Model for COVID 19

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1 The model for the spreading of COVID-19

pedestrian only have two status in our model, which are susceptible and infective. Susceptible pedestrians have the probability to be infected by infective pedestrians.

Figure 1 shows the process of infection. Pedestrians are represented by circles with radius r. There are three pedestrians in the figure. Pedestrian j and k are infective pedestrians represented by red circles. Pedestrian i is a susceptible pedestrian represented by a blue circle.

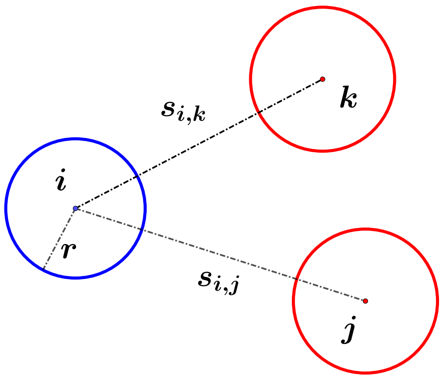


Figure 1: The process of infection

These three pedestrians move in a room. After t seconds, the probability that i is infected is calculated as following.

The first step is calculating A_i, which is the total amount of the virus contacted by pedestrian i during this period of time. We make two assumptions here. Firstly, the closer a susceptible pedestrian to an infective pedestrian, the more virus contacted by the susceptible pedestrian. Secondly, the virus contacted by a susceptible pedestrian accumulates as the increasing contact time with infective pedestrians. Therefore, A_i is defined as

$$A_i = \int_0^t f(t)\, dt, \tag{1}$$

where $f(t)$ denotes the amount of virus contacted by pedestrians i at time t.

The definition of $f(t)$ is

$$f(t) = \sum_{j \in J} \alpha_j \cdot P_j \cdot e^{(2 \cdot r - s_{i,j})/D}, \tag{2}$$

where J is the set contains all the infective pedestrians (in our case are pedestrian j and k). $s_{i,j}$ is the distance between the center of pedestrian i and j, which is changing with time. D is a parameter used for fitting the model with real data.

α_j is a value between 0 and 1. It represents the measures taken by pedestrian j (infective pedestrians). An infective pedestrian taking more effective measures has smaller α. If pedestrian j doesn't take any measures, $\alpha_j = 1$.

P_j, which is also between 0 and 1, is the probability that pedestrian j is releasing virus. According to [1] P_j is changing with the infected days of pedestrian j, which means the probability of releasing virus are different for different infective pedestrians. The curve of P_j should looks like Fig. 2, which is the probability density function of a normal distribution.

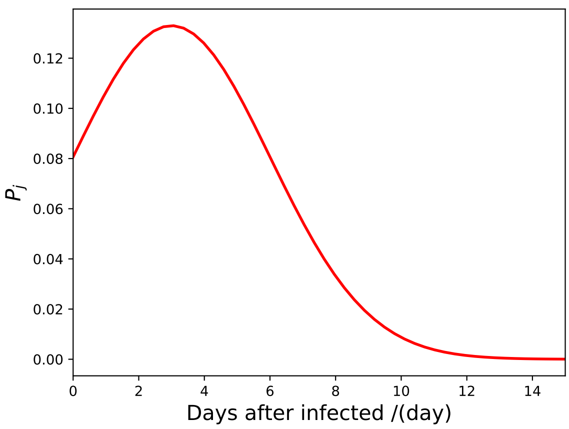


Figure 2: The curve of P_j.

The second step is calculating B_i, which is the probability of virus entering the body of pedestrian i. We assume that not all the virus contacted by a susceptible pedestrian can enter his body, and B_i is influenced by the amount of virus contacted by pedestrian i. Therefore the definition of B_i is

$$B_i = (1 - e^{-A_i \cdot \alpha_i \cdot K}), \tag{3}$$

where α_i represents the measures taken by pedestrian i. K is a parameter used for fitting the model with real data. The curve of B_i is shown in Fig. 3. With the increasing of A_i, B_i increases and gradually towards 1.

