Interval Counterexamples for Loop Invariant Learning

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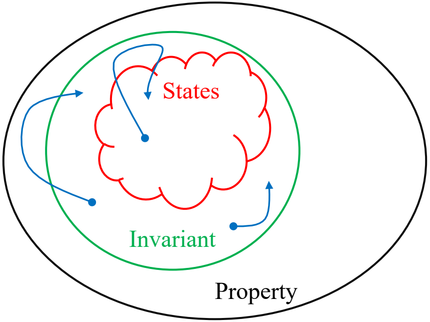
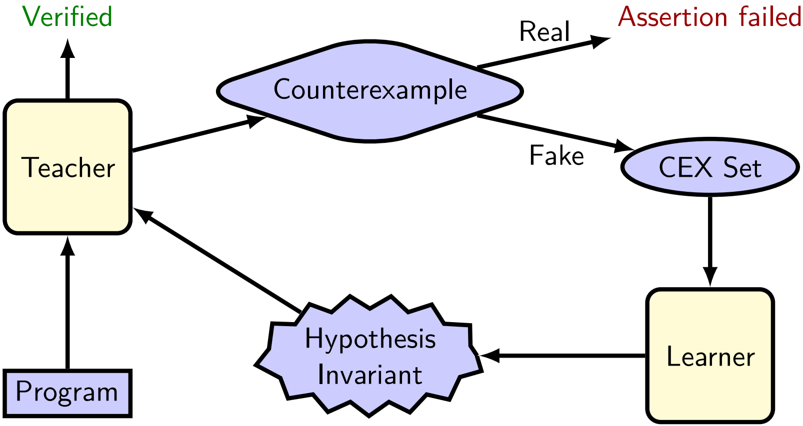
1. Background
2. Motivation
3. Approach
4. Evaluation
5. Conclusion

Loop invariants

- The loop invariant is a property that is always maintained when the program enters, iterates, and exits the loop.

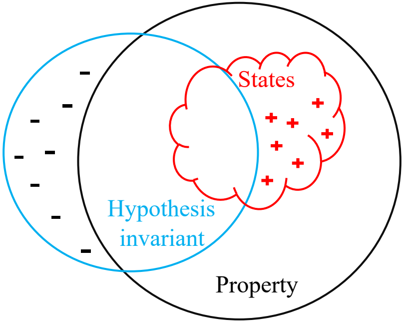


Black-box loop invariant inference



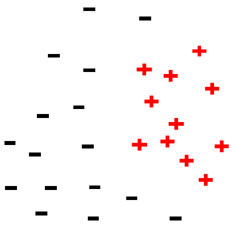
Positive and negative counterexamples

- Positive counterexample weakens the hypothesis invariant.
- Negative counterexample strengthens the hypothesis invariant.



Decision tree learner

- We use decision tree to split all positive/negative examples and use the predicate of the positive as hypothesis invariant.



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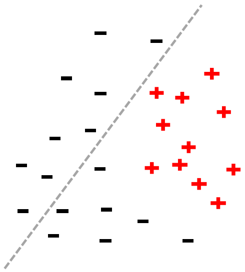

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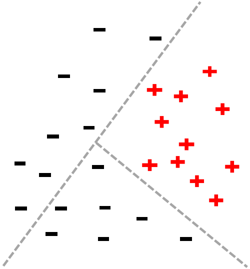

