US Spine Simulation and Results

Rui Xu
MBP rotation
Sept 8-Oct 14

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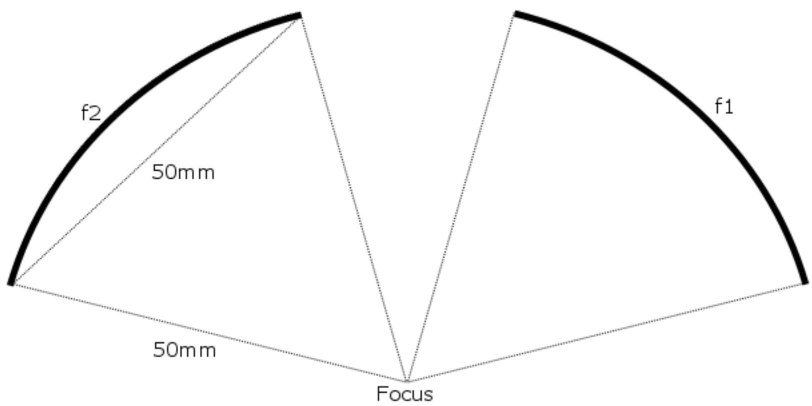
1 Introduction

This document will outline my ultrasound code and results. I have constructed a simple two-dimensional simulation using existing code from the k-wave package to construct a two-transistor system.

2 System 1: Homogeneous Medium, Two Elements, Two Frequencies

The first system I created was a system of two curved transducers, each with a radius of curvature of 50mm, and each subtending $\pi/2$ radians. The two transducers are driven at frequencies f_1 and f_2. The focal point of the two transducers are the same. I create the transducers by defining source points at a 50mm radius for $\pi/12 \leq \Theta_1 \leq 5\pi/12$ and $7\pi/12 \leq \Theta_2 \leq 11\pi/12$. The system setup is shown in Figure 2.



2.1 Half-Maximum Profiles

The first test was to vary the frequency f_1 of the right hand side transducer, while keeping the frequency of the second transducer constant $f_2 = 0.25$MHz. Implementing multiple frequencies in k-wave require that each source point have its frequency assigned to it. In the current implementation, then I've simply looped over all points, and set the frequency of the source points on the left half of the computational box to f_2 and those on the right half to f_1. Take care with the current code if you move the transducers around. It may be advisable to write a foolproof version of the frequency assignment part of the code.

We are interested in the pressure profile of the system near the focal point, for different frequencies $0.15\text{MHz} \leq f_1 \leq 0.35\text{MHz}$, tested in increments of 10kHz. The following figures show the 2D root-mean-square pressure profiles. The points of half-maximum pressure are outlined in black.

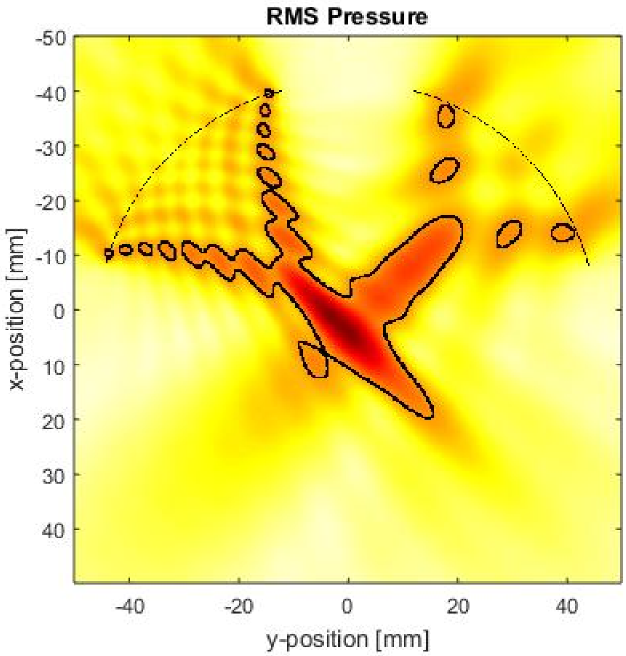


Figure 1: RMS pressure profile for $f_1 = 0.15\text{MHz}$, $f_2 = 0.25\text{MHz}$

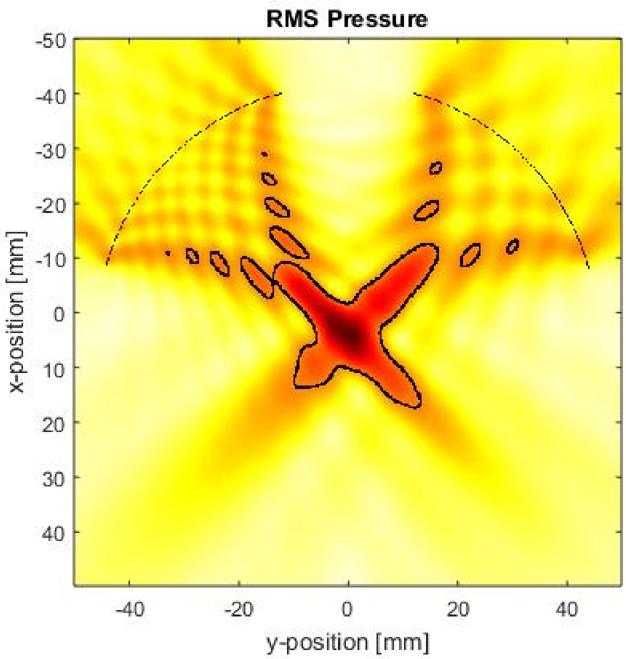


Figure 2: RMS pressure profile for $f_1 = 0.16\text{MHz}$, $f_2 = 0.25\text{MHz}$

