

1. My implementation here depends on A . I take A as an input and find n from A . The implementation is the same as the algorithm in 17-2 on note with simplification excluding all 0 entries in A .

The weight $w = \frac{2}{3}$, the eigenvalues are $1 - \frac{4}{3} \sin^2 \left[\frac{k\pi}{2(n+1)} \right]$. For $\frac{n}{2} \leq k \leq n$ these lie in the interval $[-\frac{2}{3}, \frac{2}{3}]$, which is the closest to 0 among $w \in [0, 1]$.

The implementation is in the Matlab files.

2. The reason it is a good test is because the numbers are random and there would be extreme cases with enough tests. Also, the actual answer is easy to calculate ($x=0$).

The converging ratio $R = \frac{\|b - Ax^{k+1}\|}{\|b - Ax^k\|}$ seems to converge to a number $\alpha \leq 1$, and α seems related to n . This might be explained by the change of eigen values with the growth in size of matrix. When $n \rightarrow \infty$, $\alpha \rightarrow 1$. In general, the speed of convergent is quite related to h . $h \downarrow$, speed \downarrow .

3. The ratio $R = \frac{\|b - Ax^{k+1}\|}{\|b - Ax^k\|}$ seems to be related to both k and n . $k \uparrow R \downarrow$; $n \uparrow R \downarrow$ (the jumps should be ignored, it might reach the min the computer could recognized).
What's more for weighted Jacobi $\frac{2}{3}$ converges ^{most} quickly.