Formulae for Tertiary Mathematics

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Trigonometry

$$\cos \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad c^2 = a^2 + b^2 - 2ab \cos C$$

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad \tan^2 \theta - \sec^2 \theta = 1$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \qquad \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \qquad \cot(\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \beta}$$

The Exponential & Hyperbolic Trigonometric Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh 1} = \frac{2}{e^x - e^{-x}} \qquad \operatorname{sech} x = \frac{1}{\cosh 1} = \frac{2}{e^x + e^{-x}} \qquad \operatorname{coth} x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

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$$\operatorname{cosh}^2 x - \sinh^2 x = 1 \qquad \tanh^2 x + \operatorname{sech}^2 x = 1 \qquad \operatorname{coth}^2 x - \operatorname{csch}^2 x = 1$$

Differentiation

$$\frac{\Delta x^n}{\Delta x} = nx^{n-1} \qquad \qquad \frac{\Delta uv}{\Delta x} = v\frac{\Delta v}{\Delta x} + u\frac{\Delta u}{\Delta x} \qquad \qquad \frac{\Delta e^u}{\Delta x} = \frac{\Delta u}{\Delta x} \times e^u$$

$$\frac{\Delta f(u)}{\Delta x} = \frac{\Delta f}{\Delta x}(u) \times \frac{\Delta u}{\Delta x} \qquad \qquad \frac{\Delta \frac{u}{v}}{\Delta x} = \frac{v\frac{\Delta u}{\Delta x} - u\frac{\Delta v}{\Delta x}}{v^2} \qquad \qquad \frac{\Delta \ln(u)}{\Delta x} = \frac{\Delta u}{\Delta x}$$

$$\frac{\Delta \sin(u)}{\Delta x} = \frac{\Delta u}{\Delta x} \cos u \qquad \qquad \frac{\Delta \cos(u)}{\Delta x} = -\frac{\Delta u}{\Delta x} \sin u \qquad \qquad \frac{\Delta \tan(u)}{\Delta x} = \frac{\Delta u}{\Delta x} \sec^2 u$$

Infinitesimal Calculus

finitesimal Calculus
$$\int (\lambda f(x) + \mu g(x))\Delta x = \lambda \int f(x)\Delta x + \mu \int g(x)\Delta x$$

$$\int f(u)\frac{\Delta u}{\Delta x}\Delta x = \int f(u)\Delta u$$

$$\int u\frac{\Delta v}{\Delta x}\Delta x = uv - \int \frac{\Delta v}{\Delta u}\Delta x$$

$$x \neq -1 \Rightarrow \int x^n \Delta x = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x \Delta x = e^x + C \qquad \int \frac{\frac{\Delta u}{\Delta x}}{u}\Delta x = \ln|u| + C$$

$$\int \sin(x)\Delta x = -\cos x + C \qquad \int \tan x\Delta x = -\ln|\cos x| + C$$

$$\int \cos x\Delta x = \sin x + C \qquad \int \cot x\Delta x = \ln|\sin x| + C$$

$$\int \sec^2 x\Delta x = \tan x + C \qquad \int \csc^2 x\Delta x = \ln|\cos x + \cot x| + C$$

$$\int \cos^2 x\Delta x = -\cot x + C \qquad \int \csc x\Delta x = \ln|\cos x + \cot x| + C$$

$$\int \frac{1}{a^2 + x^2}\Delta x = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \qquad |x| < a \Rightarrow \int \frac{1}{\sqrt{a^2 - a^2}}\Delta x = \sin^{-1}\frac{x}{a} + C$$

$$\int \sinh x\Delta x = \cosh x + C \qquad \int \cosh x\Delta x = \sinh x + C \qquad \int \tanh x\Delta x = \ln\cosh x + C$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\Delta x = \sinh^{-1}\frac{x}{a} + C = \ln(x + \sqrt{x^2 + a^2}) + \frac{\Delta C}{\Delta x}$$

$$x > a \Rightarrow \int \frac{1}{\sqrt{x^2 - a^2}}\Delta x = \cosh^{-1}\frac{x}{a} + C = \ln(x + \sqrt{x^2 - a^2}) + \frac{\Delta C}{\Delta x}$$