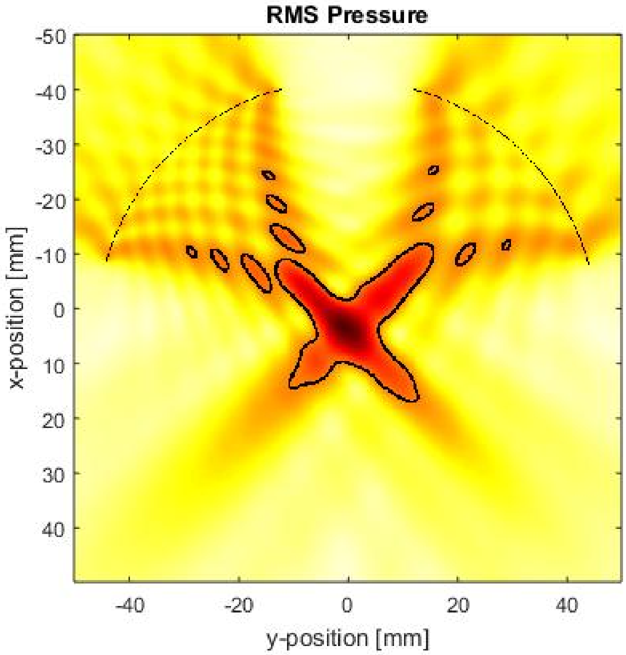


Figure 3: RMS pressure profile for $f_1 = 0.17$MHz, $f_2 = 0.25$MHz

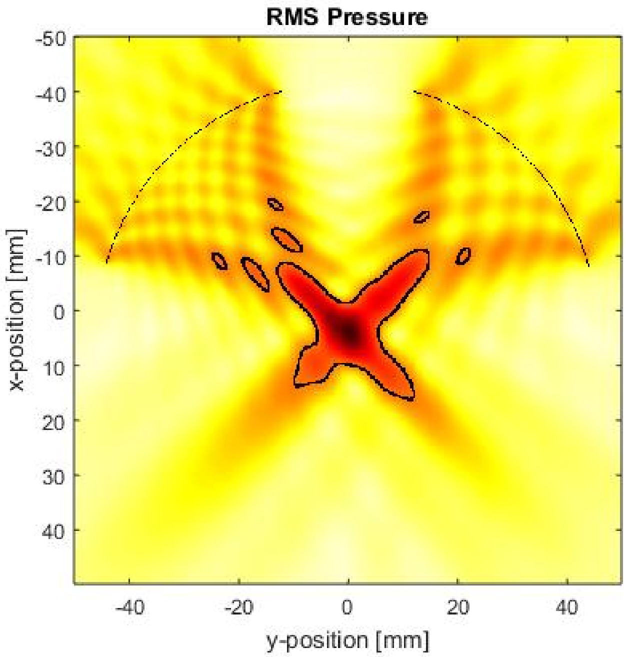


Figure 4: RMS pressure profile for $f_1 = 0.18$MHz, $f_2 = 0.25$MHz

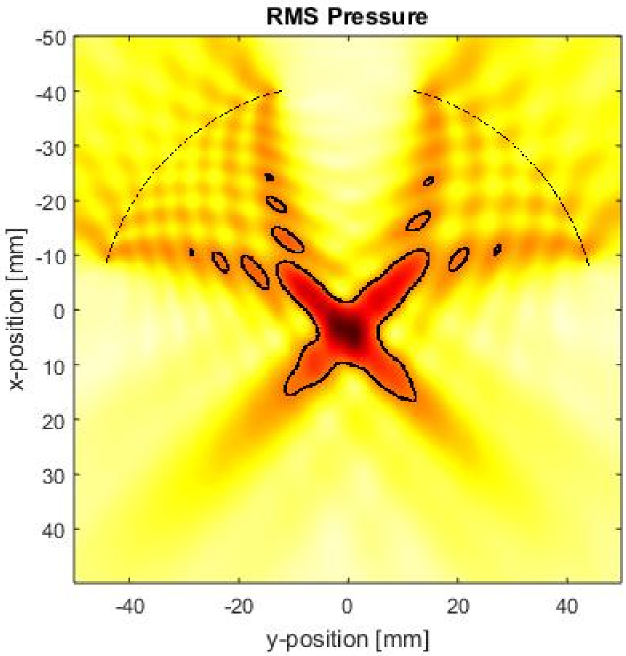


Figure 5: RMS pressure profile for $f_1 = 0.19$MHz, $f_2 = 0.25$MHz

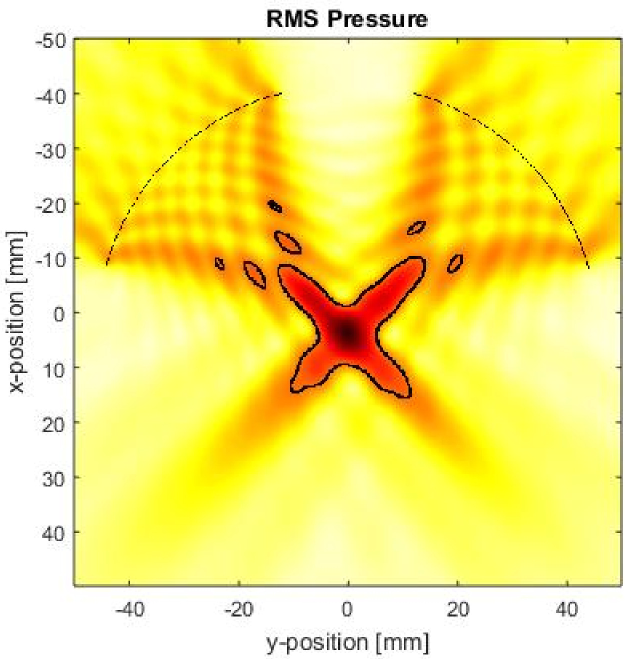


Figure 6: RMS pressure profile for $f_1 = 0.20$MHz, $f_2 = 0.25$MHz

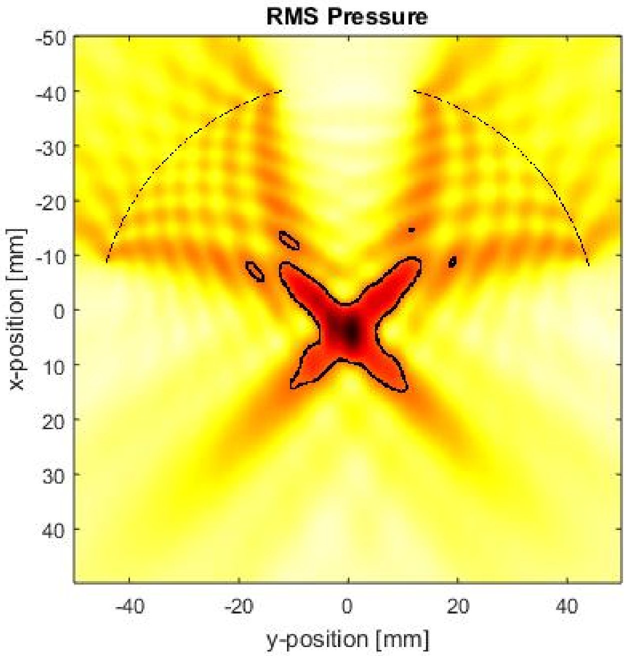


Figure 7: RMS pressure profile for $f_1 = 0.21\text{MHz}$, $f_2 = 0.25\text{MHz}$

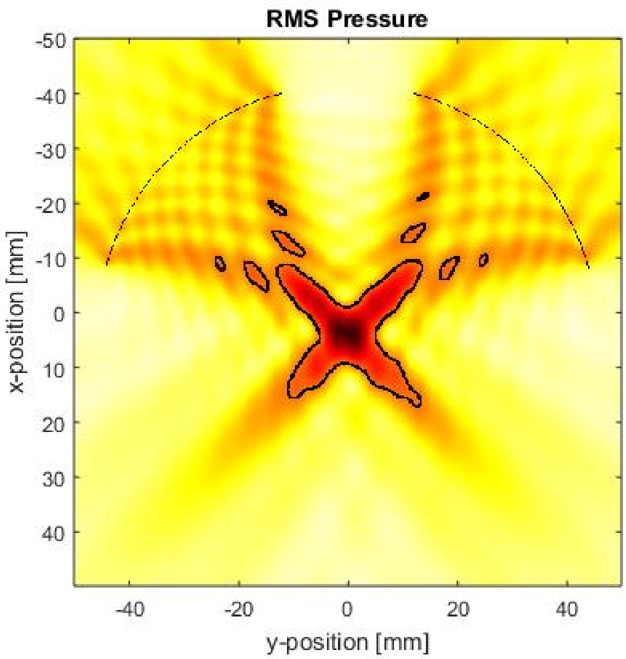


Figure 8: RMS pressure profile for $f_1 = 0.22\text{MHz}$, $f_2 = 0.25\text{MHz}$

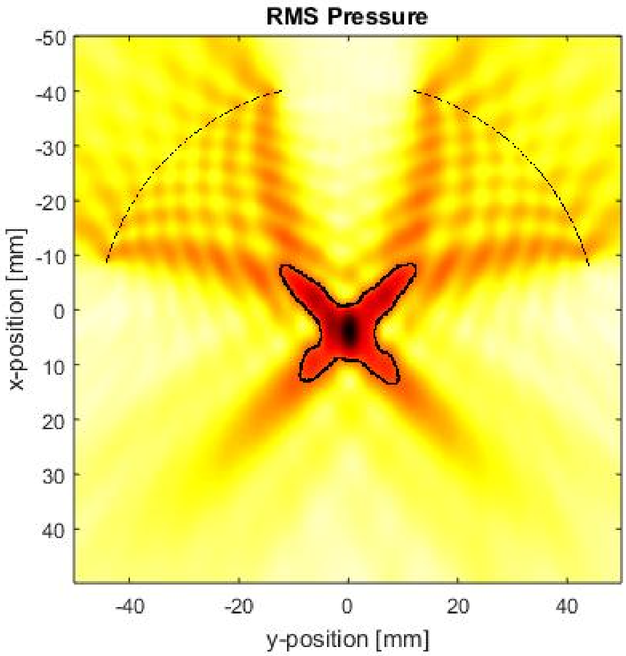


Figure 9: RMS pressure profile for $f_1 = 0.23\text{MHz}$, $f_2 = 0.25\text{MHz}$

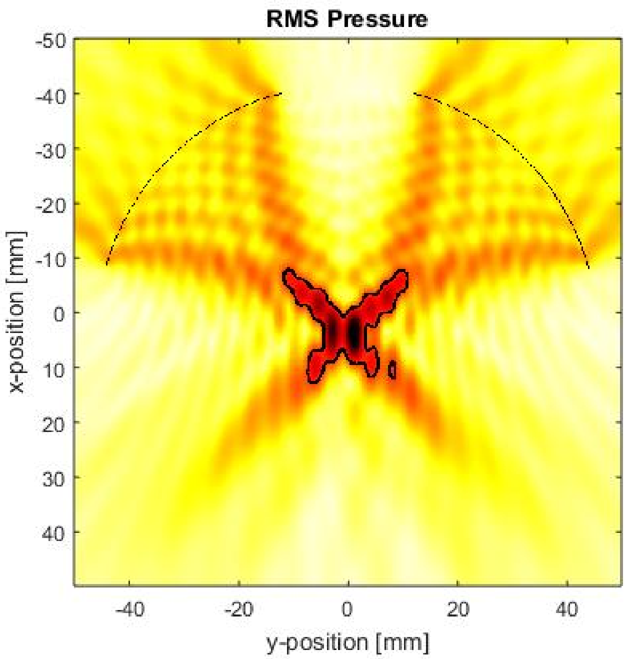


Figure 10: RMS pressure profile for $f_1 = 0.24\text{MHz}$, $f_2 = 0.25\text{MHz}$