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Motivation

```
int x, y;
assume(x <= 2);
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while(*){
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assert(y >= 0 && y <= 3);
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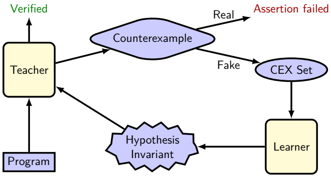

- Invariant hypothesis: *True*.

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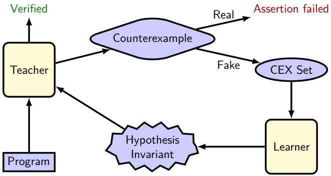

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$$\langle x = 0, y = 9 \rangle$$

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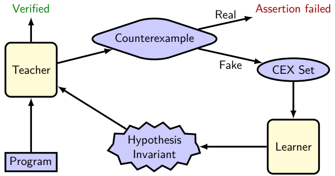

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- Current cex set:

x	0
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Label	Neg

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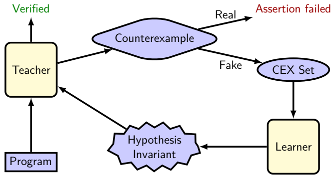

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- Modified hypothesis: *False*.

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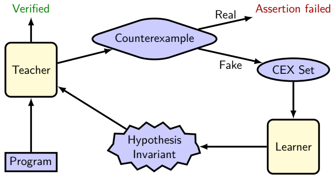

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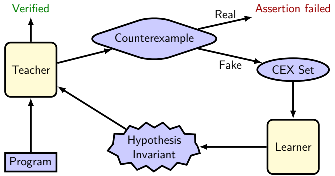

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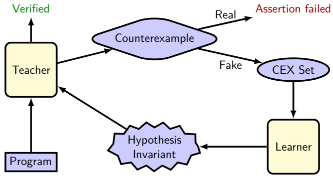

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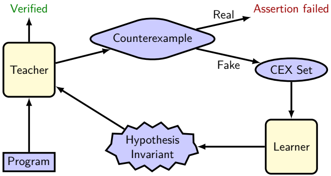

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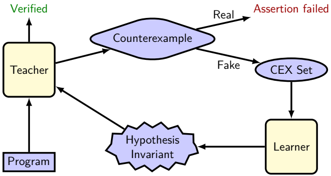

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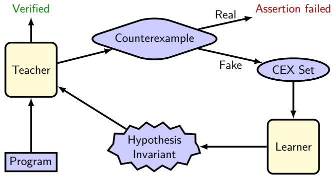

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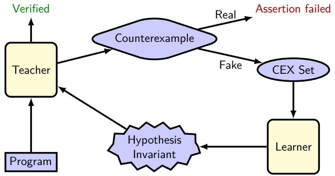

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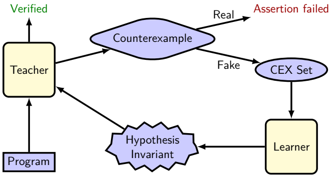

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Label	Neg	Pos	Neg

- Modified hypothesis: $y \leq 7$.

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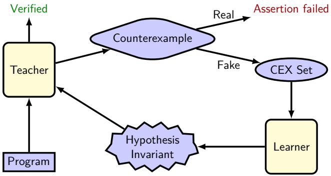

- Invariant hypothesis: $y \leq 7$
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- Current cex set:

x	0	2	0	0
y	9	1	8	7
Label	Neg	Pos	Neg	Neg

