

# Formulae for Tertiary Mathematics

Curtis Millar <curtis@xurtis.pw>

25 January, 2019

## Trigonometry

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad c^2 = a^2 + b^2 - 2ab \cos C$$

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad \tan^2 \theta - \sec^2 \theta = 1$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \qquad \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \qquad \cot(\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha}$$

## The Exponential & Hyperbolic Trigonometric Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2} \qquad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} \qquad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} \qquad \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad \tanh^2 x + \operatorname{sech}^2 x = 1 \qquad \coth^2 x - \operatorname{csch}^2 x = 1$$

## Differentiation

$$\frac{\Delta x^n}{\Delta x} = nx^{n-1}$$

$$\frac{\Delta uv}{\Delta x} = v \frac{\Delta v}{\Delta x} + u \frac{\Delta u}{\Delta x}$$

$$\frac{\Delta e^u}{\Delta x} = \frac{\Delta u}{\Delta x} \times e^u$$

$$\frac{\Delta f(u)}{\Delta x} = \frac{\Delta f}{\Delta x}(u) \times \frac{\Delta u}{\Delta x}$$

$$\frac{\Delta \frac{u}{v}}{\Delta x} = \frac{v \frac{\Delta u}{\Delta x} - u \frac{\Delta v}{\Delta x}}{v^2}$$

$$\frac{\Delta \ln(u)}{\Delta x} = \frac{\frac{\Delta u}{\Delta x}}{u}$$

$$\frac{\Delta \sin(u)}{\Delta x} = \frac{\Delta u}{\Delta x} \cos u$$

$$\frac{\Delta \cos(u)}{\Delta x} = -\frac{\Delta u}{\Delta x} \sin u$$

$$\frac{\Delta \tan(u)}{\Delta x} = \frac{\Delta u}{\Delta x} \sec^2 u$$

$$\frac{\delta F(x_1, x_2, \dots, x_n)}{\delta t} = \frac{\delta F}{\delta x_1} \frac{\Delta x_1}{\Delta t} + \frac{\delta F}{\delta x_2} \frac{\Delta x_2}{\Delta t} + \dots + \frac{\delta F}{\delta x_n} \frac{\Delta x_n}{\Delta t}$$

## Infinitesimal Calculus

$$\int (\lambda f(x) + \mu g(x)) \Delta x = \lambda \int f(x) \Delta x + \mu \int g(x) \Delta x$$

$$\int f(u) \frac{\Delta u}{\Delta x} \Delta x = \int f(u) \Delta u$$

$$\int u \frac{\Delta v}{\Delta x} \Delta x = uv - \int \frac{\Delta v}{\Delta u} \Delta x$$

$$x \neq -1 \Rightarrow \int x^n \Delta x = \frac{x^{n+1}}{n+1} + C$$

$$\int e^x \Delta x = e^x + C$$

$$\int \frac{\frac{\Delta u}{\Delta x}}{u} \Delta x = \ln |u| + C$$

$$\int \sin(x) \Delta x = -\cos x + C$$

$$\int \tan x \Delta x = -\ln |\cos x| + C$$

$$\int \cos x \Delta x = \sin x + C$$

$$\int \cot x \Delta x = \ln |\sin x| + C$$

$$\int \sec^2 x \Delta x = \tan x + C$$

$$\int \sec x \Delta x = \ln |\sec x + \tan x| + C$$

$$\int \operatorname{cosec}^2 x \Delta x = -\cot x + C$$

$$\int \operatorname{cosec} x \Delta x = \ln |\operatorname{cosec} x + \cot x| + C$$

$$\int \frac{1}{a^2 + x^2} \Delta x = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$|x| < a \Rightarrow \int \frac{1}{\sqrt{a^2 - x^2}} \Delta x = \sin^{-1} \frac{x}{a} + C$$

$$\int \sinh x \Delta x = \cosh x + C$$

$$\int \cosh x \Delta x = \sinh x + C$$

$$\int \tanh x \Delta x = \ln \cosh x + C$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \Delta x = \sinh^{-1} \frac{x}{a} + C = \ln(x + \sqrt{x^2 + a^2}) + \frac{\Delta C}{\Delta x}$$

$$x > a \Rightarrow \int \frac{1}{\sqrt{x^2 - a^2}} \Delta x = \cosh^{-1} \frac{x}{a} + C = \ln(x + \sqrt{x^2 - a^2}) + \frac{\Delta C}{\Delta x}$$

## Trigonometric and Hyperbolic Substitutions

Expression in integrand	Trigonometric substitution	Hyperbolic substitution
$\sqrt{a^2 - x^2}$	$x = a \sin \theta$	$x = a \tanh \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta$	$x = a \sinh \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$	$x = a \cosh \theta$