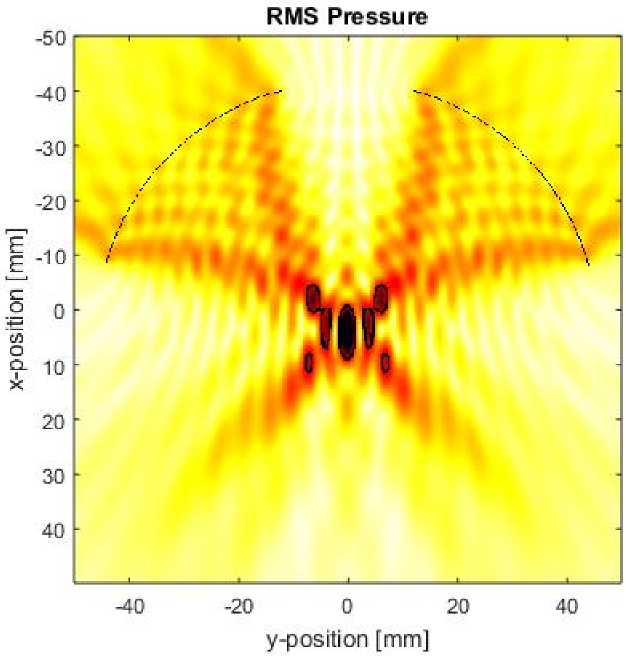


Figure 11: RMS pressure profile for $f_1 = 0.25$MHz, $f_2 = 0.25$MHz

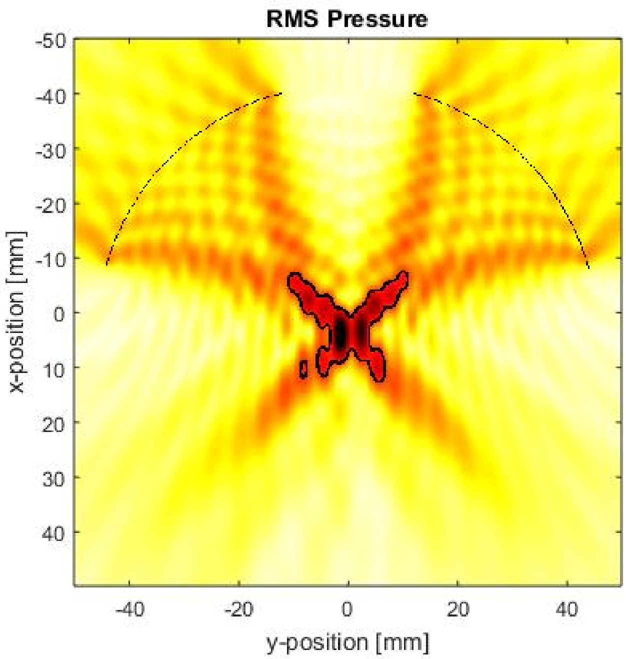


Figure 12: RMS pressure profile for $f_1 = 0.26$MHz, $f_2 = 0.25$MHz

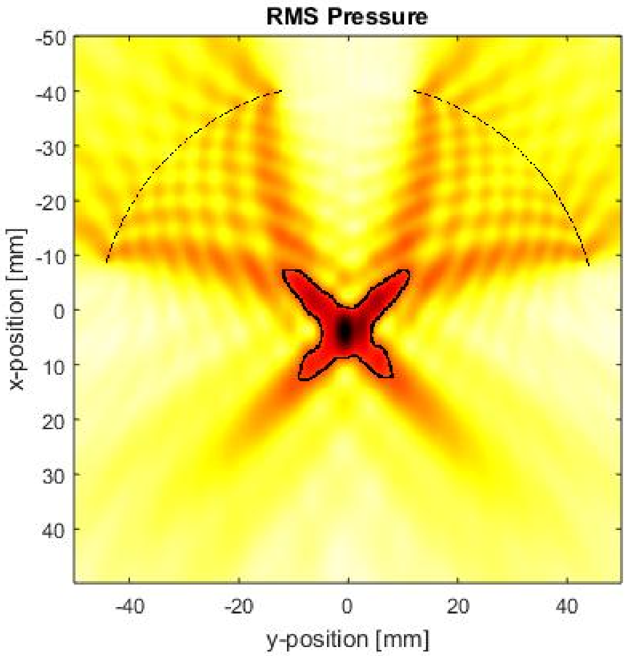


Figure 13: RMS pressure profile for $f_1 = 0.27\text{MHz}$, $f_2 = 0.25\text{MHz}$

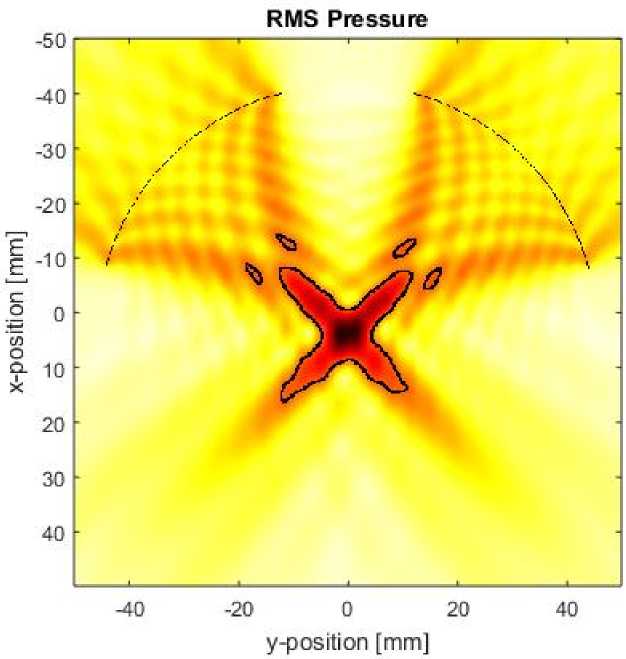


Figure 14: RMS pressure profile for $f_1 = 0.28\text{MHz}$, $f_2 = 0.25\text{MHz}$

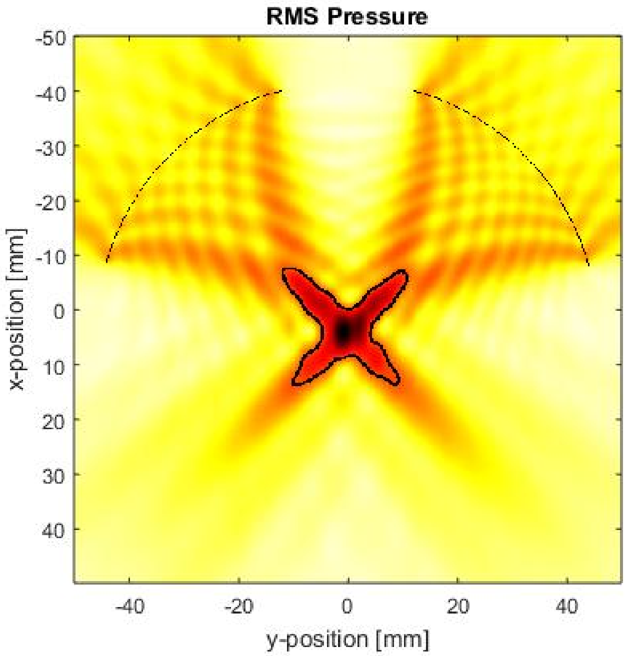


Figure 15: RMS pressure profile for $f_1 = 0.29$MHz, $f_2 = 0.25$MHz

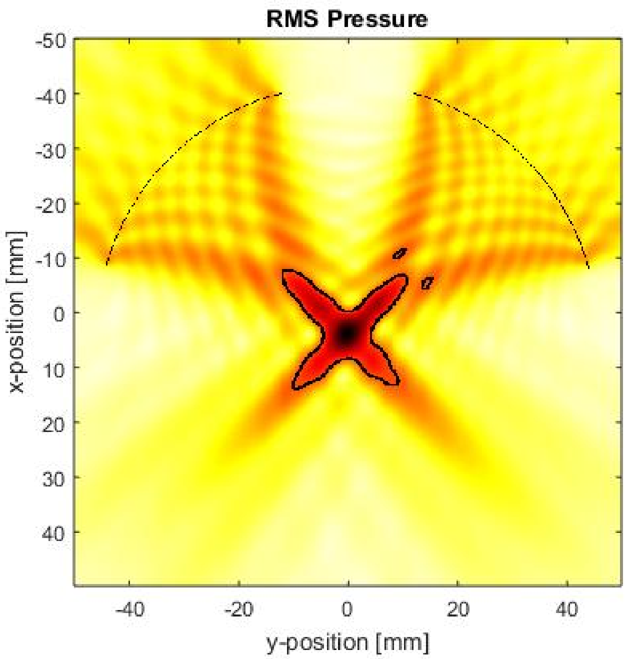


Figure 16: RMS pressure profile for $f_1 = 0.30$MHz, $f_2 = 0.25$MHz

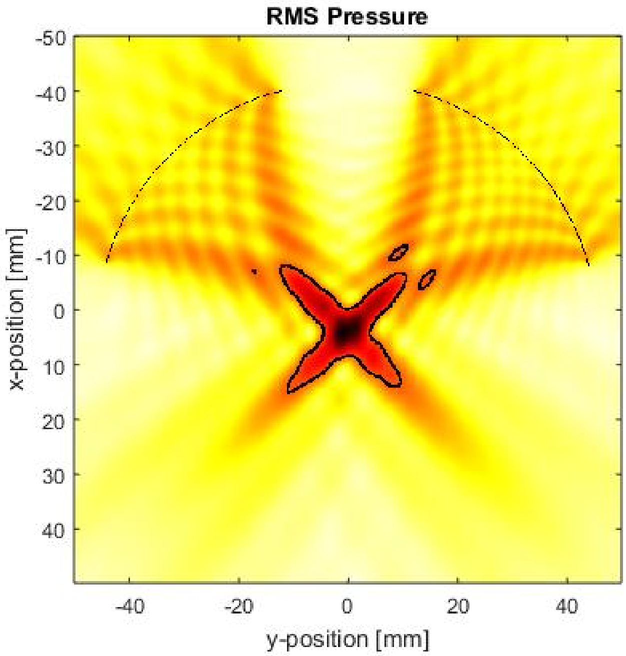


Figure 17: RMS pressure profile for $f_1 = 0.31\text{MHz}$, $f_2 = 0.25\text{MHz}$

