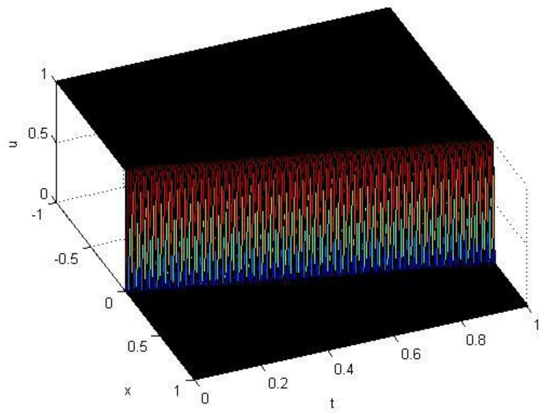
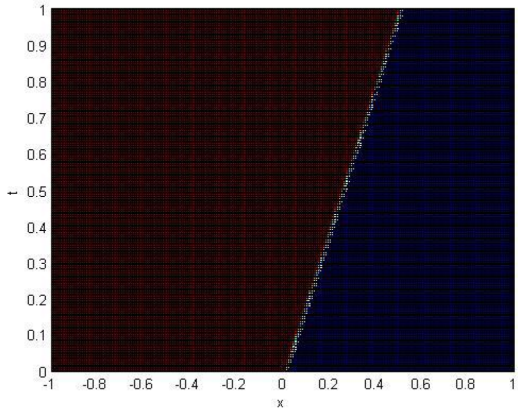
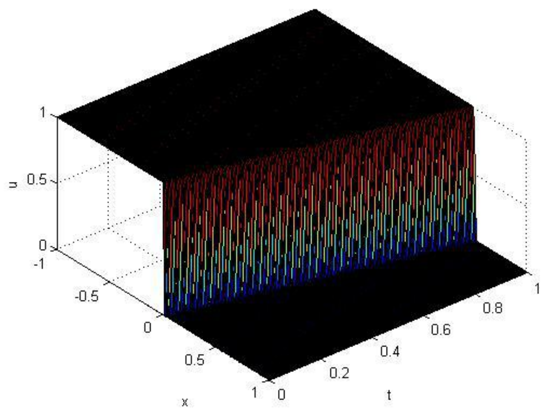
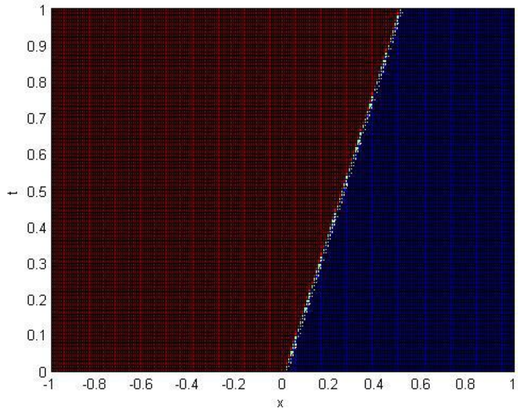
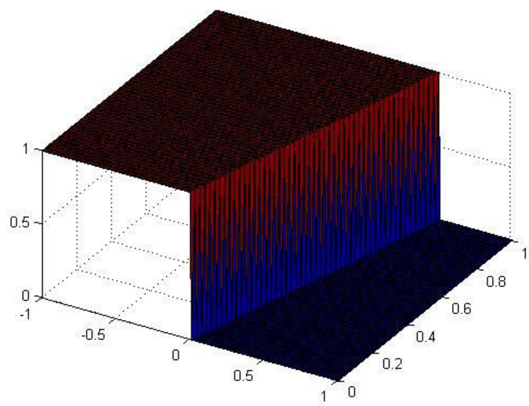
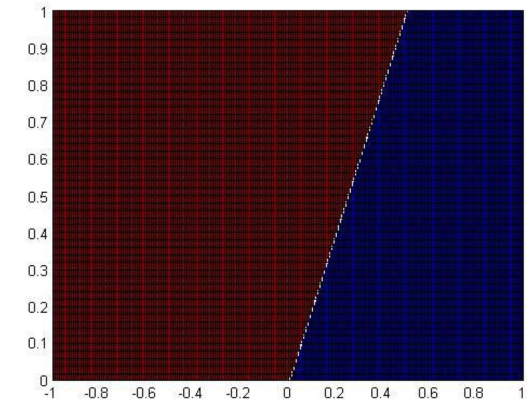
Problem 4

(a) Checking the stable condition with $u_l = 1$. $u_r = 0$.

