Homework Assignment 1
Due date: Sep. 21 請印出來以紙本繳交
#請參考 Douglas C. Montgomery, (2012). Introduction to Linear Regression Analysis, 5th ed.

1. (#2.23) Consider the simple linear regression model $y = 50 + 10x + \epsilon$ where ϵ is NID(Normally and Independently Distributed)(0, 16). Suppose that $n = 20$ pairs of observations are used to fit this model.
Generate 500 samples of 20 observations, drawing one observation for each level of $x = 1, 1.5, 2, \ldots, 10$ for each sample.

a. For each sample compute the least - squares estimates of the slope and intercept. Construct histograms of the sample values of $\hat{\beta}_0$ and $\hat{\beta}_1$. Discuss the shape of these histograms.

b. For each sample, compute an estimate of $E(y|x = 5)$. Construct a histogram of the estimates you obtained. Discuss the shape of the histogram.

c. For each sample, compute a 95% CI on the slope. How many of these intervals contain the true value $\beta_1 = 10$? Is this what you would expect?

d. For each estimate of $E(y|x = 5)$ in part b, compute the 95% CI. How many of these intervals contain the true value of $E(y|x = 5) = 100$? Is this what you would expect?

2. (#2.25) Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, with $E(\varepsilon) = 0$ $\text{Var}(\varepsilon) = \sigma^2$, and ε uncorrelated.

a. Show that $\text{Cov}\left(\hat{\beta}_0, \hat{\beta}_1\right) = -\bar{x}\sigma^2/S_{xx}$

b. Show that $\text{Cov}\left(\bar{y}, \beta_1\right) = 0$

3. (#2.27) Suppose that we have fit the straight-line regression model $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1$ but the response is affected by a second variable x_2 such that the true regression function is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

a. Is the least-squares estimator of the slope in the original simple linear regression model unbiased?

b. Show the bias in $\hat{\beta}_1$

4. (#2.32) Consider the simple linear regression model

$$y = \beta_0 + \beta_1 x + \epsilon$$

where the intercept β_0 is known.

a. Find the least-squares estimator of β_1 for this model. Does this answer seem reasonable?

b. What is the variance of the slope $\left(\hat{\beta}_1\right)$ for the least-squares estimator found in part a?

c. Find a $100(1 - \alpha)$ percent Cl for β_1. Is this interval narrower than the estimator for the case where both slope and intercept are unknown?

5. (#2.33) Consider the least-squares residuals $e_i = y_i - \hat{y}_i, i = 1, 2, \ldots, n$, from the simple linear regression model. Find the variance of the residuals $\text{Var}\left(e_i\right)$. Is the variance of the residuals a constant? Discuss.