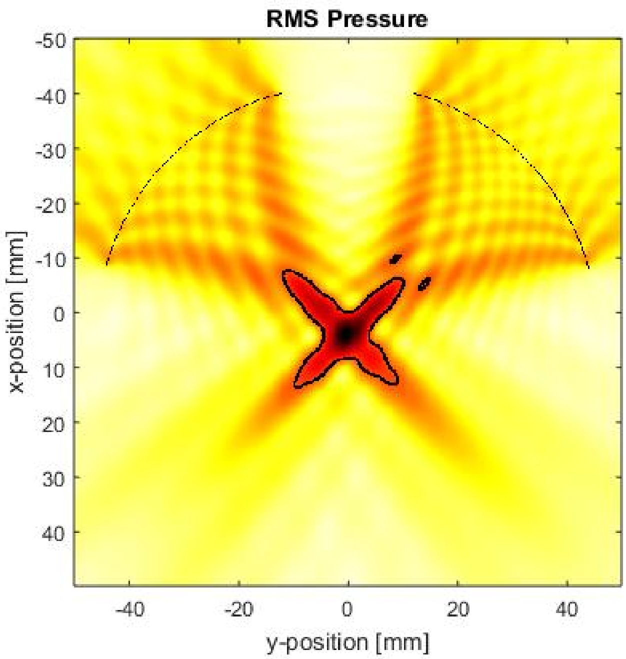


Figure 18: RMS pressure profile for $f_1 = 0.32\text{MHz}$, $f_2 = 0.25\text{MHz}$

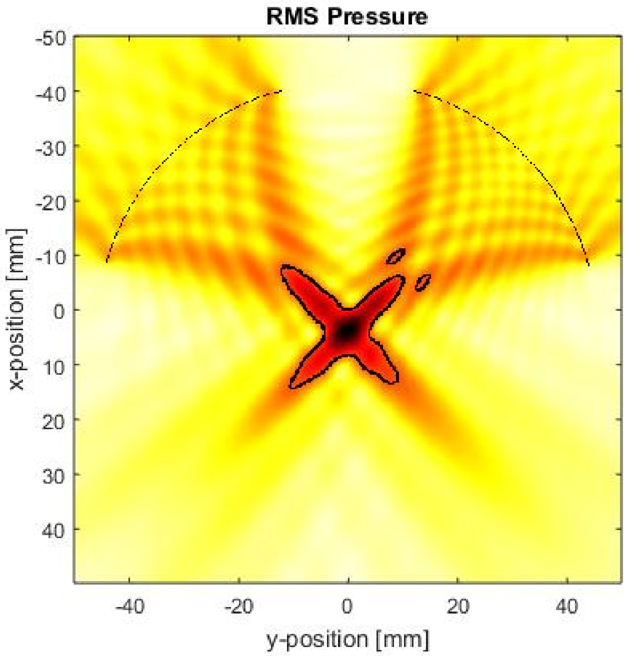


Figure 19: RMS pressure profile for $f_1 = 0.33$MHz, $f_2 = 0.25$MHz

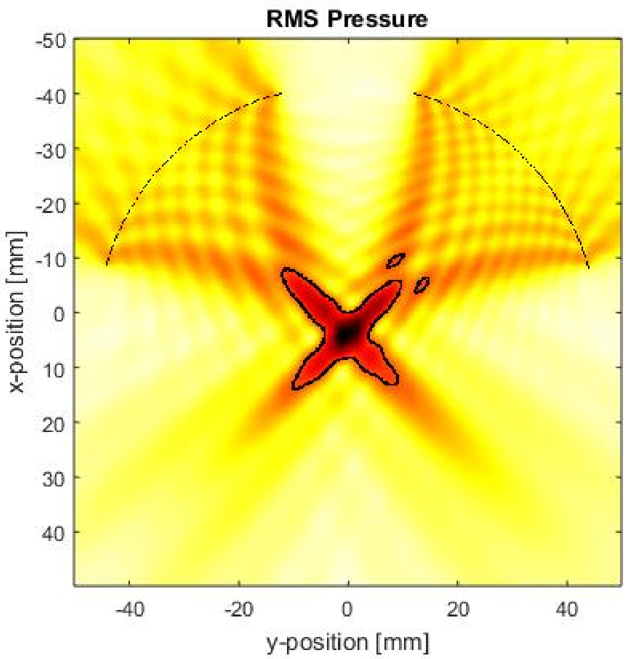


Figure 20: RMS pressure profile for $f_1 = 0.34$MHz, $f_2 = 0.25$MHz

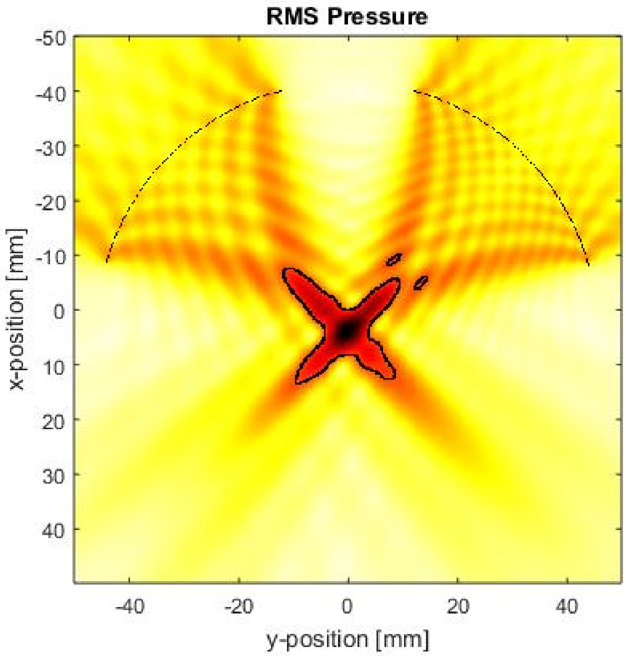


Figure 21: RMS pressure profile for $f_1 = 0.35\text{MHz}$, $f_2 = 0.25\text{MHz}$

2.2 FWHM for different frequencies

Another informative metric can be the FWHM of the profile. However, this is more useful for pressure profiles that are approximately Gaussian. The method I used for finding the points with half maximum pressure returns coordinates for all points, not just those around the central maximum. This is remediable, but the profiles remain non-circulate. The FWHM of the distributions can easily be calculated from the coordinates (x_{HM}, y_{HM}) by first calculating the coordinate of the centre of the HM points (x_{HMc}, y_{HMc}):

$$x_{HMc} = \frac{1}{N}\sum_{i=1}^{N} x_{HM_i} \cdot dx, \qquad x_{HMc} = \frac{1}{N}\sum_{i=1}^{N} x_{HM_i} \cdot dx \tag{1}$$

where N is the number of HM points. We can calculate the average distance d of the HM points from the HM centre using:

$$d = \frac{1}{N}\sum_{i=1}^{N} \sqrt{(x_{HM_i} - x_{HMc})^2 + (y_{HM_i} - y_{HMc})^2} \cdot dx\alpha\beta \tag{2}$$

The FWHM is then approximately equal to two times the average distance between an HM point and the centre of the HM point distribution. For HM distributions that are highly non-Gaussian, then there is little purpose in calling it the FWHM - it's simply the average width of the half-maximum RMS pressure profile. Despite this, I'll continue calling it FWHM for the time being. The FWHM is shown for 200kHz$\leq f_1 \leq$ 300kHz in Figure 2.2.

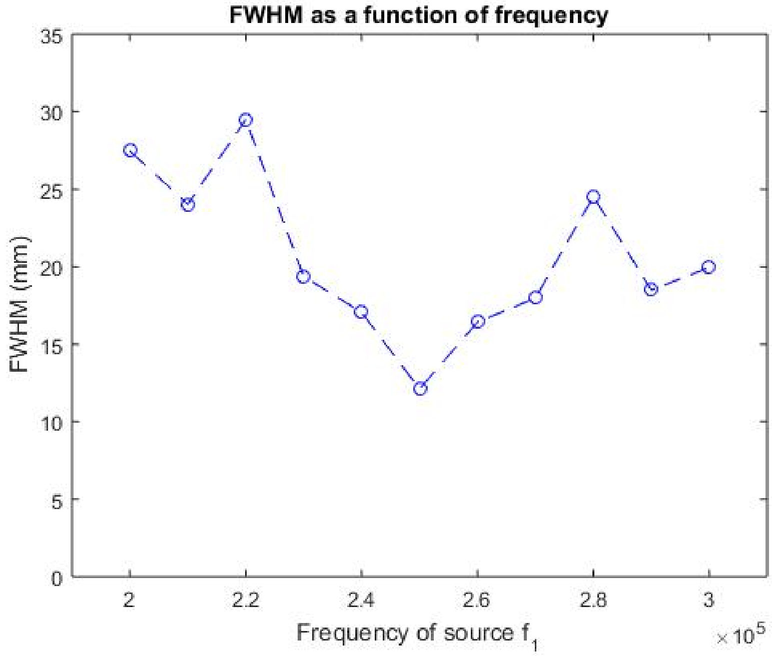


Figure 22: The FWHM of the RMS pressure distribution for 200kHz$\leq f_1 \leq$ 300kHz, $f_2 = 250$kHz.