The domain is [-5, 5]. Time is [0,1]. $u_l = 1$. $u_r = 0$.

Plotting x from [-1, 1], Time from [0,1].

	3D	X-T
Hx = 0.01 Ht = 0.005		
Hx = 0.01 Ht = 0.01		
Hx = 0.01 Ht = 0.02		

When the stable condition is violated (Hx = 0.01 Ht = 0.02), we cannot see shocks appearing (only red and blue colors in 3D plot). When the stable condition is valid, we can see shocks appearing (rainbow colors in 3D plot).

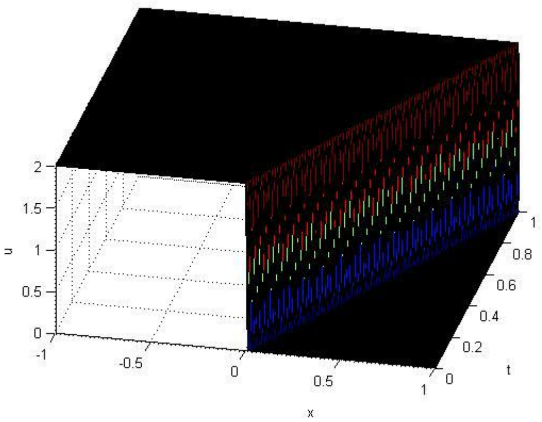
In x-t plot, we can conclude the shocking is traveling at $v = 0.5 = 0.5*(u_l + u_r)$.

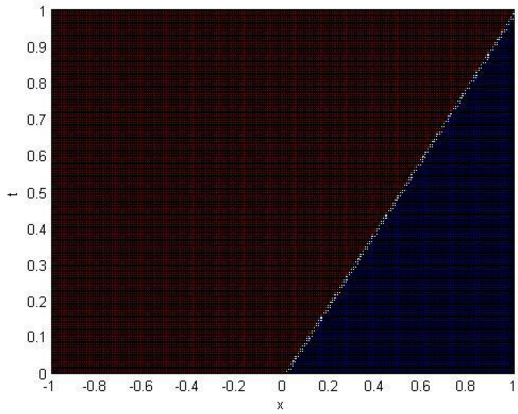
Problem 4

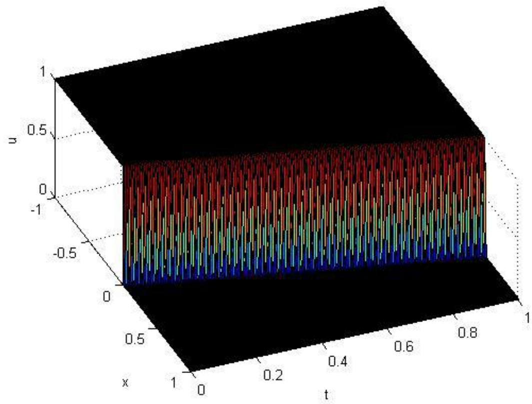
(b) Investigating the speed of shock.

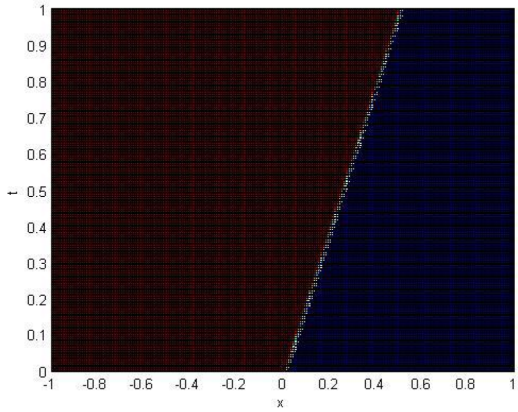
The domain is [-5, 5]. Time is [0,1]. $h_x = 0.01$. $h_t = 0.005$.

