

# A Modal Symbolic Classifier for Interval Data

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**Abstract.** A modal symbolic classifier for interval data is presented. The proposed method needs a previous pre-processing step to transform interval symbolic data into modal symbolic data. The presented classifier has then as input a set of vectors of weights. In the learning step, each group is also described by a vector of weight distributions obtained through a generalization tool. The allocation step uses the squared Euclidean distance to compare two modal descriptions. To show the usefulness of this method, examples with synthetic symbolic data sets are considered.

## 1 Introduction

In many data analysis problems, the individuals are described by vectors of continuous-value data that are points. However, sometimes, these points to treat can not be quite localized and their positions are then imprecise. A solution is to define uncertainty zones around the imprecise points provided the acquisition system and their positions are then to estimate, for example the parameters of a regression or classification model. The concept of uncertainty zones data constitutes a generalization of interval-valued data which are quite natural in many application where they represent uncertainty on measurements (confidence interval for instance), variability (minimum and maximum temperatures during a day).

*Symbolic Data Analysis* (SDA) [2] is a new domain in the area of knowledge discovery and data management, related to multivariate analysis, pattern recognition and artificial intelligence. It aims to provide suitable methods (clustering, factorial techniques, decision tree, etc.) for managing aggregated data described through multi-valued variables, where there are sets of categories, intervals, or weight (probability) distributions in the cells of the data table (for more details about SDA, see [www.jsda.unina2.it](http://www.jsda.unina2.it)). A symbolic variable is defined according to its type of domain. For example, for an object, an interval variable takes an interval of  $\mathbb{R}$  (the set of real numbers). A symbolic modal takes, for a object, a non-negative measure (a frequency or a probability distribution or a system of weights). If this measure is specified in terms of a *histogram*, the modal variable is called *histogram variable*.

Several supervised classification tools has been extended to handle interval and modal data. Ichino et al. [6] introduced a symbolic classifier as a region

oriented approach for multi-valued data. In this approach, the classes of examples are described by a region (or set of regions) obtained through the use of an approximation of a *Mutual Neighbourhood Graph (MNG)* and a symbolic join operator. Souza et al. [11] proposed a *MNG* approximation to reduce the complexity of the learning step without losing the classifier performance in terms of prediction accuracy. D'Oliveira et al. [4] presented a region oriented approach in which each region is defined by the convex hull of the objects belonging to a class. Ciampi et al. [3] introduced a generalization of binary decision trees to predict the class membership of symbolic data. Prudencio et al. [9] proposed a supervised classification method from symbolic data for the model selection problem. Rossi and Conan-Guez [10] have generalized Multi-Perceptrons to work with interval data. Mali and Mitra [8] extended the fuzzy radial basis function (FRBF) network to work in the domain of symbolic data. Appice et al. [1] introduced a lazy-learning approach (labeled Symbolic Objects Nearest Neighbor SO-SNN) that extends a traditional distance weighted k-Nearest Neighbor classification algorithm to interval and modal data.

In this paper, we present a modal symbolic classifier for interval data. This method assumes a previous pre-processing step to transform interval data into modal data. In the learning step, each class of items is represented by a weight distribution obtained through a generalization tool. In the allocation step, the new items are classified using the squared Euclidean distance between modal data. Section 2 describes modal and interval symbolic data. Section 3 introduces the modal symbolic classifier based on weight distributions. Section 4 describes the evaluation experimental considering synthetic symbolic data sets. A comparative study involving the proposed classifier and the SO-SNN approach introduced by Appice et al [1] is presented. The evaluation of the performance of these classifiers is based on the accuracy prediction that is assessed in the framework of a Monte Carlo experience with 100 replications of each set. In Section 5, the concluding remarks are given.

## 2 Modal and Interval Symbolic Data

In classical data analysis, the items to be grouped are usually represented as a vector of quantitative or qualitative measurements where each column represents a variable. In particular, each individual takes just one single value for each variable. In practice, however, this model is too restrictive to represent complex data since to take into account variability and/or uncertainty inherent to the data, variables must assume sets of categories or intervals, possibly even with frequencies or weights.

