Homework Assignment 3
Due date: Oct. 19 請印出來以紙本繳交

1. Consider the following two models where $E(\varepsilon) = 0$ and $Var(\varepsilon) = \sigma^2 I$:

- Model A: $y = X_1\beta_1 + \varepsilon$
- Model B: $y = X_1\beta_1 + X_2\beta_2 + \varepsilon$

Show that $R_A^2 \le R_B^2$.

2. Suppose we need to compare the effects of two drugs each administered to n subjects. The model for the effect of the first drug is

$$y_{1i} = \beta_0 + \beta_1 x_{1i} + \varepsilon_{1i}$$

while for the second drug it is

$$y_{2i} = \beta_0 + \beta_{2i} x_{2i} + \varepsilon_{2i}$$

and in each case $i = 1, \ldots, n$ and $\bar{x}_1 = \bar{x}_2 = 0$. Assume that all observations are independent and that for each i both ε_{1i} and ε_{2i} are normally distributed with mean 0 and variance σ^2.

1. Obtain the least squares estimator for $\beta = (\beta_0, \beta_1, \beta_2)'$ and its covariance matrix.
2. Estimate σ^2.
3. Write the test statistic for testing $\beta_1 = \beta_2$ against the alternative that $\beta_1 \ne \beta_2$.

3. Consider the two models $y_1 = X_1\beta_1 + \varepsilon_1$ and $y_2 = X_2\beta_2 + \varepsilon_2$ where the $X_i's$ are $n_i \times p$ matrices. Suppose that $\varepsilon_i \sim N(0, \sigma_i^2 I)$ where $i = 1, 2$ and that ε_1 and ε_2 are independent.

1. Assuming that the $\sigma_i's$ are known, obtain a test for the hypothesis $\beta_1 = \beta_2$·
2. *Assume that $\sigma_1 = \sigma_2$ but they are unknown. Derive a test for the hypothesis $\beta_1 = \beta_2$.

4. Moore (1975) reported the results of an experiment to construct a model for total oxygen demand in dairy wastes as a function of five laboratory measurements (Data is attached in the mail). Data were collected on samples kept in suspension in water in a laboratory for 220 days. Although all observations reported here were taken on the same sample over time, assume that they are independent. The measured variables are:

y log(oxygen demand, mg oxygen per minute)
x_1 biological oxygen demand, mg/liter
x_2 total Kjeldahl nitrogen, mg/ liter
x_3 total solids, mg/ liter
x_4 total volatile solids, a component of x_3, mg/ liter
x_5 chemical oxygen demand, mg/liter

1. Fit a multiple regression model using y as the dependent variable and all x_j 's as the independent variables.
2. Now fit a regression model with only the independent variables x_3 and x_5. How do the new parameters, the corresponding value of R^2 and the t-values compare with those obtained from the full model?

5. Consider the data given in 4. Suppose the model is

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + \beta_5 x_{i5} + \varepsilon_i$$

where $i = 1, \ldots, n$ and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)' \sim N(0, \sigma^2 I_n)$.

1. Test the hypothesis $\beta_2 = \beta_4 = 0$ at the 5 per cent level of significance.
2. Find a 95 per cent C.I. for β_1,
3. Find a 95 per cent C.I. for $\beta_3 + 2\beta_5$.