Numerical Analysis on Black-Scholes Model: An FEM Method

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1 Introduction

The Black-Scholes Model is a mathematical model of financial market containing derivative investment instruments. From the model, we can find a theoretical estimate of the price of European Options. The formula is called the "Black-Scholes Formula". Many empirical tests have shown that the BlackCScholes price is "fairly close" to the observed prices, although there are well-known discrepancies such as the "option smile".

Currently, many numerical methods in Partial Differential Equations, such as simulation-based methods, the lattice methods, have been well applied to the "Black-Scholes Formula", delaying the study of other numerical methods like finite elements methods, which has been widely used in many fields of science and engineering.

2 Goal of this poject

In this project, we will adapt the FEM methods into the "Black-Scholes Formula":

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0...[1]$$

where S is a real asset value, $0 \leq S \leq \infty$, V is the (real) option price, r is the risk-free rate, t is the time since the option was issued, $0 \leq t \leq T$, and is the real asset volatility. Eq. (1) is a backward moving equation, i.e. it is solved from the future to the present time.
For an European call option the time condition becomes a final condition because its value is known at the maturity date t = T and it is defined as its intrinsic value by:

$$V(S,T) = max(S-K,0), \forall S.$$

We will explore the way to discrete the variables and assemble the matrix to solve for the solution and compare the solution the theoretical results of options. Meanwhile, we also want to choose the boundary condition for the equation. The study is mainly focused on European Options. If time permitted, we might also study the formula of American Options.

Figure 1: Time to exercise

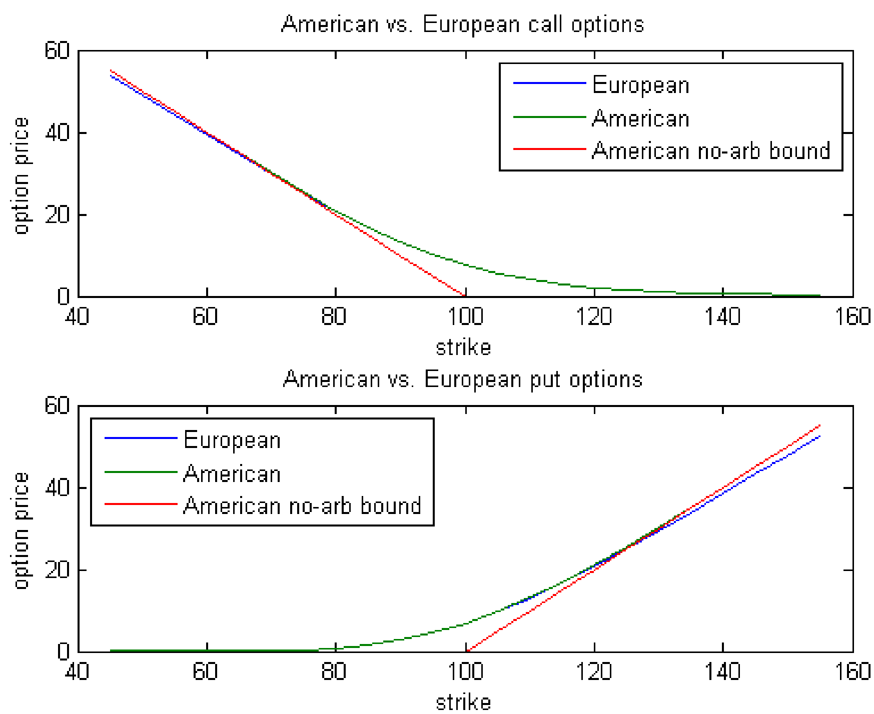

Figure 2: Different Value at Strikes

3 Reference

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