The last step is calculating I_i, which is the infective probability of pedestrian i. Even the virus enter the body of pedestrian i, the infective probability of pedestrian i is not hundred percent. Since the virus may be wiped out by the immune system of pedestrian i. Therefore the definition of I_i is

$$I_i = B_i \cdot Q_i, \tag{4}$$

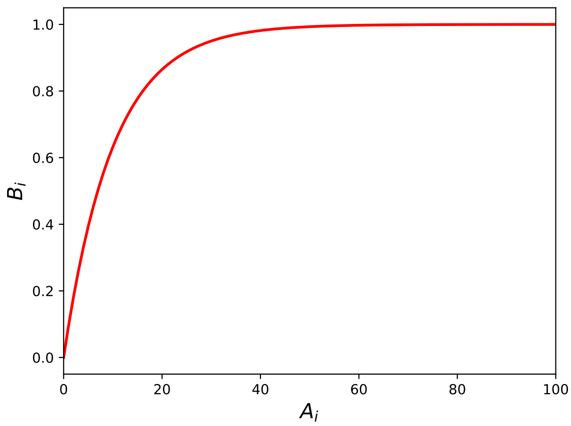
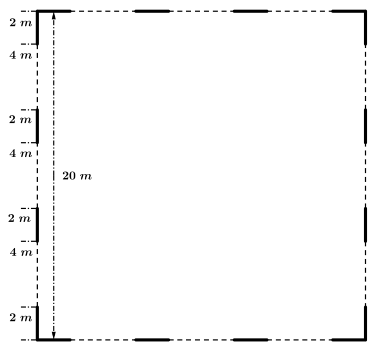


Figure 3: The curve of B_i.

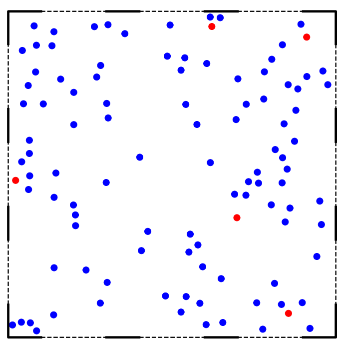
where Q_i is the probability that pedestrian i is infected after the virus enter his body. Q_i, which is between 0 to 1, depends on the human immune system and be influenced by the gender, age and other thing. Pedestrian with stronger immune system has smaller Q.

2 Simulation

The simulation scenario is a big square room as shown in Fig. 4(a).



(a)



(b)

Figure 4: Geometry for simulation. (a): The information of the room for simulation. (b): The distribution of pedestrians in the room.

The four sides of the room are exactly the same. The length of each side is 20 meters and there are three 4 meters exits on each side.

For each simulation, N susceptible pedestrians and M infective pedestrians are putted in the room. For example, in Fig. 4(b),5 infective pedestrians represented by red circles and 100 susceptible pedestrians represented by blue circles are distributed in the room.

The initial goal of each pedestrian is one of these exits. After arriving the exit, the pedestrian choose another exit randomly as next goal. Therefore, pedestrians move randomly in the room and contact with each other, but can not leave the room.

The mean value of A, B and I for all the susceptible pedestrians in the simulation are calculated then as $\bar{A}$, $\bar{B}$ and $\bar{I}$.

Since we don't have any data now, we using $D = 1$, $K = 1$, $P = 0.3$, $Q = 0.3$ in the following simulations.

In the first simulation, we investigate the influence of the number of the infective pedestrians at the beginning of the simulation. We set $N = 100$ and $M = 1, 2, 3, 4, 5$ in five runs, respectively. In these simulations we set $\alpha = 1$, which means no any measures taken by pedestrians. The simulation result is shown in Fig. 5.

With the increasing number of the infective pedestrians at the beginning of simulation, the mean probability of susceptible pedestrians been infected increases and tends to the maximum value Q.