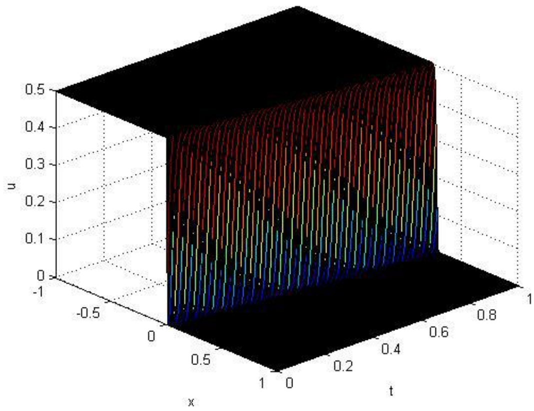
Plotting x from [-1, 1], Time from [0,1].

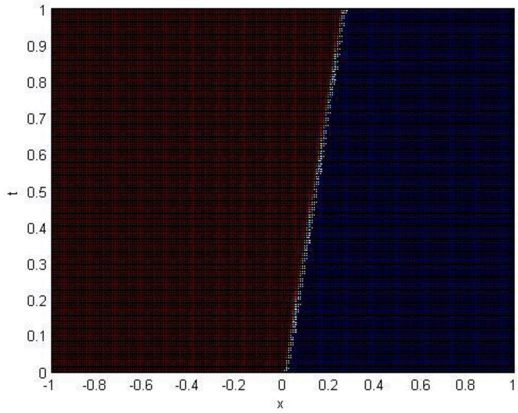
<table>
<tr><td></td><td>3D</td><td>X-T</td></tr>
<tr><td>$u_l = 2$
$u_r = 0$</td><td>


</td><td>


$v = 1 = 0.5*(u_l + u_r)$.</td></tr>
<tr><td>$u_l = 1$
$u_r = 0$</td><td>


</td><td>


$v = 0.5 = 0.5*(u_l + u_r)$.</td></tr>
<tr><td>$u_l = 0.5$
$u_r = 0$</td><td>


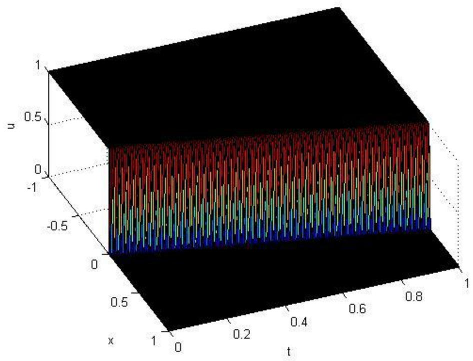
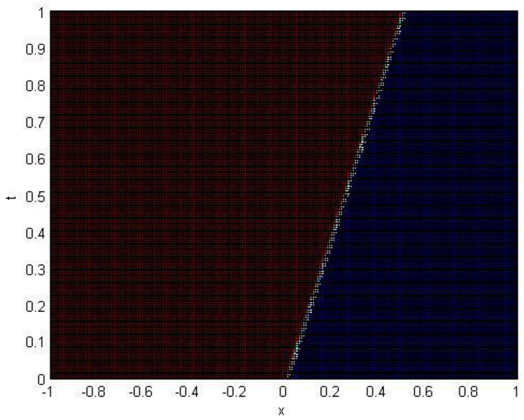
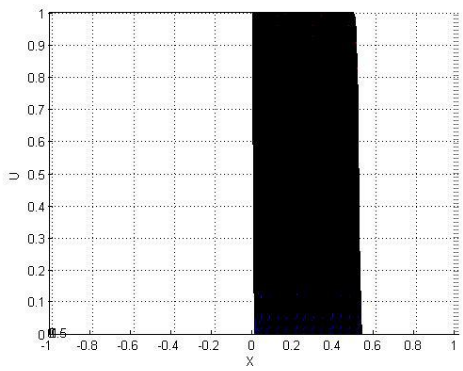
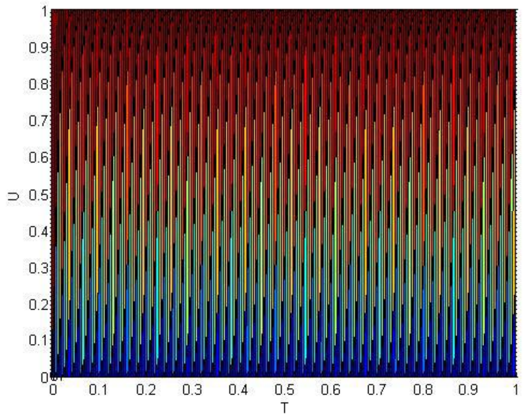
</td><td>