Let  $C_k, k = 1, \dots, K$ , be a class of  $n_k$  items indexed by  $ki$  ( $i = 1, \dots, n_k$ ) with  $C_k \cap C_{k'} = \emptyset$  if  $k \neq k'$  and  $\cup_{k=1}^K C_k = \Omega$  a training set of size  $n = \sum_{k=1}^K n_k$ . Each item  $ki$  ( $i = 1, \dots, n_k$ ) is described by  $p$  symbolic variables  $X_1, \dots, X_p$ . A symbolic variable  $X_j$  is an interval variable when, given an item  $i$  of  $C_k$

( $k = 1, \dots, K$ ),  $X_j(ki) = x_{ki}^j = [a_{ki}^j, b_{ki}^j] \subseteq \mathcal{A}_j$  where  $\mathcal{A}_j = [a, b]$  is an interval. A symbolic variable  $X_j$  is a histogram modal variable if, given an item  $i$  of  $C_k$  ( $k = 1, \dots, K$ ),  $X_j(ki) = (S(ki), \mathbf{q}(ki))$  where  $\mathbf{q}(ki)$  is a vector of weights defined in  $S(ki)$  such that a weight  $w(m)$  corresponds to each category  $m \in S(ki)$ .  $S(ki)$  is the support of the measure  $\mathbf{q}(ki)$ .

*Example:* An individual may have as description for a symbolic variable the modal data vector  $d = (0.7[60, 65[, 0.3[65, 80])$  where the symbolic variable, which takes values in  $[60, 65[$  and  $[65, 80[$ , is represented by a histogram (or modal) variable, where 0.7 and 0.3 are relative frequencies of the two intervals of values.

### 3 A Symbolic Classifier

In this section, a modal symbolic classifier for interval data is presented. Two main steps are involved in the construction of this classifier.

1. *Learning step.* Construction of a symbolic modal description for each class of items:
  - (a) *Pre-processing:* Transformation of interval symbolic data into modal symbolic data (vector of weight distribution) for each item of the training set.
  - (b) *Generalization:* Using the pre-processed items to obtain a modal description for each class.
2. *Allocation step.* Assignment of a new item to a class according to the proximity between the modal description of this item and the modal description of a class.
  - (a) *Pre-processing:* Transforming new interval data into modal data.
  - (b) *Affectation:* Computing the dissimilarity between each class and a new item.

#### 3.1 Learning Step

This step aims to construct a modal symbolic description for each class synthesizing the information given by the items associated to this class.

Two steps constitute the learning process: pre-processing and generalization.

**Pre-processing.** In this paper we consider a data transformation approach which the aim is to obtain modal symbolic data from interval data. So, the presented symbolic classifier has as input data vectors of weight distributions.

The variable  $X_j$  is transformed into a modal symbolic variable  $\tilde{X}_j$  in the following way [5]:  $\tilde{X}_j(ki) = \tilde{x}_{ki}^j = (\tilde{\mathcal{A}}_j, \mathbf{q}^j(ki))$ , where  $\tilde{\mathcal{A}}_j = \{I_1^j, \dots, I_{H_j}^j\}$  is a set of elementary intervals,  $\mathbf{q}^j(ki) = (q_1^j(ki), \dots, q_{H_j}^j(ki))$  and  $q_h^j(ki)$  ( $h = 1, \dots, H_j$ ) is defined as:

$$q_h^j(ki) = \frac{l(I_h^j \cap x_{ki}^j)}{l(x_{ki}^j)} \quad (1)$$

$l(I)$  being the length of a closed interval  $I$ .

The bounds of these elementary intervals  $I_h^j$  ( $h = 1, \dots, H_j$ ) are obtained from the ordered bounds of the  $n + 1$  intervals  $\{x_{11}^j, \dots, x_{1n_1}^j, \dots, x_{k1}^j, \dots, x_{kn_k}^j, \dots, x_{K1}^j, \dots, x_{Kn_K}^j, [a, b]\}$ . They have the following properties:

1.  $\bigcup_{h=1}^{H_j} I_h^j = [a, b]$
2.  $I_h^j \cap I_{h'}^j = \emptyset$  if  $h \neq h'$
3.  $\forall h \exists ki \in \Omega$  such that  $I_h^j \cap x_{ki}^j \neq \emptyset$
4.  $\forall ki \exists S^j(ki) \subset \{1, \dots, H_j\} : \bigcup_{h \in S^j(ki)} I_h^j = x_{ki}^j$

Table 1 shows items of a training data set from two classes. Each item is described by an interval variable.