2.3 Centre of HM profile

I've included this for curiosity's sake. Here I plot the centre of the FWHM profile, for 200kHz$\leq f_1 \leq$ 300kHz.

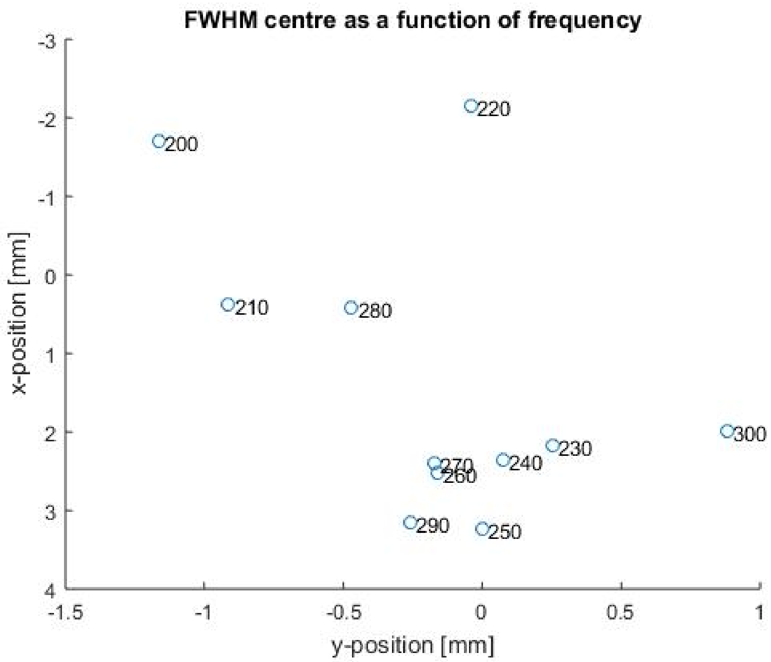


Figure 23: The coordinates of the centre of the FWHM profile of the RMS pressure distribution for 200kHz$\leq f_1 \leq$ 300kHz, $f_2 = 250$kHz.

2.4 Maximum RMS Pressure

We are also interested in the location of the maximum RMS intensity for different frequencies. These are shown in the following Figure 2.4.

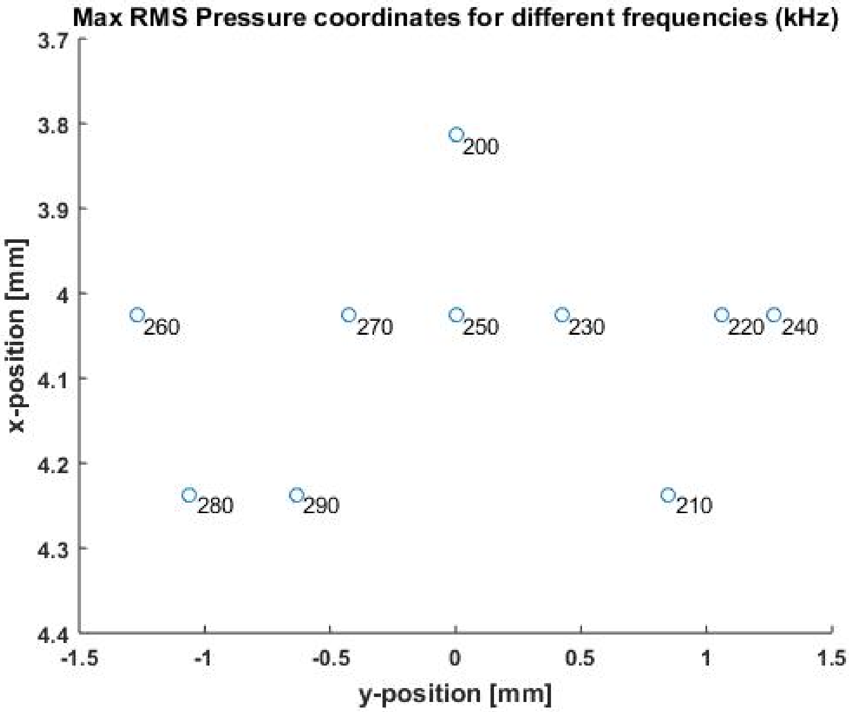


Figure 24: The Maximum RMS pressure location for 200kHz$\leq f_1 \leq$300kHz.

There does not appear to be any pattern in the location of the maximum RMS pressure coordinates. In the x-dimension, the maxima are confined to within three computational grid points, approximately +4mm away from the focal point of the independent transducers. In the y-dimension, the maxima remain closely clustered around the focal point. Generally, the maximum RMS pressure is much more highly clustered than the FWHM centre.

3 System 2: Adding Inclusions

The next step to increasing the complexity of the simulation is to add an inclusion near the focal point of the two transducers. The acoustic properties that k-wave includes are the sound speed within the medium, the ambient density distribution within the medium, a nonlinearity parameter, the power law absorption prefactor α_0, and the power law absorption exponent y, where the power law is:

$$\alpha(f) = \alpha_0 \omega^y, \quad \omega = 2\pi f \tag{3}$$

The simulation package, k-wave, doesn't include all non-linear effects, but does include two additional non-linear terms to attempt to account for all non-linear behaviour. These terms are defined as BonA in k-wave, where A and B are the first and second terms in the Taylor series expansion of the relation between the materials pressure and density.

All of these variables can be set as spatially heterogeneous in k-wave. In order to make an accurate simulation, I will need the acoustic properties of the medium I'll be simulating - namely water, bone, fat, nerve tissue, etc. Some of these are available in "Physical Properties of Tissue", a reference book by Francis A. Duck, and referenced by Chris Diederich in his interstitial spine simulation work [1].

For this simulation, it will also be important to be able to account for shear wave propagation. Simulation results from [2]. Luckily, there is the option to use an updated file from k-wave called PSTDELASTIC2D that allows for the separation of the compressional and shear components of the ultrasound wave. The details of the algorithm used for the propagation of shear waves in elastic media are described in full in their paper 'Modelling Elastic Wave Propagation Using the k-Wave MATLAB Toolbox' [3].

In order to understand the effect of shear wave propagation in a bone, I built another simulation, starting from the Snell's law example. In this simulation I use an inclusion centred near the focal point of the two transducers such that the ultrasound waves intersect at the proximal side of the inclusion. I set the radius of the circular inclusion to 1cm, and set the compressional speed of sound to be $v_P = 2820$m/s, the shear speed of sound to $v_S = 1500$m/s, the compressional alpha factor to $\alpha_P = 9$, and the shear alpha factor to $\alpha_S = 20$. In the rest of the simulation medium then I use $v_P = 1580$m/s, $v_s = 0$m/s, $\alpha_P = 0.57$, and $\alpha_S = 0$. I'm not sure if I should be setting α_S to zero, but since there should not be any shear waves propagating through the non-bone medium, then it should be a reasonable input. The setup for this simulation is shown in Figure **??**

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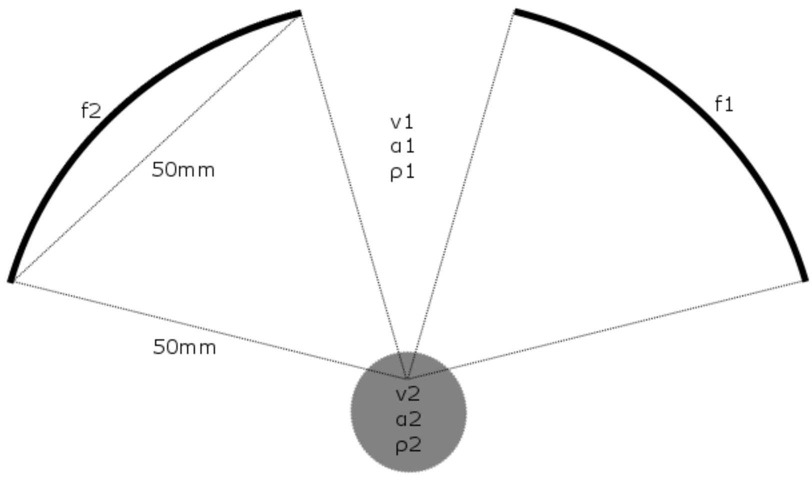


Figure 25: An inclusion is added to the system, with the same acoustic properties as those used by [2].

The code is set up to run the fluid simulation first, and then run the elastic simulation after to allow for comparison between the two models. The results of the simulation are shown in Figure 3. In this figure, the quantity plotted is the logarithm of squared particle velocity, normalized by the maximum squared particle velocity $|\mathbf{u} \cdot \mathbf{u}|/|\mathbf{u}_{max} \cdot \mathbf{u}_{max}|$. For this first test case, the frequencies of the two transducers are both 0.25MHz.