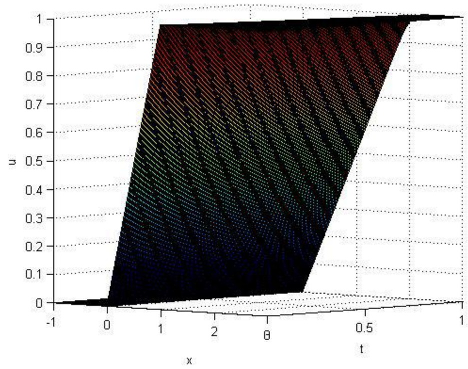
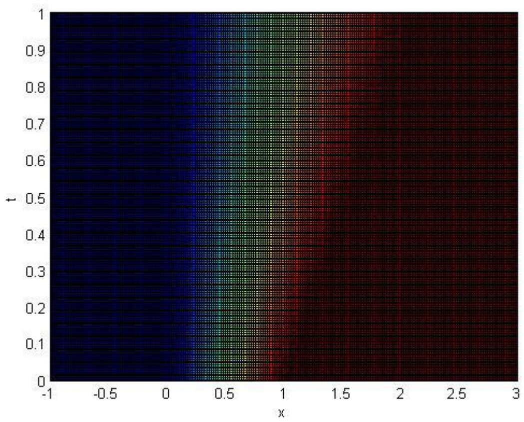
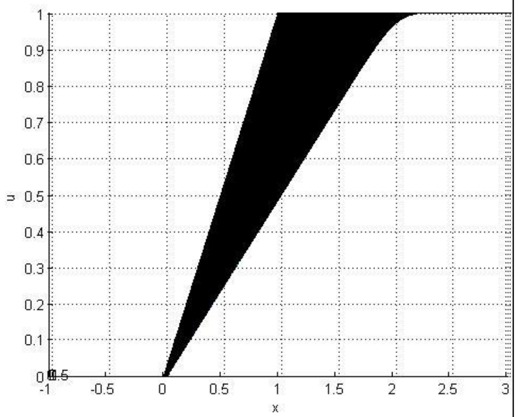
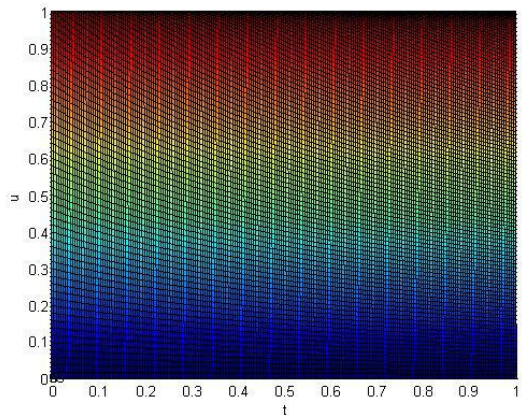
$v = 0.25 = 0.5*(u_l + u_r)$.</td></tr>
</table>

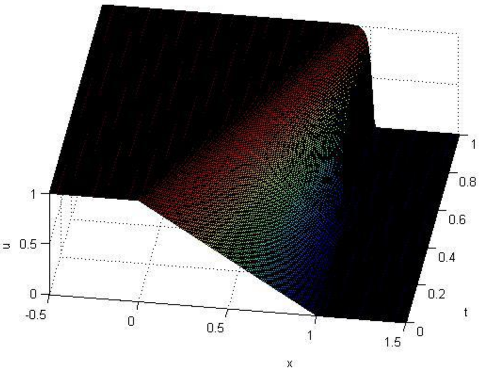
This shows $v = 0.5*(u_l + u_r)$.