- Modified hypothesis: $y \leq 6$

Potential improvement

x	0	2	0	0	$\cdots$	0
y	9	1	8	7	$\cdots$	4
Label	Neg	Pos	Neg	Neg	$\cdots$	Neg

- A original counterexample is an *information-limited* single program state.

Potential improvement

x	0	2	0	0	$\cdots$	0
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- A original counterexample is an *information-limited* single program state.
- The generalized counterexamples we call them **interval counterexamples** look like:

$$\langle x \in (-\infty, \infty), y \in [4, \infty) \rangle$$

is a **set** of program states but not a single.

Core issues

- How to get a generalized interval counterexample?
- How to learn an invariant using interval counterexamples?

Generalization

- Suppose current hypothesis invariant by Learner: *True*.

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int x, y;
assume(x <= 2);
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while(*){ // Inv: True
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Generalization

- Suppose current hypothesis invariant by Learner: *True*.
- The verification condition to prove the assertion via the loop invariant: $True \rightarrow y \geq 0 \wedge y \leq 3$. (Simplify to $y \geq 0 \wedge y \leq 3$)

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Generalization by variable elimination

- Suppose the negative counterexample provided by Teacher:

$$x = 0 \wedge y = 9$$

Generalization by variable elimination

- Suppose the negative counterexample provided by Teacher:

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- Consider the constraint:

$$\underbrace{y \geq 0 \land y \leq 3}_{\text{(VC)}} \land x = 0 \land y = 9$$

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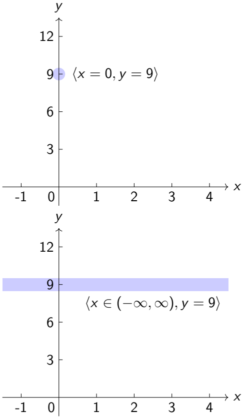

Generalization by boundary constraints

- Variable elimination: Rough and not widely general.

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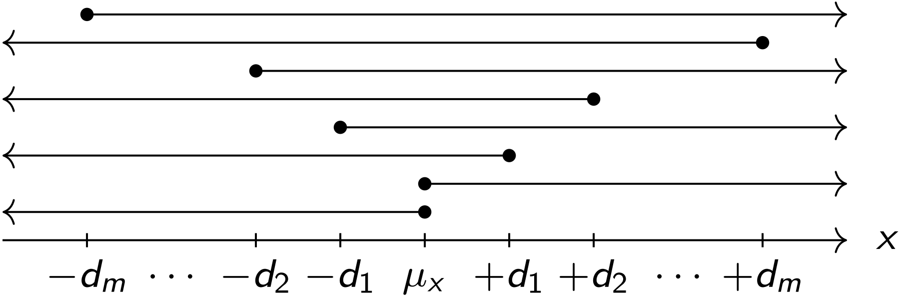
- Intuition: Find the generalization boundaries among some pre-defined distances.



Generalization by boundary constraints

