253 Sheng Xu HW1 P3

1(b) ② Stability For C-N

$$-\frac{a\Delta t}{4h_x} V_{k-1,l+1} + V_{k,l+1} + \frac{a\Delta t}{4h_x} V_{k+1,l+1} = \frac{a\Delta t}{4h_x} V_{k-1,l} + V_{k,l} - \frac{a\Delta t}{4h_x} V_{k+1,l}$$

so $\xi(j) = \dfrac{1 - i2\lambda \sin\theta}{1 + i2\lambda \sin\theta}$ $\left(\lambda = \dfrac{a\Delta t}{4h_x} > 0\right)$

so $|g(j)| = 1$ unconditionally stable.

Add one step jumped:

$$-\lambda e_{l+1}^{(j)} \exp[ij(k-1)h_x] + e_{l+1}^{(j)} \exp[ijkh_x] + \lambda e_{l+1}^{(j)} \exp[ij(k+1)h_x]$$
$$= \lambda e_l^{(j)} \exp[ij(k-1)h_x] + e_l^{(j)} \exp[ijkh_x] - \lambda e_l^{(j)} \exp[ij(k+1)h_x]$$