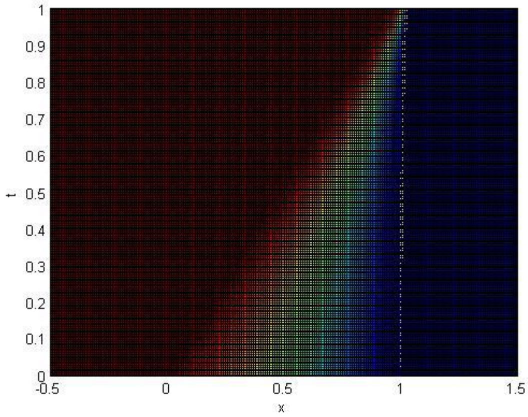
Problem 4: Plots of all 3 different u_0. T in [0,1] (u: Initial Condition; V: Speed of Shock)

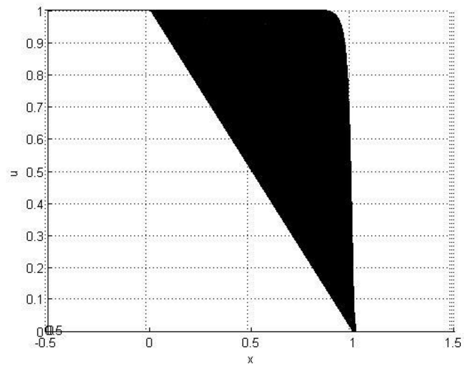
<table>
<tr><td rowspan="2">u_1= 0 (x<0)
1 (x>0)

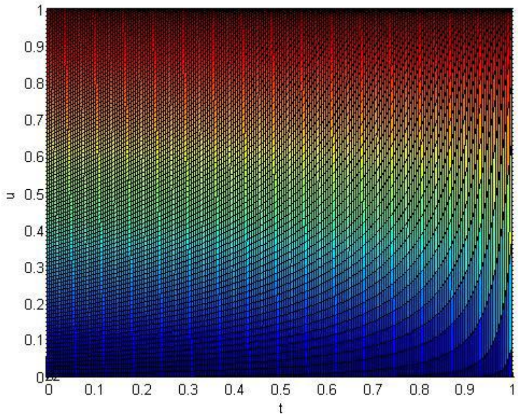
V= 0.5</td><td>
3D</td><td>
X-T</td></tr>
<tr><td>
U-X</td><td>
U-T</td></tr>
<tr><td rowspan="2">u_2= 0 (x<0)
X [0,1]
1 (x>1)

No Shock</td><td>
3D</td><td>
X-T</td></tr>
<tr><td>
U-X</td><td>
U-T</td></tr>
</table>

<table>
<tr>
<td rowspan="2">u_3= 1 (x<0)
1-X [0,1]
0 (x>1)</td>
<td>

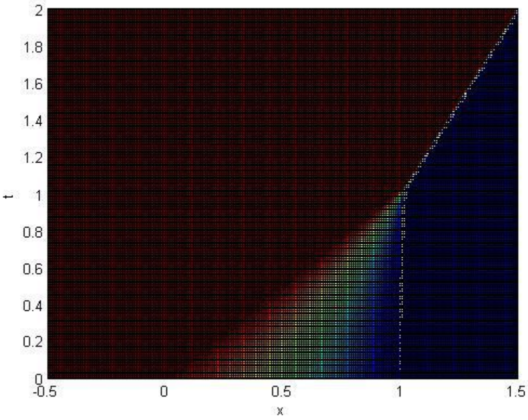
3D</td>
<td>

X-T (Inverse Fan)</td>
</tr>
<tr>
<td>

U-X</td>
<td>

U-T</td>
</tr>
</table>

For u_3 of Question 1(b), we plot another x-t picture until T=2:



The shock appears at t=1, the speed is 0.5. The x-axis of shock (depending on time t) is x=0.5*(t+1).