**Table 1.** Items described by a symbolic interval variable

Item	Interval Data ( $X_1$ )	Class
$e_1$	[10,30]	1
$e_2$	[25,35]	1
$e_3$	[90,130]	2
$e_4$	[125,140]	2

From the interval data describing the items, we create a set of elementary intervals  $\tilde{\mathcal{A}}_1 = \{I_1^1, \dots, I_{H_1}^1\}$  as follows: at first, we take the set of values formed by every bound (lower and upper) of all the intervals associated to the items. Then, such set of bounds is sorted in a growing way. This set of elementary intervals is:  $\tilde{\mathcal{A}}_1 = \{I_1^1, I_2^1, I_3^1, I_4^1, I_5^1, I_6^1, I_7^1\}$  where  $I_1^1 = [10, 25[$ ,  $I_2^1 = [25, 30[$ ,  $I_3^1 = [30, 35[$ ,  $I_4^1 = [35, 90[$ ,  $I_5^1 = [90, 125[$ ,  $I_6^1 = [125, 130[$  and  $I_7^1 = [130, 140]$ .

Using the transformation approach into modal data, we got the following modal data table:

**Table 2.** Items described by a modal symbolic variable

Item	Modal Data ( $\tilde{X}_1$ )	Class
$e_1$	$((0.75[10,25[), (0.25[25,30[), (0.0[30,35[), (0.0[35,90[), (0.0[90,125[), (0.0[125,130[), (0.0[130,140[))$	1
$e_2$	$((0.0[10,25[), (0.50[25,30[), (0.50[30,35[), (0.0[35,90[), (0.0[90,125[), (0.0[125,130[), (0.0[130,140[))$	1
$e_3$	$((0.0[10,25[), (0.0[25,30[), (0.0[30,35[), (0.0[35,90[), (0.88[90,125[), (0.12[125,130[), (0.0[130,140[))$	2
$e_4$	$((0.0[10,25[), (0.0[25,30[), (0.0[30,35[), (0.0[35,90[), (0.0[90,125[), (0.33[125,130[), (0.67[130,140[))$	2

**Generalization.** This step aims to represent each class as a modal symbolic example. The symbolic description of each class is a generalization of the modal symbolic description of its items.

Let  $C_k$  be a class of  $n_k$  items. Each item of  $C_k$  is represented as a vector of modal symbolic data. This class is also represented as a vector of modal symbolic data  $\tilde{\mathbf{g}}_k = (\tilde{g}_k^1, \dots, \tilde{g}_k^p)$ ,  $\tilde{g}_k^j = (\tilde{\mathcal{A}}_j, \mathbf{v}^j(k))$  ( $j = 1, \dots, p$ ), where  $\mathbf{v}^j(k) = (v_1^j(k), \dots, v_{H_j}^j(k))$  is a vector of weights. Notice that for each variable the modal symbolic data presents the same support  $\tilde{\mathcal{A}}_j = \{I_1^j, \dots, I_{H_j}^j\}$  for all individuals and prototypes.

The weight  $v_h^j(k)$  is computed as follows:

$$v_h^j(k) = \frac{1}{n_k} \sum_{i \in C_k} q_h^j(ki) \quad (2)$$

Table 3 shows the modal symbolic description for each class of the Table 2.

**Table 3.** Classes described as a modal symbolic description

Class	Modal Data ( $\tilde{X}_1$ )
1	$((0.375[10,25[), (0.375[25,30[), (0.25[30,35[), (0.0[35,90[)$ $(0.0[90,125[), (0.0[125,130[), (0.0[130,140[))$
2	$((0.0[10,25[), (0.0[25,30[), (0.0[30,35[), (0.0[35,90[)$ $(0.44[90,125[), (0.225[125,130[), (0.335[130,140[))$

### 3.2 Allocation Step

The allocation of a new item to a group is based on a dissimilarity function, which compares the modal description of the new item and the modal description of a class. Two steps also constitute the allocation process.

