

$$2. (a2) S(k) = \frac{1-2i\lambda \sin\theta}{1+2i\lambda \sin\theta} = \frac{1-4\lambda^2 \sin^2\theta}{1+4\lambda^2 \sin^2\theta} - i \frac{4\lambda \sin\theta}{1+4\lambda^2 \sin^2\theta} \quad (\theta = khx)$$

Here $|S(k)|=1$ so there is no dissipation. (Fig. 2-a2-1)
 For $S(k) = e^{i\phi}$
 so $\phi = i \ln \frac{1-2i\lambda \sin\theta}{1+2i\lambda \sin\theta} / (kht)$

On the other hand;

$$S(k) = e^{i\phi} = \cos\phi + i \sin\phi$$

$$\text{so } \phi = -\arcsin\left(\frac{4\lambda \sin khx}{1+4\lambda^2 \sin^2 khx}\right)$$

$$\text{so } V_{ph} = \frac{\arcsin\left[\frac{4\lambda \sin khx}{1+4\lambda^2 \sin^2 khx}\right]}{k ht} \quad \lambda = \frac{aht}{hx}$$

For plot please see Fig 2-a2-2 2-a2-3 2-a2-4

If we set $a=1$ (we can always re-scale x to get that),

For small hx , when $ht \rightarrow 0$ ($\lambda \rightarrow 0$) $V_{ph} \rightarrow 1$ Fig 2-a2-2

For small ht , when $hx \rightarrow 0$ $V_{ph} \rightarrow 1$ Fig 2-a2-3

For small λ , when $hx \rightarrow 0$, $V_{ph} \rightarrow 1$ Fig 2-a2-4