- Consider distances (0,5) and new constraints to replace $x = 0$ and $y = 9$:

$$\underset{(VC)}{y \geq 0 \wedge y \leq 3} \wedge \bigwedge \begin{Bmatrix} x \geq -5 \\ x \geq 0 \\ x \leq 0 \\ x \leq 5 \end{Bmatrix} \wedge \bigwedge \begin{Bmatrix} y \geq 4 \\ y \geq 9 \\ y \leq 9 \\ y \leq 14 \end{Bmatrix}$$

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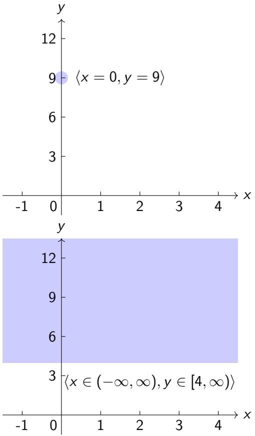

Generalization by interval digging

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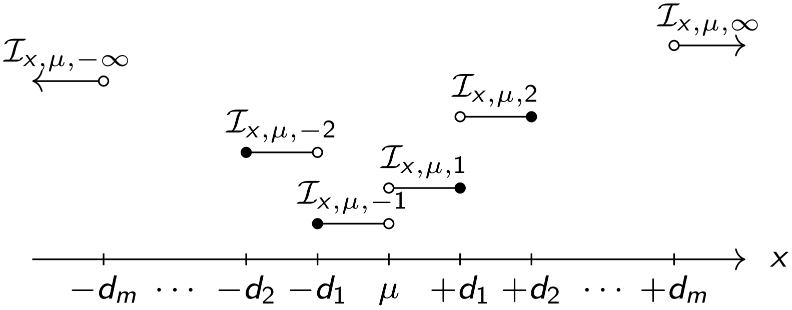
- UNSAT core may not the best range. ($y \geq 4$ or $y \geq 9$?)
- Intuition: Dig out all intervals which contains program states can **not** be generalized.



Generalization by interval digging

- Rearrange the constraints:

$$\underset{\text{(VC)}}{y \geq 0 \land y \leq 3} \land \bigwedge \left\{ \begin{array}{c} \neg(x > 5) \\ \neg(x > 0 \land x \leq 5) \\ \neg(x < 0 \land x \geq -5) \\ \neg(x < -5) \end{array} \right\} \land \bigwedge \left\{ \begin{array}{c} \neg(y > 14) \\ \neg(y > 9 \land y \leq 14) \\ \neg(y < 9 \land y \geq 4) \\ \neg(y < 4) \end{array} \right\}$$

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- Now the UNSAT core is **unique**: $\neg(y < 4)$.

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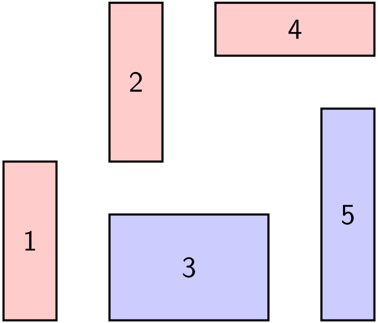
- Now the UNSAT core is **unique**: $\neg(y < 4)$.
- The counterexample is $\langle x \in (-\infty, \infty), y \in [4, \infty) \rangle$.

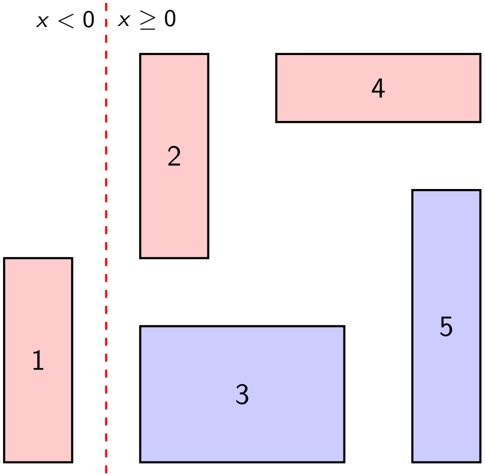
How to split interval examples?

- Avoid crossing examples first
- But also allow examples crossing

	x	y	Label
1	[-2,-1]	[-2, 1]	+
2	[0,1]	[1,4]	+
3	[0,3]	[-2,0]	-
4	[2,5]	[3,4]	+
5	[4,5]	[-2,2]	-



Interval decision tree



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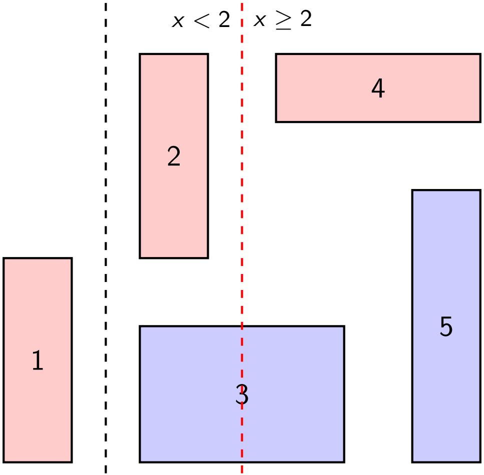

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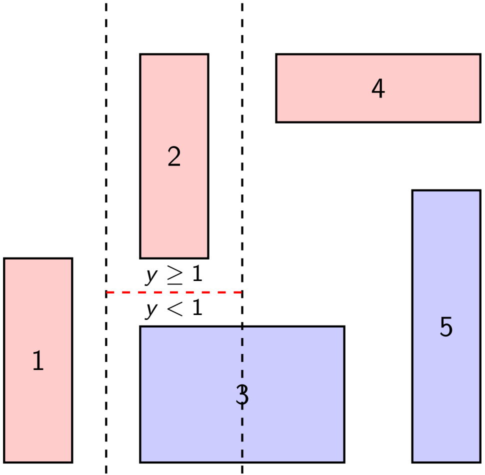

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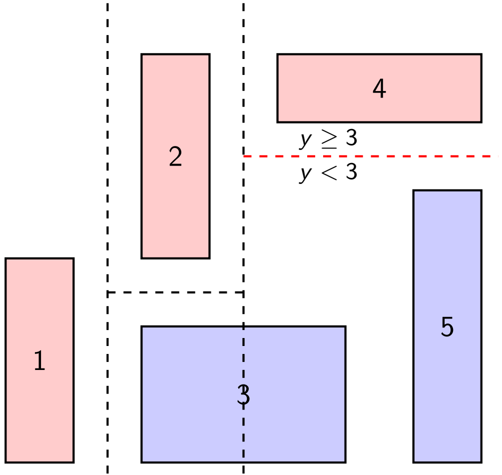

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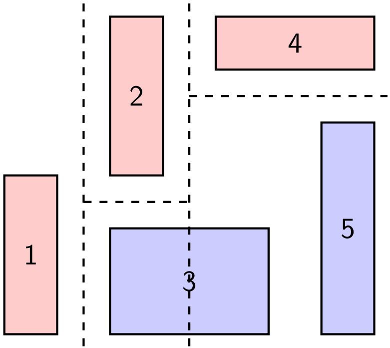

