Coffee Medal problems

September 18, 2021

1 Calculations in QFT

Given

$$\mathcal{L} = \sum_{j=1,2} \overline{\psi_j}(i\partial\!\!\!/ - m_j - y_j\phi)\psi_j + \frac{1}{2}\phi(\partial^2 - m_\phi^2)\phi\,,$$

compute the cross sections of $\psi_1 + \overline{\psi_1} \to \psi_2 + \overline{\psi_2}$, $\psi_1 + \overline{\psi_2} \to \psi_1 + \overline{\psi_2}$, and $\psi_1 + \overline{\psi_1} \to \phi + \phi$.

2 Calculations in cosmology

In an expanding universe where all particles are absent except for $e^\pm$ and γ, formulate the Boltzmann equations and compute the relic density and the freeze-out temperature of $e^\pm$.

3 Numerical skills

Consider the harmonic oscillator in quantum mechanics with the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$. Now add a quartic term λx^4, set $m = k = 1$ and $\lambda \in \{0.1, 1, 10\}$. Numerically solve the Schrödinger equation to find the energy levels and plot the wavefunctions.