Tufts University - Department of Mathematics
Math 253 - Homework 5

Let A be the finite-element matrix that you found for the one-dimensional diffusion equation in HW 3, discretized on a uniform mesh with n unknowns, but with Dirichlet boundary conditions on both ends. That is, A corresponds to the linear finite-element discretization of $-u''(x) = f(x)$ on $(0,1)$ with $u(0) = 0$ and $u(1) = 0$.

1. Implement the weighted-Jacobi and Gauss-Seidel iterations to solve $Ax = b$.
   Note: There are several possible ways to do this, including to have generic Jacobi and Gauss-Seidel functions that take A as an input, or to have functions that are specific to this choice of A for a given value of n. You are free to implement this in any way that you choose. However, please indicate which approach you took and how you did it. Which weight should you choose for weighted-Jacobi and why?

2. Test the convergence of your method by using it to solve $Ax = 0$ with a random initial guess for x. (Why is this a good test problem?) Plot the norms of the residual and error as a function of iteration for one realization of the initial guess. Also plot the relative reduction in residual norm per iteration, $\frac{\|0-Ax^{(\ell)}\|}{\|0-Ax^{\ell-1}\|}$. What can you say about the "asymptotic" convergence as ℓ gets large? Does this depend on n?

3. Test the convergence of your method with initial guesses $x_i^{(0)} = \sin(k\pi\frac{i}{n})$. What can you say about how convergence depends on k? Does this depend on n? For weighted-Jacobi does the weight have an effect?