**Pre-processing.** Let  $\mathbf{x}_\omega = (x_\omega^1 = [a_\omega^1, b_\omega^1], \dots, x_\omega^p = [a_\omega^p, b_\omega^p])$  be the interval description of a item to be classified  $\omega$ . The aim of this step is to transform the interval description of this item into a modal symbolic description.

Here, this is achieved through the following steps:

1. Update the bounds of the set of elementary intervals  $\tilde{\mathcal{A}}_j = \{I_1^j, \dots, I_{H_j}^j\}$  considering the bounds of the interval  $[a_\omega^j, b_\omega^j]$  to create the new elementary intervals  $\tilde{\mathcal{A}}_j^* = \{I_1^{*j}, \dots, I_{H_j^*}^{*j}\}$ .
2. Compute the vector of weights  $\mathbf{q}^j(\omega) = (q_1^j(\omega), \dots, q_{H_j^*}^j(\omega))$  from the new set of elementary intervals  $I_t^{*j}$  ( $t = 1, \dots, H_j^*$ ) as follow:

$$q_t^j(\omega) = \frac{l(I_t^{*j} \cap x_\omega^j)}{l(x_\omega^j)} \quad (3)$$

3. Update the vector of weights  $\mathbf{v}^j(k) = (v_1^j(k), \dots, v_{H_j}^j(k))$  ( $k = 1, \dots, K$ ) of  $C_k$  from the new set of elementary intervals  $I_t^{*j}$  ( $t = 1, \dots, H_j^*$ ) as follow:

$$v_t^j(k) = v_h^j(k) * \frac{l(I_h^j \cap I_t^{*j})}{l(I_h^j)} \quad (4)$$

for  $h \in \{1, \dots, H_j\} / I_h^j \cap I_t^{*j} \neq \emptyset$ . Otherwise,  $v_t^j(k) = 0$ .

**Affectation step.** Let  $\omega$  be a new item, which is candidate to be assigned to a class  $C_k$  ( $k = 1, \dots, K$ ), and its corresponding modal description for the variable  $j$  ( $j = 1, \dots, p$ ) is:  $\tilde{x}_\omega^j = (\tilde{\mathcal{A}}_j^*, \mathbf{q}^j(\omega))$ . Let  $\tilde{\mathbf{g}}_k^j = (\tilde{\mathcal{A}}_j^*, \mathbf{v}^j(k))$  be the corresponding modal description of  $C_k$  for the variable  $j$  ( $j = 1, \dots, p$ ).

Here, the comparison between two vectors of cumulative weights  $\mathbf{q}^j(\omega)$  and  $\mathbf{v}^j(k)$  for the variable  $j$  is accomplished by a suitable squared Euclidean distance:

$$d^2(\mathbf{q}^j(\omega), \mathbf{v}^j(k)) = \sum_{h=1}^{H_j^*} (q_h^j(\omega) - v_h^j(k))^2 \quad (5)$$

The *classification rule* is defined as follow:  $\omega$  is affected to the class  $C_k$  if

$$\phi(\omega, C_k) \leq \phi_1(\omega, C_m), \forall m \in \{1, \dots, K\} \quad (6)$$

where

$$\phi(\omega, C_k) = \sum_{j=1}^p d^2(\mathbf{q}^j(\omega), \mathbf{v}^j(k)) \quad (7)$$

*Example:* Let  $\omega$  be a new item with the description [8, 28] for an interval variable. Considering the modal description of the classes 1 and 2 of the Table 3, we have  $\phi(\omega, C_1) = 0.2145$  and  $\phi(\omega, C_2) = 1.3554$ . Therefore, this new item  $\omega$  will be affected to the class  $C_1$ .

## 4 Experimental Evaluation

In order to show the usefulness of the proposed symbolic classifier, this section presents an experimental evaluation based on prediction accuracy with two synthetic interval data sets. Our aim is to compare the modal symbolic classifier presented in this paper with the Symbolic Objects Nearest Neighbor (SO-NN) method introduced by Appice et al. [1] based on an extension of the traditional weighted  $k$ -Nearest Neighbor classifier ( $k$ -NN) to modal and interval symbolic data.

