

Count of Range Sum

Problem Description

Given an integer array **nums**, return the number of range sums that lie in **[lower, upper]** inclusive.

- Range sum **$S(i, j)$** is defined as the sum of the elements in **nums** between indices **i** and **j** (**$i \leq j$**), inclusive.

Example

Input: **nums** = [-2, 5, -1], **lower** = -2, **upper** = 2,

Output: 3

Explanation: The three ranges are : [0, 0], [2, 2], [0, 2] and their respective sums are: -2, -1, 2.

Requirement on Complexity

A naïve algorithm of $O(n^2)$ is trivial. You **MUST** do better than that.

- Just searching over all $S(i, j)$ will not pass the tests!
- We will check your code

Try to write an algorithm of $O(n \log n)$

Due

Dec. 12

Prefix Sums

To calculate any $S(i, j)$ in $O(1)$ time

$$P[i] = \sum_{k=0}^i \text{nums}[k]$$
$$S(i, j) = P[j] - P[i - 1]$$

$O(n)$ time for generating array P

Divide and Conquer

How about borrowing the idea of merge sort?

Divide:

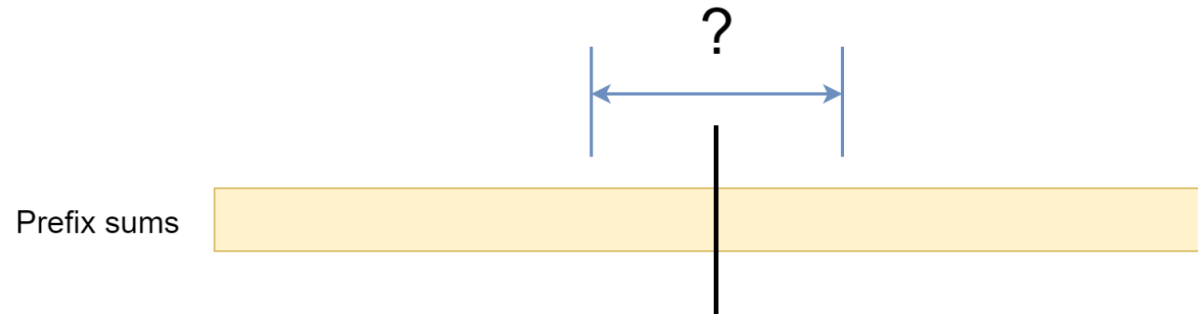
- 2 halves

Conquer:

- # of ranges within left half that meet the condition
- # of ranges within right half that meet the condition

Combine:

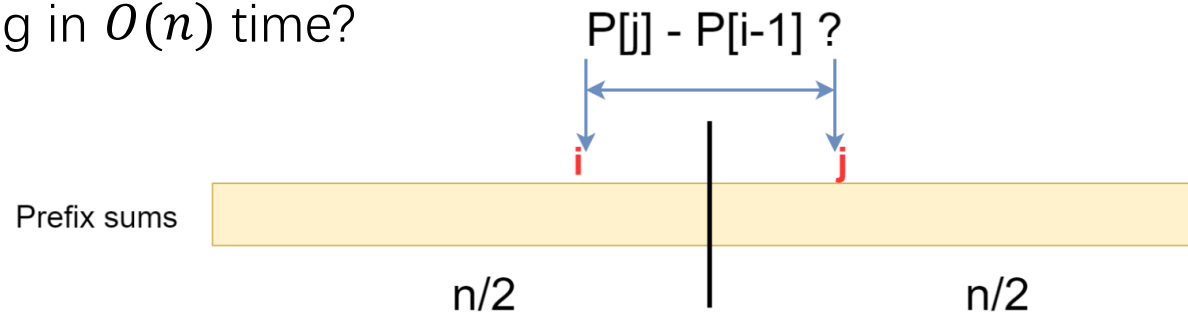
- And the ranges that cross the middle line?



Combine

We want to achieve $T(n) = O(n \log n)$. $T(n) = 2T\left(\frac{n}{2}\right) + O(n)$

How to do the combining in $O(n)$ time?



Check every pair of (i, j) : unfortunately it's $\left(\frac{n}{2}\right) * \left(\frac{n}{2}\right) = O(n^2) \dots$

\dots if we make no assumption on the array P that you have at this stage.

“How about borrowing the idea of **merge sort**?”