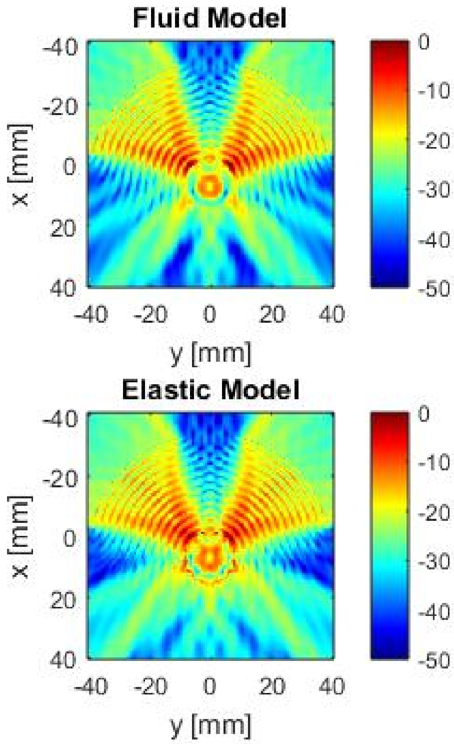
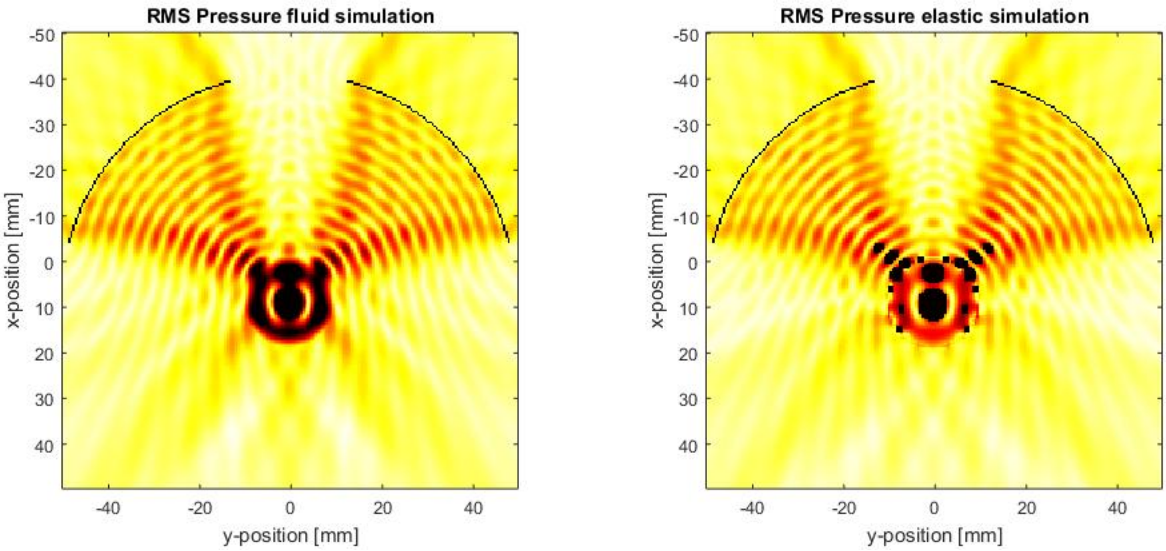
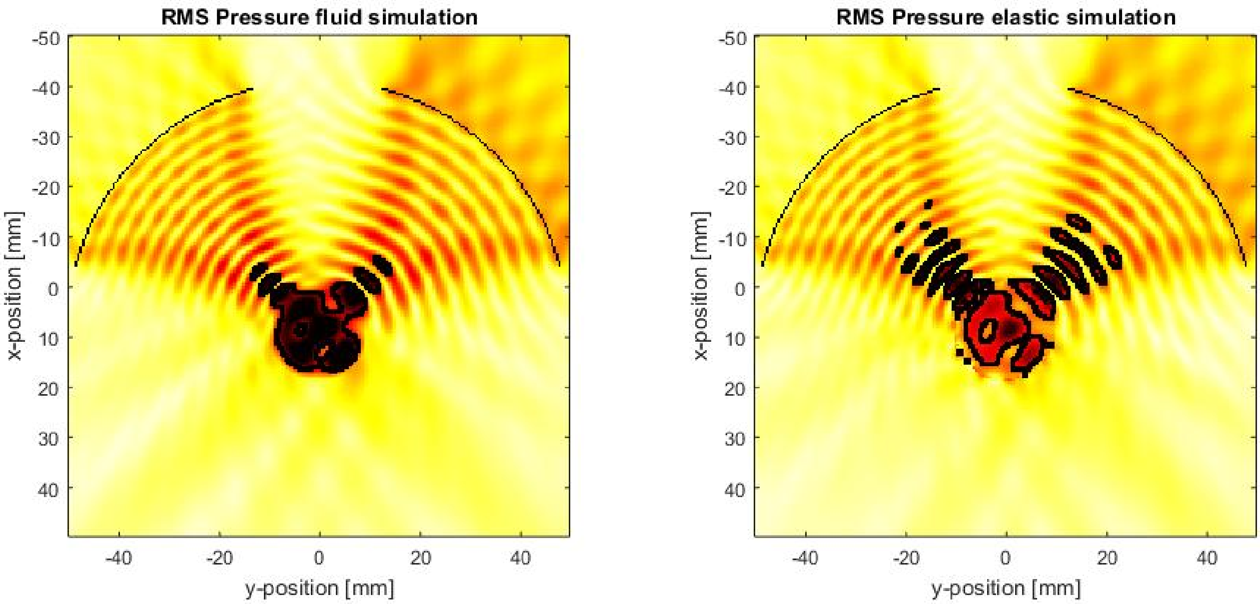


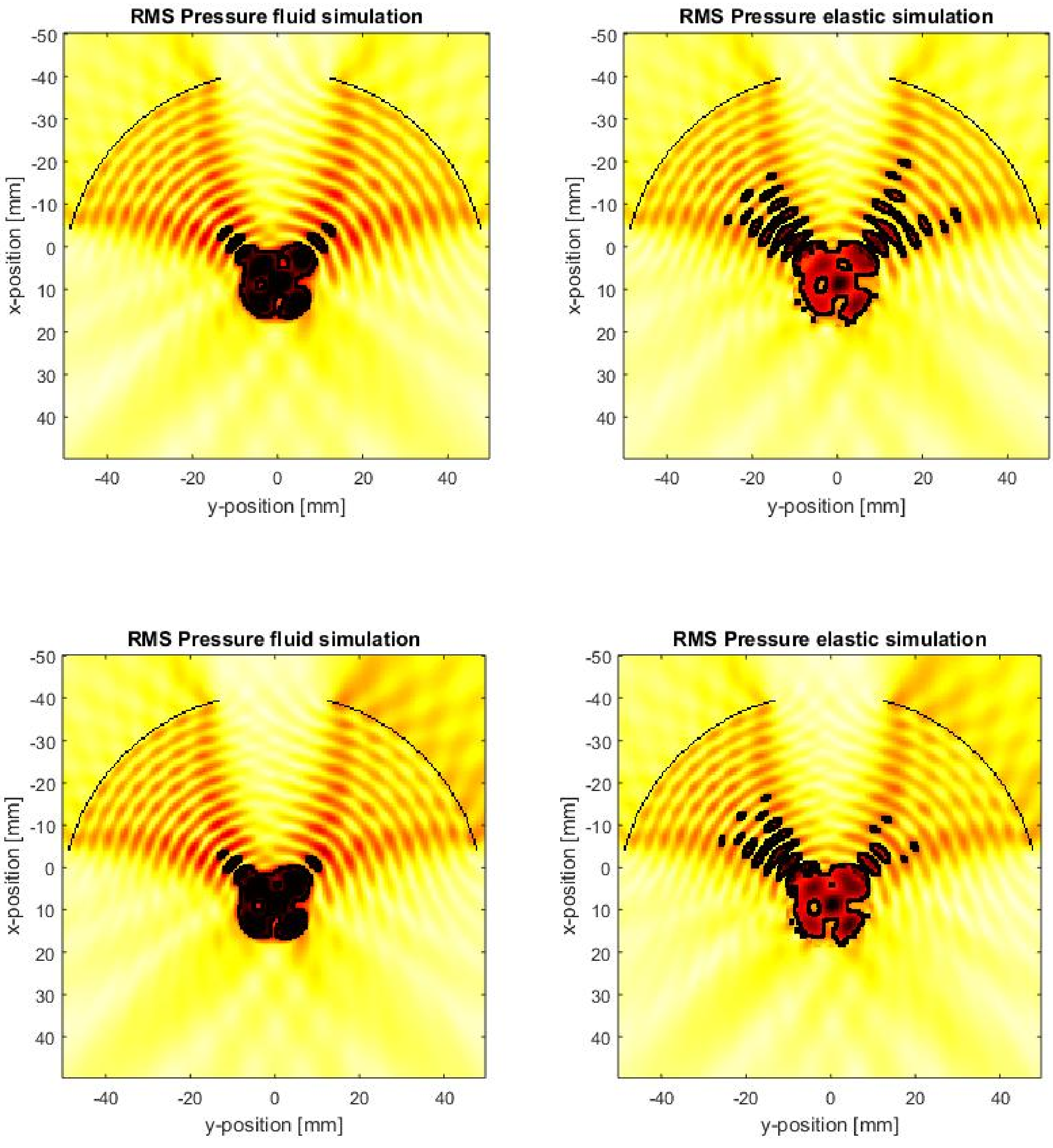
Figure 26: In this figure, the logarithm of normalized, squared particle velocity is plotted for the fluid simulation and the elastic simulation. This quantity is proportional to heat deposition, where the proportionality constants are the speed of sound in the medium, the medium density, and the absorption coefficient of the medium. This quantity, thermal deposition, is important for thermal ultrasound applications.

In the following figure (Figure 3) , the half-maximum contours of the RMS pressure profiles are outlined for both fluid and elastic simulations. It is somewhat difficult to see the HM lines in the figure because the pressure profiles are highly peaked.