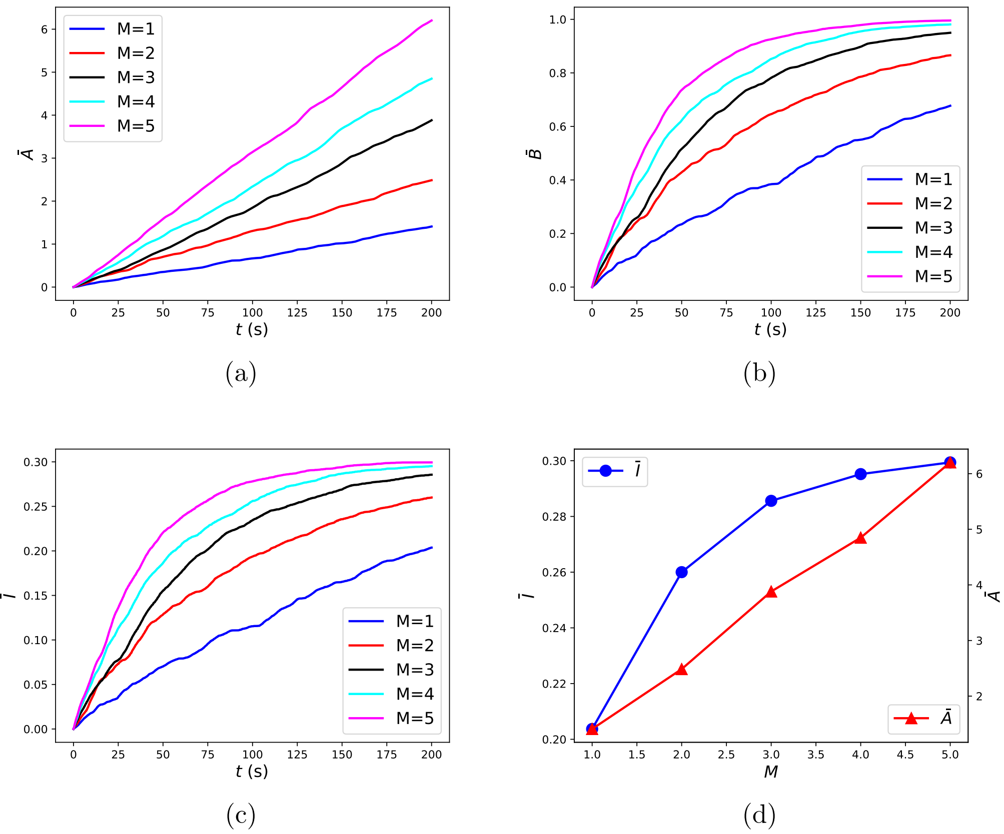
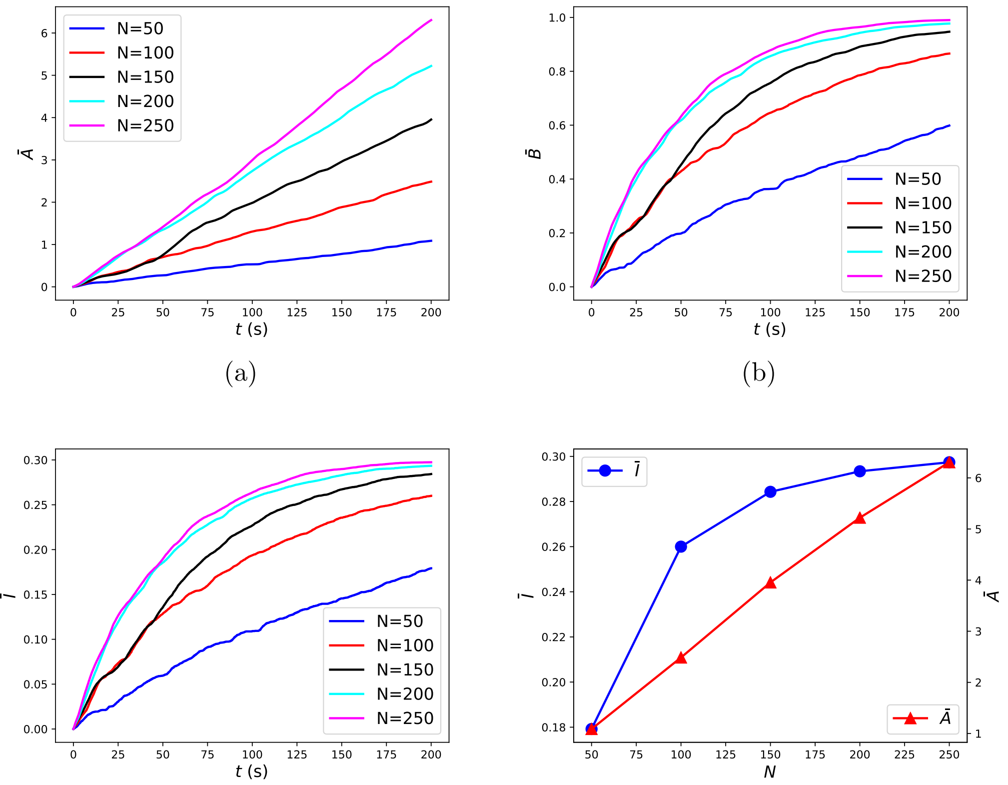


Figure 5: The simulation to investigate the influence of the number of infective pedestrians at the beginning of the simulation. (a): The trend of $\bar{A}$ over time. $\bar{A}$ is the mean amount of virus contacted by susceptible pedestrians. (b): The trend of $\bar{B}$ over time. $\bar{B}$ is the mean probability of virus entering the body of susceptible pedestrians. (c): The trend of $\bar{I}$ over time. $\bar{I}$ is the mean infected probability of susceptible pedestrians. (d): The trend of the value of $\bar{I}$ and $\bar{A}$ at the end of the simulation over the number of infective pedestrians at the beginning of the simulation.

In the second simulation, we investigate the influence of the pedestrians density in the room. We run five simulations with different number of pedestrian inside the room. In these five runs, we set $N = 50, 100, 150, 200, 250$ and $M = 2\% \cdot N$, respectively. In these simulation, we set $\alpha = 1$ as well. The simulation result is shown in Fig. 6.

