

Coffee Medal problems

July 9, 2024

1 Calculations in QFT

Given

$$\mathcal{L} = \sum_{j=1,2} \bar{\psi}_j (i\not{\partial} - m_j - y_j \phi) \psi_j + \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} m_\phi^2 \phi^2,$$

compute the cross sections of $\psi_1 + \bar{\psi}_1 \rightarrow \psi_2 + \bar{\psi}_2$, $\psi_1 + \bar{\psi}_2 \rightarrow \psi_1 + \bar{\psi}_2$, and $\psi_1 + \bar{\psi}_1 \rightarrow \phi + \phi$.

2 Loop calculations in QFT+SM

For massive Dirac neutrinos with SM interactions, compute neutrino magnetic moments in the R_ξ gauge and show explicitly the cancellation of gauge dependence.

3 Calculations in cosmology

In an expanding universe where all particles are absent except for e^\pm and γ , formulate the Boltzmann equations and compute the relic density and the freeze-out temperature of e^\pm (assuming no matter-antimatter asymmetry).

4 Calculations in cosmology II

Consider a 100 MeV hypothetical scalar boson only coupled to the electron, with the Yukawa coupling $y = 10^{-12}$. Assuming its initial abundance in the early universe at a sufficiently high temperature is zero, compute its subsequent cosmological evolution.

5 Numerical skills

Consider the harmonic oscillator in quantum mechanics with the Hamiltonian $H = \frac{p^2}{2m} + \frac{1}{2} k x^2$. Now add a quartic term λx^4 , set $m = k = 1$ and $\lambda \in \{0.1, 1, 10\}$. Numerically solve the Schrödinger equation to find the energy levels and plot the wavefunctions.

6 Numerical and analytical skills

Compute the following Fourier transform

$$\int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p} \cdot \mathbf{x}}}{\mathbf{p}^2 \pm m^2} F(\mathbf{p}^2)$$

for $F(\mathbf{p}^2) = 1$, $\mathbf{p}^2 + m^2$, and $\exp[-(\mathbf{p}^2/m^2)^2]$. Here m is a real number.