- Predicted invariant:

$$x < 0 \vee (x \geq 0 \wedge x < 2 \wedge y \geq 1) \vee (x \geq 0 \wedge x \geq 2 \wedge y \geq 3)$$

Experiment on generalization methods

- We implement prototype[1] of three generalization methods and also an interval counterexample supported decision tree.

[1] Available at: https://doi.org/10.5281/zenodo.3898483.

[2] Pranav Garg et al. Learning Invariants Using Decision Trees and Implication Counterexamples. POPL 2016

[3] Available at: https://github.com/sosy-lab/sv-benchmarks/

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- Total 94 benchmarks are from ICE-DT[2] and SV-COMP 2019[3].

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	Solved	R	$T(s)$	$T_C(s)$	$T_L(s)$
ICE-DT	90	3007	135.79	20.95	97.51
Elim	91	1916(36.3%)	87.47(35.6%)	17.48(16.6%)	52.46(46.2%)
Boundary	91	1717(42.9%)	72.27(46.8%)	15.15(27.7%)	42.41(56.5%)
Digging	91	1712(43.1%)	73.41(45.9%)	15.87(24.2%)	42.55(56.4%)

Table 1: Summary results on interval generalization methods

[1] Available at: https://doi.org/10.5281/zenodo.3898483.

[2] Pranav Garg et al. Learning Invariants Using Decision Trees and Implication Counterexamples. POPL 2016

[3] Available at: https://github.com/sosy-lab/sv-benchmarks/

Experiment on learning methods

- How much efficiency improvement comes from the interval decision tree?

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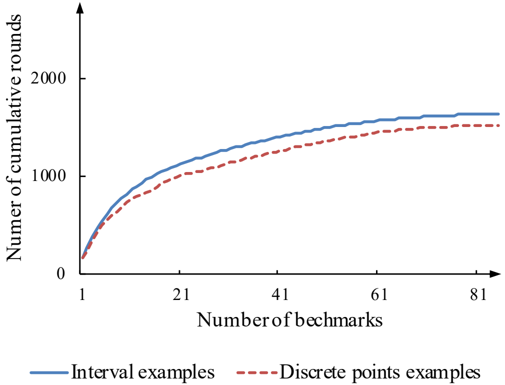
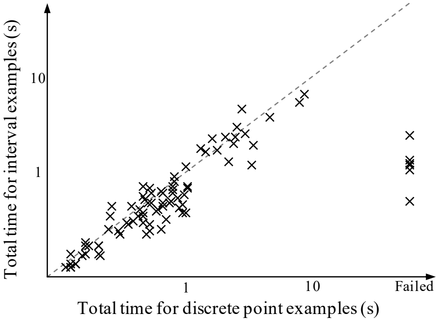
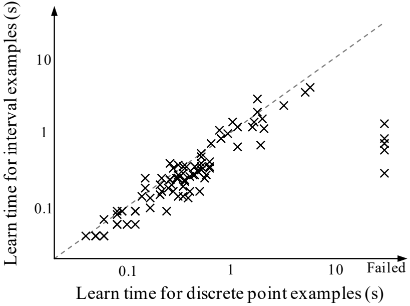


Experiment on learning methods

- Iteration rounds are similar, but time?



(a) Total time comparison



(b) Learn time comparison

1. Background
2. Motivation
3. Approach
4. Evaluation
5. Conclusion

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- Three different generalization techniques to compute interval counterexamples.
- An improved decision tree algorithm to adapt interval counterexamples.
- A prototype and over 40% improvement on learning rounds and verification time compared with existing methods.

Thanks!