With more pedestrians in the room, $\bar{I}$ increases and tends to the maximum value Q.



(a) (b)

(c) (d)

Figure 6: The simulation to investigate the influence of the pedestrians density in the room. (a): The trend of $\bar{A}$ over time. $\bar{A}$ is the mean amount of virus contacted by susceptible pedestrians. (b): The trend of $\bar{B}$ over time. $\bar{B}$ is the mean probability of virus entering the body of susceptible pedestrians. (c): The trend of $\bar{I}$ over time. $\bar{I}$ is the mean infected probability of susceptible pedestrians. (d): The trend of the value of $\bar{I}$ and $\bar{A}$ at the end of the simulation over the pedestrians density in the room.

In the third simulation, we investigate the influence of the protective measures taken by pedestrians. we set $N = 100$ and $M = 5$, and run fives simulations with $\alpha = 0.2, 0.4, 0.6, 0.8, 1.0$, respectively. The smaller the value of α, the more efficient the protective measure. The simulation result is shown in Fig. 7.

Smaller value of α results in lower $\bar{I}$.

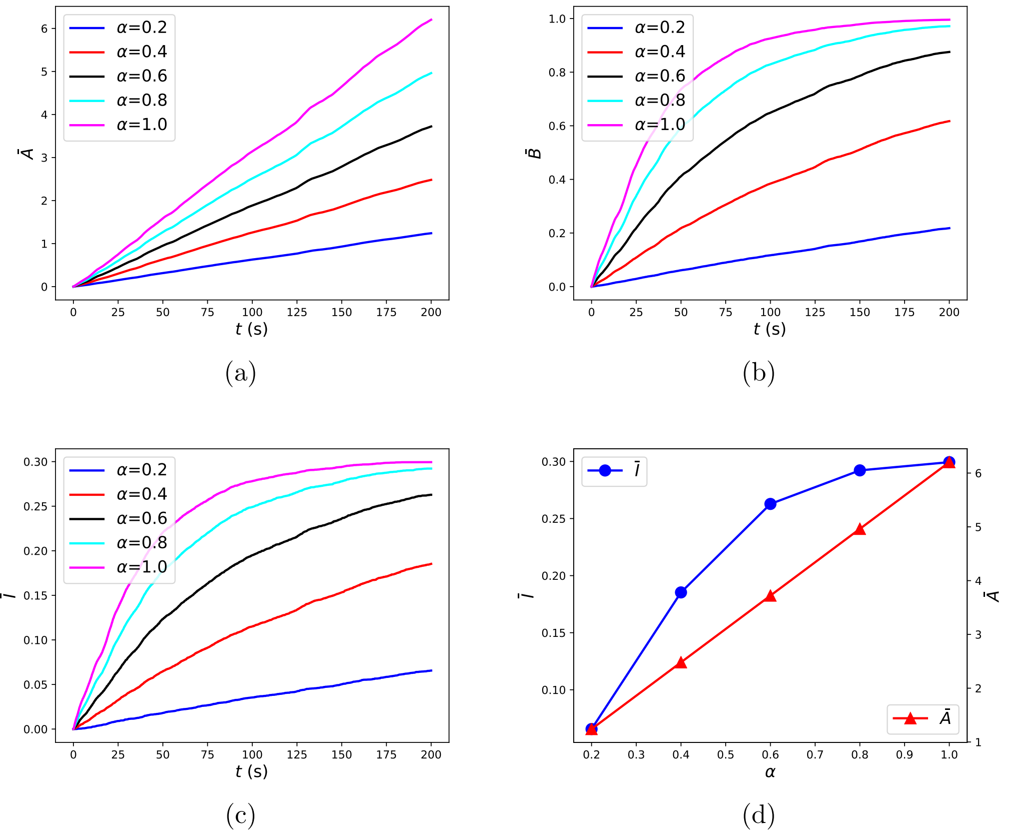


Figure 7: The simulation to investigate the influence of the protective measures taken by pedestrians. (a): The trend of $\bar{A}$ over time. $\bar{A}$ is the mean amount of virus contacted by susceptible pedestrians. (b): The trend of $\bar{B}$ over time. $\bar{B}$ is the mean probability of virus entering the body of susceptible pedestrians. (c): The trend of $\bar{I}$ over time. $\bar{I}$ is the mean infected probability of susceptible pedestrians. (d): The trend of the value of $\bar{I}$ and $\bar{A}$ at the end of the simulation over the value of α.

References

[1] P. Derjany, E. Riddle, S. Namilae, D. Liu, A. Srinivasan, Computational modeling framework for the study of infectious disease spread through commercial air-travel.