Like the traditional classifier ( $k$ -NN), the SO-NN classifier also requires only a dissimilarity measure and a positive integer  $k$  to define the number of the neighborhood used to classifier a new item of the test set. So, Appice et al. [1]

investigated the performance of the SO-NN classifier using different dissimilarity measures for symbolic data and selecting the optimal  $k$  from the interval  $[1, \sqrt{n}]$ . Moreover, for modal data, they used the KT-estimate [7] to estimate the probability (or weight) distribution of modal variables when the distribution has a zero-valued probability for some categories.

In this evaluation, the accuracy of the SO-NN classifier will be performed by using the squared Euclidean distance of the equation (7) and the following values to determine the neighborhood of a test item:  $k = 5$ ,  $k = 10$  and  $k = 15$ .

#### 4.1 Synthetic Interval Data Sets

In each experiment, we considered two standard quantitative data sets in  $\mathbb{R}^2$ . Each data set has 250 points scattered among three classes of unequal sizes: two classes with ellipse shapes and sizes 70 and 80 and one class with spherical shape of size 100. Each class in these quantitative data sets were drawn according to a bi-variate normal distribution with vector  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}$  represented by:

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \text{ and } \boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

We will consider two different configurations for the standard quantitative data sets: 1) data drawn according to a bi-variate normal distribution with well separated classes and 2) data drawn according to a bi-variate normal distribution with overlapping classes.

Each data point  $(z_1, z_2)$  of each one of these synthetic quantitative data sets is a seed of a vector of intervals (rectangle):  $([z_1 - \gamma_1/2, z_1 + \gamma_1/2], [z_2 - \gamma_2/2, z_2 + \gamma_2/2])$ . These parameters  $\gamma_1, \gamma_2$  are randomly selected from the same predefined interval. The intervals considered in this paper are:  $[1, 10]$ ,  $[1, 20]$ ,  $[1, 30]$ ,  $[1, 40]$  and  $[1, 50]$ .

Standard data set 1 was drawn according to the following parameters (configuration 1):

- a) Class 1:  $\mu_1 = 17$ ,  $\mu_2 = 34$ ,  $\sigma_1^2 = 36$ ,  $\sigma_2^2 = 64$  and  $\rho_{12} = 0.85$ ;
- b) Class 2:  $\mu_1 = 37$ ,  $\mu_2 = 59$ ,  $\sigma_1^2 = 25$ ,  $\sigma_2^2 = 25$  and  $\rho_{12} = 0.0$ ;
- c) Class 3:  $\mu_1 = 61$ ,  $\mu_2 = 31$ ,  $\sigma_1^2 = 49$ ,  $\sigma_2^2 = 100$  and  $\rho_{12} = -0.85$ ;

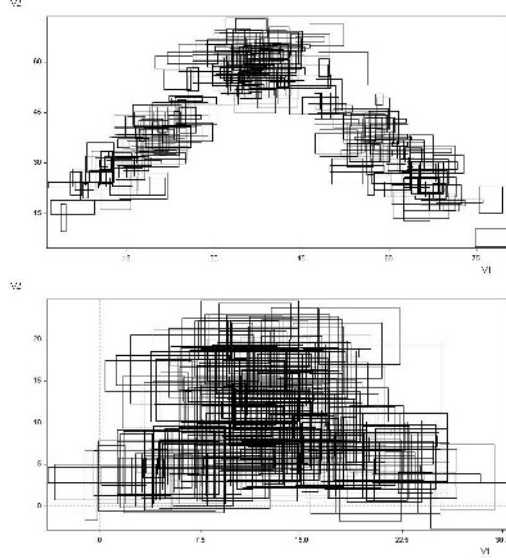
Standard data set 2 was drawn according to the following parameters (configuration 2):

- a) Class 1:  $\mu_1 = 8$ ,  $\mu_2 = 5$ ,  $\sigma_1^2 = 16$ ,  $\sigma_2^2 = 1$  and  $\rho_{12} = 0.85$ ;
- b) Class 2:  $\mu_1 = 12$ ,  $\mu_2 = 15$ ,  $\sigma_1^2 = 9$ ,  $\sigma_2^2 = 9$  and  $\rho_{12} = 0.0$ ;
- c) Class 3:  $\mu_1 = 18$ ,  $\mu_2 = 7$ ,  $\sigma_1^2 = 16$ ,  $\sigma_2^2 = 9$  and  $\rho_{12} = -0.85$ ;

From these configurations of standard data sets and the predefined intervals for the parameters  $\gamma_1$  and  $\gamma_2$ , interval data sets are obtained considering two

different cases of classification with overlapping classes: 1) symbolic interval data set 1 shows a moderate case of classification with class overlapping along one interval variable and 2) symbolic interval data set 2 shows a difficult case of classification with overlapping classes along two interval variables.

