

P1.

$$\text{cov}(Z, Z^2) = E[Z^3] - E[Z^2]E[Z]$$

$$Z \sim N(0,1) \Rightarrow E[Z] = 0$$

$$\text{var}[Z] = E[Z^2] - E[Z]^2 = 1 \Rightarrow E[Z^2] = 1$$

$$E[Z^3] = 0 \text{ due to symmetric distribution of normal distr.}$$

$$\therefore \text{cov}(Z, Z^2) = 0 - 1 \times 0 = 0$$

$$\therefore \text{var}(Z) = 1 > 0$$

$$\text{var}(Z^2) = E[Z^4] - E[Z^2]^2 = 3 - 1^2 = 2 > 0$$

$$\therefore \text{Corr}(Z, Z^2) = \frac{\text{cov}(Z, Z^2)}{\sigma_Z \sigma_{Z^2}} = \frac{0}{\sqrt{1} \cdot \sqrt{2}} = 0$$

They are not independent.

$$P[Z > 1, Z^2 > 1] = P[Z > 1] \text{ since } Z > 1 \text{ implies } Z^2 > 1$$

$$\neq P[Z > 1] P[Z^2 > 1]$$

$\therefore Z$ & Z^2 are not independent.