3.1 Half-Maximum Profiles




RMS Pressure fluid simulation
RMS Pressure elastic simulation
RMS Pressure fluid simulation
RMS Pressure elastic simulation
x-position [mm]
y-position [mm]
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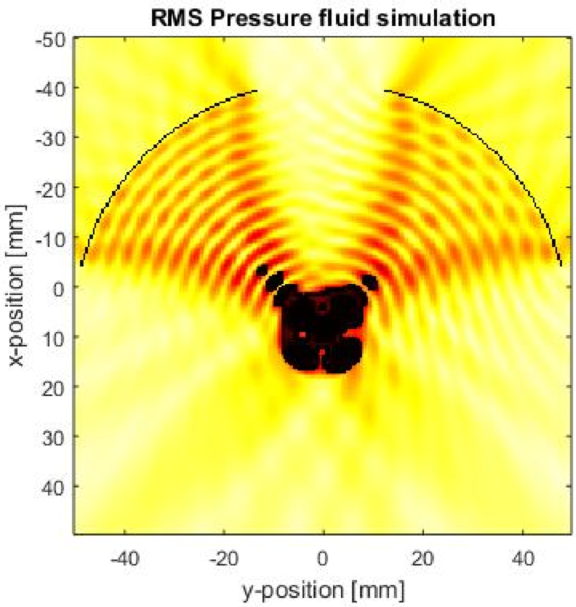
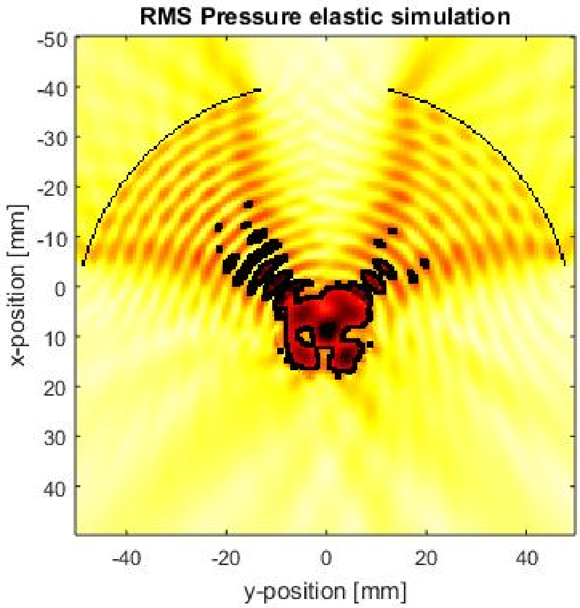
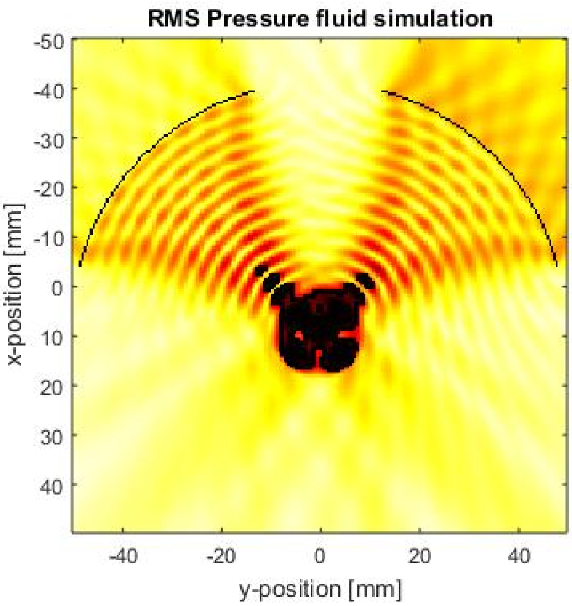

RMS Pressure fluid simulation
x-position [mm]
y-position [mm]


RMS Pressure elastic simulation
x-position [mm]
y-position [mm]


RMS Pressure fluid simulation
x-position [mm]
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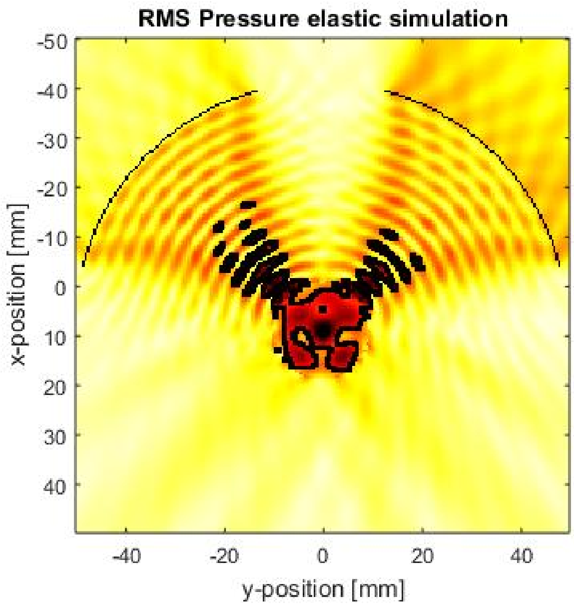
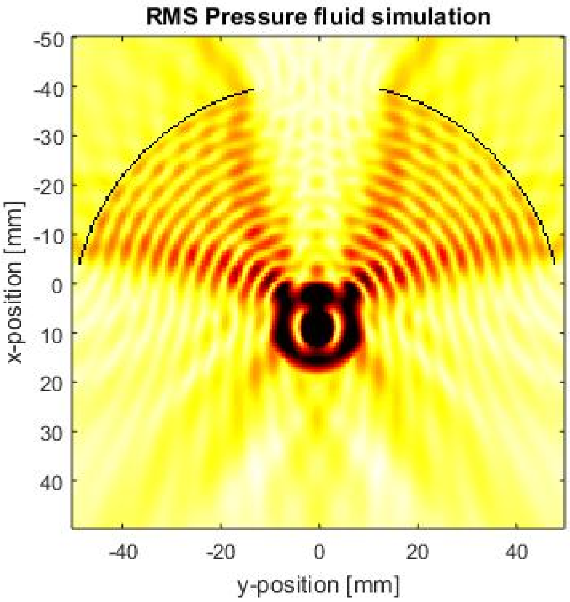

RMS Pressure elastic simulation
x-position [mm]
y-position [mm]


