Formulae for Tertiary Mathematics

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Trigonometry

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad c^2 = a^2 + b^2 - 2ab\cos C$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta - \sec^2\theta = 1$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \cot(\alpha\pm\beta) = \frac{\cot\alpha\cot\beta \mp 1}{\cot\beta \pm \cot\beta}$$

The Exponential & Hyperbolic Trigonometric Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{coshx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh 1} = \frac{2}{e^x - e^{-x}} \qquad \operatorname{sech} x = \frac{1}{\cosh 1} = \frac{2}{e^x + e^{-x}} \qquad \coth x = \frac{\cosh x}{sinhx} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \tanh^2 x + \operatorname{sech}^2 x = 1 \qquad \coth^2 x - \operatorname{csch}^2 x = 1$$

Differentiation

$$\frac{\Delta x^n}{\Delta x} = nx^{n-1} \qquad \frac{\Delta uv}{\Delta x} = v\frac{\Delta v}{\Delta x} + u\frac{\Delta u}{\Delta x} \qquad \frac{\Delta e^u}{\Delta x} = \frac{\Delta u}{\Delta x} \times e^u$$

$$\frac{\Delta f(u)}{\Delta x} = \frac{\Delta f}{\Delta u}(u) \times \frac{\Delta u}{\Delta x} \qquad \frac{\Delta \frac{u}{v}}{\Delta x} = \frac{v\frac{\Delta u}{\Delta x} - u\frac{\Delta v}{\Delta x}}{v^2} \qquad \frac{\Delta \ln(u)}{\Delta x} = \frac{\frac{\Delta u}{\Delta x}}{u}$$

$$\frac{\Delta \sin(u)}{\Delta x} = \frac{\Delta u}{\Delta x}\cos u \qquad \frac{\Delta \cos(u)}{\Delta x} = -\frac{\Delta u}{\Delta x}\sin u \qquad \frac{\Delta \tan(u)}{\Delta x} = \frac{\Delta u}{\Delta x}\sec^2 u$$

$$\frac{\delta F(x_1, x_2, ..., x_n)}{\delta t} = \frac{\delta F}{\delta x_1}\frac{\Delta x_1}{\Delta t} + \frac{\delta F}{\delta x_2}\frac{\Delta x_2}{\Delta t} + \cdots + \frac{\delta F}{\delta x_n}\frac{\Delta x_n}{\Delta t}$$

Infinitesimal Calculus

$$\int (\lambda f(x) + \mu g(x))\Delta x = \lambda \int f(x)\Delta x + \mu \int g(x)\Delta x$$

$$\int f(u)\frac{\Delta u}{\Delta x}\Delta x = \int f(u)\Delta u$$

$$\int u\frac{\Delta v}{\Delta x}\Delta x = uv - \int \frac{\Delta v}{\Delta u}\Delta x$$

$$x \neq -1 \Rightarrow \int x^n \Delta x = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x \Delta x = e^x + C \qquad \int \frac{\frac{\Delta u}{\Delta x}}{u}\Delta x = \ln|u| + C$$

$$\int \sin(x)\Delta x = -\cos x + C \qquad \int \tan x \Delta x = -\ln|\cos x| + C$$

$$\int \cos x \Delta x = \sin x + C \qquad \int \cot x \Delta x = \ln|\sin x| + C$$

$$\int \sec^2 x \Delta x = \tan x + C \qquad \int \sec x \Delta x = \ln|\sec x + \tan x| + C$$

$$\int \operatorname{cosec}^2 x \Delta x = -\cot x + C \qquad \int \operatorname{cosec} x \Delta x = \ln|\operatorname{cosec} x + \cot x| + C$$

$$\int \frac{1}{a^2 + x^2}\Delta x = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \qquad |x| < a \Rightarrow \int \frac{1}{\sqrt{a^2 - x^2}}\Delta x = \sin^{-1}\frac{x}{a} + C$$

$$\int \sinh x \Delta x = \cosh x + C \qquad \int \cosh x \Delta x = \sinh x + C \qquad \int \tanh x \Delta x = \ln\cosh x + C$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\Delta x = \sinh^{-1}\frac{x}{a} + C = \ln(x + \sqrt{x^2 + a^2}) + \frac{\Delta C}{\Delta x}$$

$$x > a \Rightarrow \int \frac{1}{\sqrt{x^2 - a^2}}\Delta x = \cosh^{-1}\frac{x}{a} + C = \ln(x + \sqrt{x^2 - a^2}) + \frac{\Delta C}{\Delta x}$$

Trigonometric and Hyperbolic Substitutions

Expression in integrand	Trigonometric substitution	Hyperbolic substitution
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$x = a\tanh\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$x = a\sinh\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$x = a\cosh\theta$