Figure 1 illustrates the interval data sets 1 and 2 with parameters  $\gamma_1$  and  $\gamma_2$  randomly selected from the interval  $[1, 10]$ .



**Fig. 1.** Symbolic interval data sets 1 (at the top) and 2 (at the bottom)

## 4.2 Performance Analysis

The evaluation of these clustering methods was performed in the framework of a Monte Carlo experience: 200 replications (100 for training set and 100 for test set) are considered for each interval data set, as well as for each predefined interval. The prediction accuracy of the classifier was measured through the error rate of classification obtained from a test set. The estimated error rate of classification corresponds to the average of the error rates found between the 100 replications of the test set.

Table 1 and 2 show the values of the average and standard deviation of the error rate for the modal and SO-NN classifiers and interval data configurations 1 and 2, respectively.

For both types of interval data configurations, the average error rates for the modal classifier are less than those for the SO-NN classifier in all situations. As it is expected, the average error rates for both classifiers increase as long as the widest intervals are considered. These results show clearly that the modal classifier outperforms the SO-NN classifier.



**Table 4.** The average (%) and the standard deviation (in parenthesis) of the error rate for interval data set 1

Range of values	Modal Classifier	SO-NN Classifier		
		$k = 5$	$k = 10$	$k = 15$
[1, 10]	2.32 (0.0104)	10.90 (0.0643)	20.74 (0.0496)	31.43 (0.0296)
[1, 20]	2.20 (0.0101)	7.78 (0.0276)	13.48 (0.0483)	19.64 (0.0690)
[1, 30]	2.50 (0.0105)	8.27 (0.0267)	13.41 (0.0410)	18.56 (0.0545)
[1, 40]	2.78 (0.0113)	8.71 (0.0277)	14.55 (0.0443)	20.36 (0.0614)
[1, 50]	2.89 (0.0105)	9.65 (0.0279)	16.21 (0.0446)	22.38 (0.0569)

**Table 5.** The average (%) and the standard deviation (in parenthesis) of the error rate for interval data set 2

Range of values	Modal Classifier	SO-NN Classifier		
		$k = 5$	$k = 10$	$k = 15$
[1, 10]	9.78 (0.0184)	20.40 (0.0294)	29.48 (0.0456)	38.25 (0.0537)
[1, 20]	10.52 (0.0187)	25.35 (0.0373)	36.65 (0.0484)	45.76 (0.0514)
[1, 30]	12.68 (0.0223)	30.80 (0.0396)	44.07 (0.0467)	53.95 (0.0462)
[1, 40]	15.34 (0.0269)	33.74 (0.0407)	48.06 (0.0418)	58.32 (0.0365)
[1, 50]	18.22 (0.0322)	37.76 (0.0262)	52.52 (0.0331)	61.46 (0.0324)

## 5 Concluding Remarks

In this paper, a modal symbolic classifier for interval data was introduced. The proposed method needs a previous pre-processing step to transform interval symbolic data into modal symbolic data. The presented classifier has then as input a set of vectors of weights. In the learning step, each class of items is also described by a vector of weight distributions obtained through a generalization tool. The allocation step uses the squared Euclidean distance to compare the modal description of a class with the modal description of a item.

Experiments with synthetic interval data sets illustrated the usefulness of this classifier. The accuracy of the results is assessed by the error rate of classification under situations ranging from moderate to difficult cases of classification in the framework of a Monte Carlo experience. Moreover, the modal classifier is compared with the lazy SO-NN classifier for symbolic data proposed by Appice et al.

[1]. Results showed that the modal classifier proposed in this paper is superior to the SO-NN classifier in terms of error rate of classification.

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