RMS Pressure fluid simulation
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y-position [mm]
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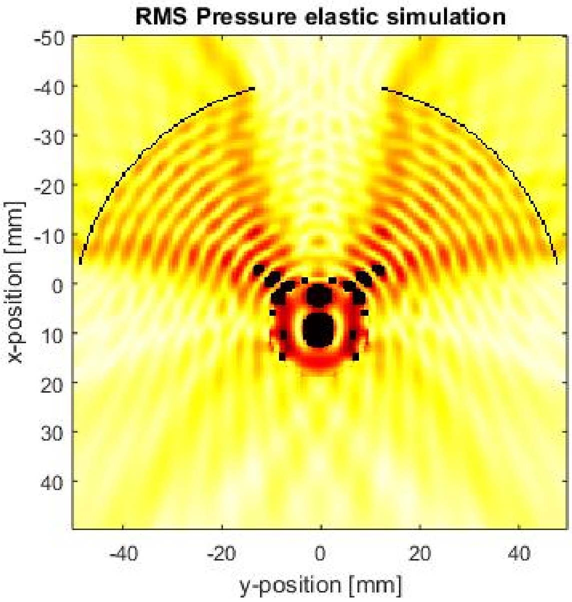
RMS Pressure elastic simulation
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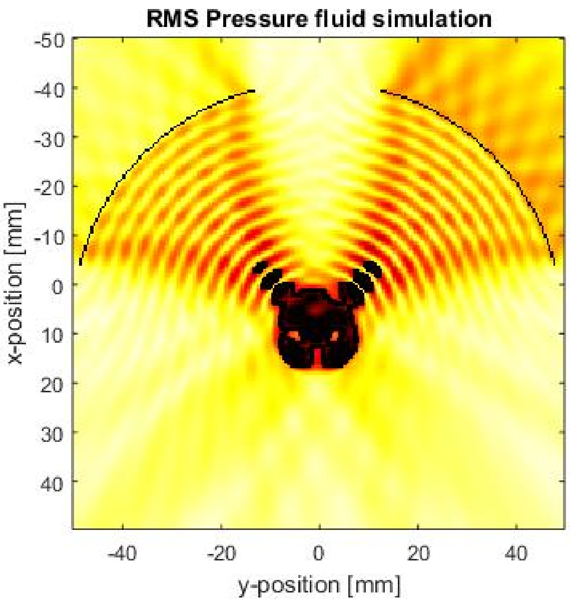
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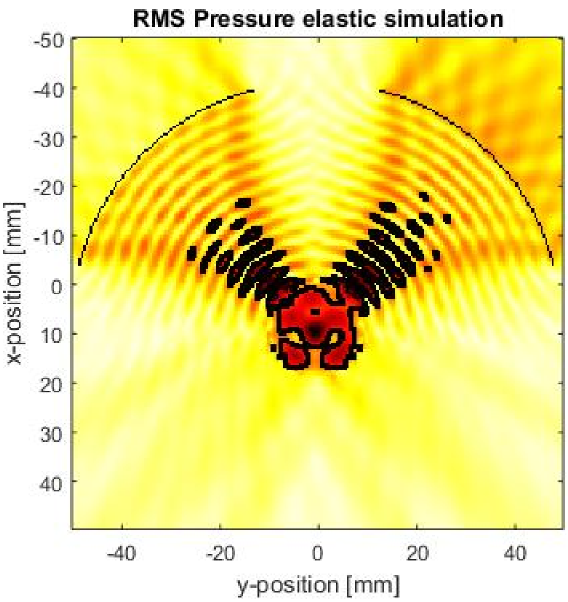
RMS Pressure elastic simulation
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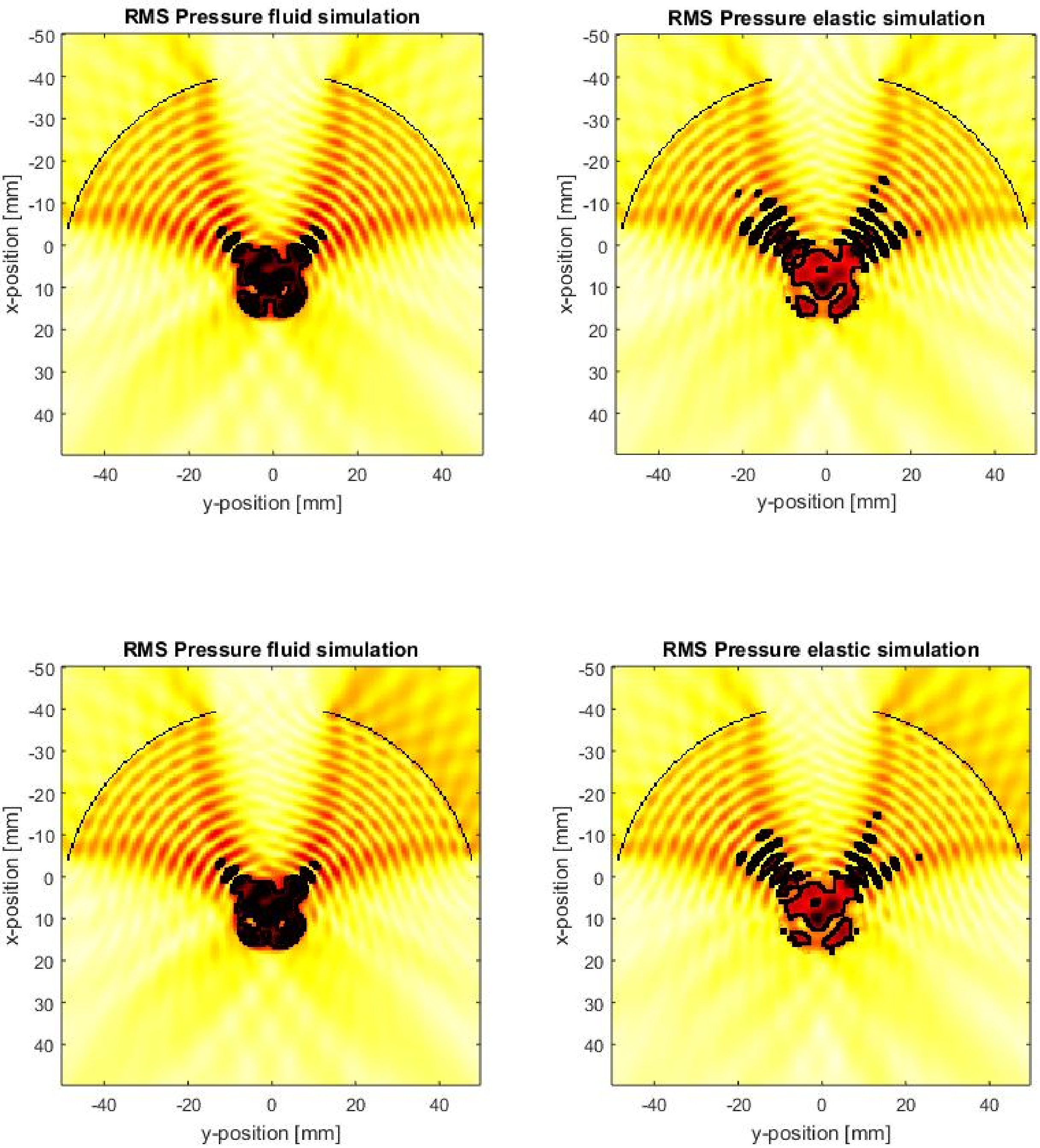
RMS Pressure fluid simulation
RMS Pressure elastic simulation
x-position [mm]
y-position [mm]
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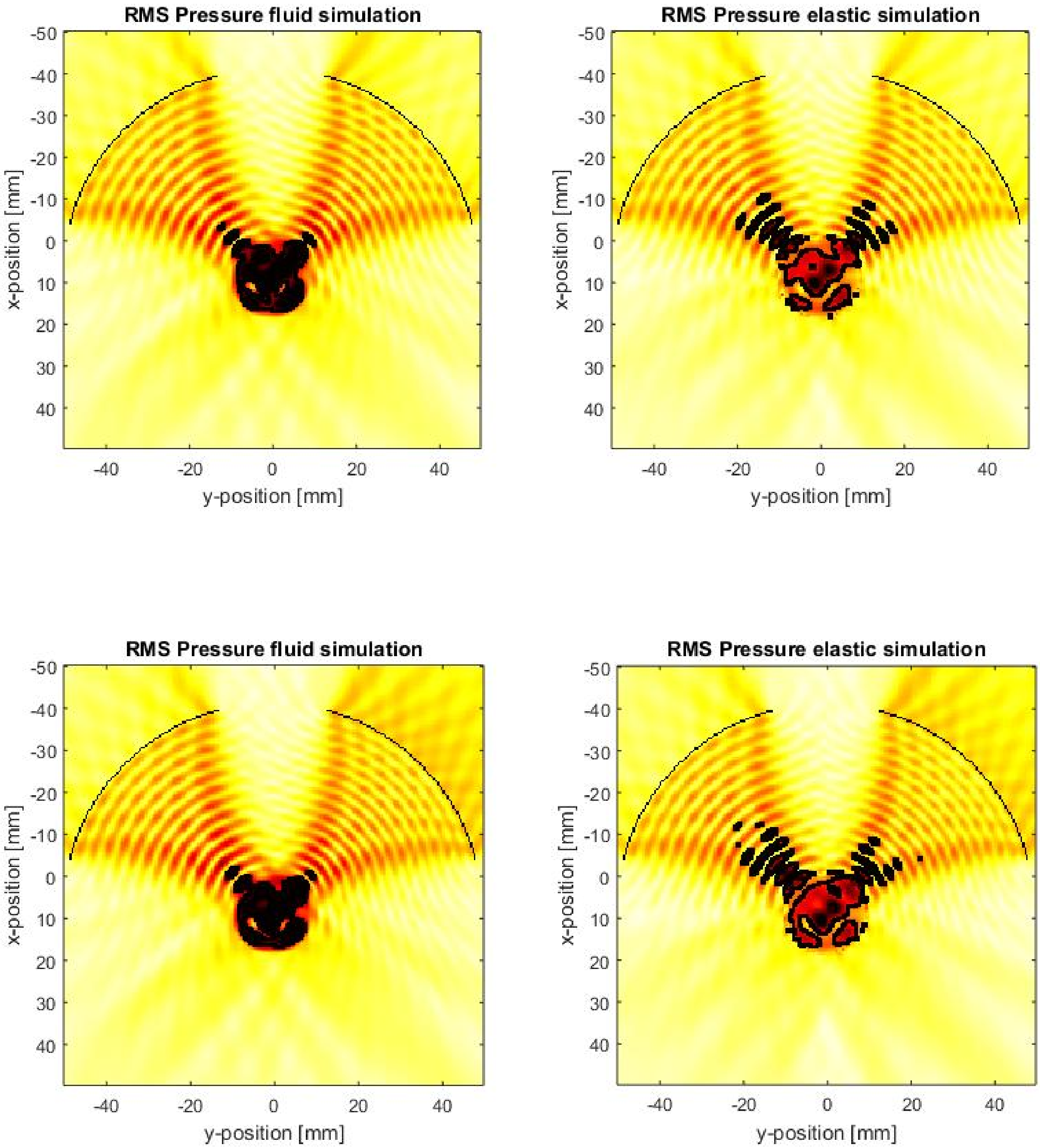
-40 -20 0 20 40
RMS Pressure fluid simulation
RMS Pressure elastic simulation
x-position [mm]
y-position [mm]


RMS Pressure fluid simulation
RMS Pressure elastic simulation
x-position [mm]
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RMS Pressure fluid simulation
RMS Pressure elastic simulation
x-position [mm]
y-position [mm]

References

[1] F. Duck, "Physical properties of tissue: A comprehensive reference book (academic, london)," 1990.

[2] B. E. Treeby and T. Saratoon, "The contribution of shear wave absorption to ultrasound heating in bones: Coupled elastic and thermal modeling," in *Ultrasonics Symposium (IUS), 2015 IEEE International*, pp. 1–4, IEEE, 2015.

[3] B. E. Treeby, J. Jaros, D. Rohrbach, and B. Cox, "Modelling elastic wave propagation using the k-wave matlab toolbox," in *2014 IEEE International Ultrasonics Symposium*, pp. 146–149, IEEE, 2014.