of M are in S but *not* all elements of S are in M.

What's an example of a set and a proper subset?

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Ordered vs. Unordered Sets

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Empty Set

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A set that contains all elements in a universe of possibilities is the *universal set*

- All the possibilities of a single roll of a six-sided die:

$$R = 1, 2, 3, 4, 5, 6$$

- Sets for the even and odd possibilities:

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Complements

The complement of set A is the set that contains the elements that are *not* contained in A

- The difference between a set and the universal set
- Denoted A' or A^C

From our above example, $E = O^C$ and $O = E^C$.

Intersection

$A \cap B$ ("A intersection B") is the set of all elements common to both sets A and B

- If $A \cap B = \emptyset$ (i.e. there are no elements in the intersection), they are *mutually exclusive* (or *disjoint*)

Union

$A \cup B$ ("A union B") is the set of all elements of either set

- Two sets' unions can't be empty (unless both sets are empty to begin with)

Size

$n(R) = 6$ means set R contains six elements

Size of unions: $n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

- X union Y is equal to the size of set X plus the size of set Y minus X intersection Y
- We need to make sure we don't double-count the elements in the intersection

Consider a problem in which you are rolling two dice.

- Create a set A consisting of all the outcomes from rolling the dice where the sum is equal to 5. (hint: an element will be an ordered pair like $(1, 2)$).
- Use set notation to denote the size of A.
- Use set notation to denote whether $(3, 1)$ is in A or not.

Practice

Let X and Y be two sets, where $n(X) = 14$ and $n(Y) = 25$. If there are twice as many objects in $X \cup Y$ as there are in $X \cap Y$, how many objects are in both X and Y?