1 (b) ① $\dfrac{V_{k,l+1} - V_{k,l}}{h_t}$

$= V_t(x_k, t_l) + \frac{1}{2} V_{tt}(x_k, t_l)\, h_t + \frac{1}{6} V_{ttt}(x_k, t_l)\, h_t^2 + O_1(h_t^3)$

$\left(\dfrac{V_{k+1,l+1} - V_{k-1,l+1}}{2h_x} \right)$

$= \frac{1}{2h_x}\Big[V_{k,l} + V_{xk,l} h_x + V_{tk,l} h_t + \frac{1}{2!} V_{xx\,k,l} h_x^2 + \frac{1}{1!1!} V_{xt\,k,l} h_x h_t$
$+ \frac{1}{2!} V_{tt\,k,l} h_t^2 + \frac{1}{3!} V_{xxx} h_x^3 + \frac{1}{2!} V_{xxt} h_x^2 h_t + \frac{1}{2!} V_{xtt} h_x h_t^2 + \frac{1}{3!} V_{ttt} h_t^3$
$- \frac{1}{2h_x}\Big[V_{k,l} - V_x h_x + V_t h_t + \frac{1}{2!} V_{xx} h_x^2 - \frac{1}{1} V_{xt} h_x h_t$
$+ \frac{1}{2!} V_{tt} h_t^2 - \frac{1}{3!} V_{xxx} h_x^3 + \frac{1}{2!} V_{xxt} h_x^2 h_t - \frac{1}{2!} V_{xtt} h_x h_t^2 + \frac{1}{3!} V_{ttt} h_t^3$

* All h_t^n term → $+\frac{1}{2h_x} O_\alpha(h_x^4, h_t h_x^3, h_t^2 h_x^2, h_t^3 h_x)$ are cancelled out.

$= \frac{1}{h_x}\Big[V_x h_x + V_{xt} h_x h_t + \frac{1}{3!} V_{xxx} h_x^3 + \frac{1}{2!} V_{xtt} h_x h_t^2 \Big]$
$+ O_2(h_x^3, h_t h_x^2, h_t^2 h_x, h_t^3)$

$\dfrac{V_{k+1,l} - V_{k-1,l}}{2h_x}$

$= \frac{1}{2h_x}\Big[V_{k,l} + V_x h_x + \frac{1}{2} V_{xx} h_x^2 + \frac{1}{3!} V_{xxx} h_x^3 \Big]$
$- \frac{1}{2h_x}\Big[V_{k,l} - V_x h_x + \frac{1}{2} V_{xx} h_x^2 - \frac{1}{3!} V_{xxx} h_x^3 \Big] + \frac{1}{2h_x} O_\beta(h_x^4)$

$= \frac{1}{h_x}\Big[V_x h_x + \frac{1}{3!} V_{xxx} h_x^3 \Big] + O_3(h_x^3)$

Now we combine everything:

$T_{xt} = V_t + \frac{1}{2} V_{tt} h_t + \frac{1}{6} V_{ttt} h_t^2 + O_1(h_t^3)$
$+ aV_x + \frac{a}{2} V_{xx} h_t + \frac{a}{6} V_{xxx} h_x^2 + \frac{a}{4} V_{xtt} h_t^2 +$
$O_2'(h_x^3, h_t h_x^2, h_t^2 h_x, h_t^3) + O_3'(h_x^3)$

$O_2' = \frac{a}{2} \cdot O_2$
$O_3' = \frac{a}{2} O_3$

$= (V_t + aV_x) + \frac{1}{2} h_t (V_{tt} + aV_{xt}) + O_4(h_t^2) + O_5(h_t^2, h_x^2)$
$O_6(h_x^3, h_t h_x^2, h_t^2 h_x, h_t^3)$

We know $V_t + aV_x = 0$, $V_{tt} + aV_{xt} = (V_t + aV_x)_t = 0_t = 0$

So $T_{xt} = O_4(h_t^2) + O_6(h_x^3, h_t h_x^2, h_t^2 h_x, h_t^3) + O_5(h_t^2, h_x^2)$
$= O(h_t^2, h_x^2)$