2-a1 For First Order Upwind:

For dissipation, one can set a=1, since we can re-scale x and t to achieve that. That means V_{ph} is only related to h_t and h_x.

We first set h_t=0.01 to see what's going on if CFL is satisfied. As h_x goes to h_t, |s(k)|goes to 1. (Figure 2-a1-1). This is true since S(k)=exp(-ikh_x) when λ=1.

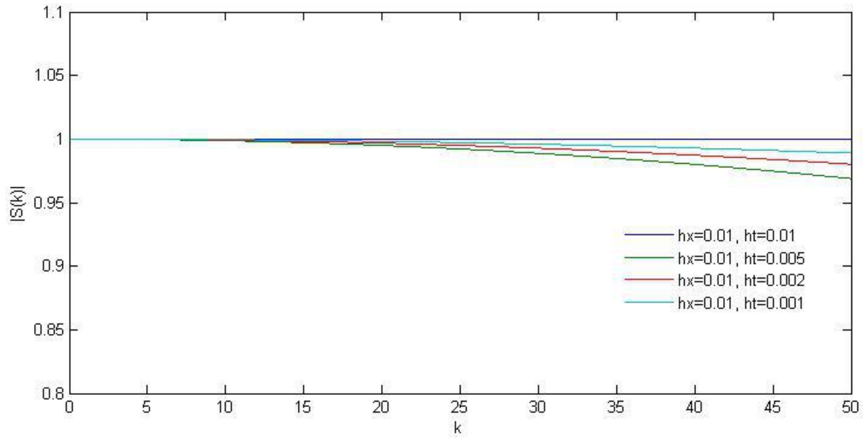


Figure 2-a1-1

We first set h_t=0.01 to see what's going on if CFL is not satisfied. As h_x goes bigger, |s(k)| blows up. (Figure 2-a1-2). This will bring unstability. (L=30.)

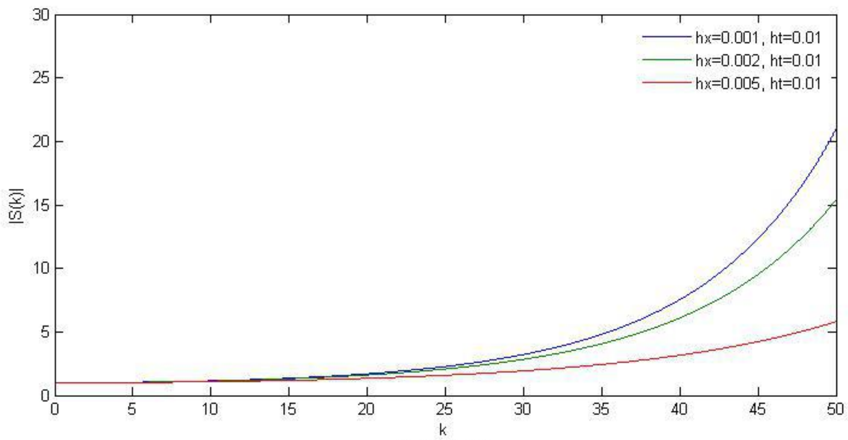


Figure 2-a1-2

For dispersion, if λ→1 or λ→0, V_{ph}→1. And λ→1 has a bigger effect in eliminating dispersion.

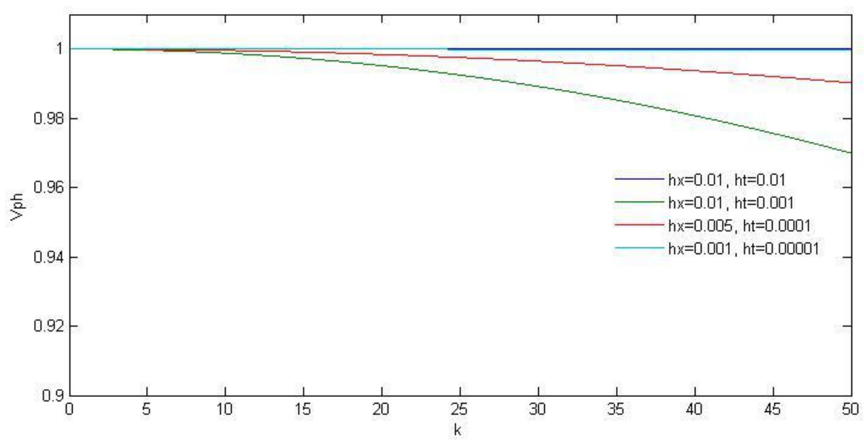


Figure 2-a1-3

2-a2 For Crank-Nicholson:

For dissipation, |s(k)|=1. So there is no dissipation in C-N. (Figure 2-a2-1)

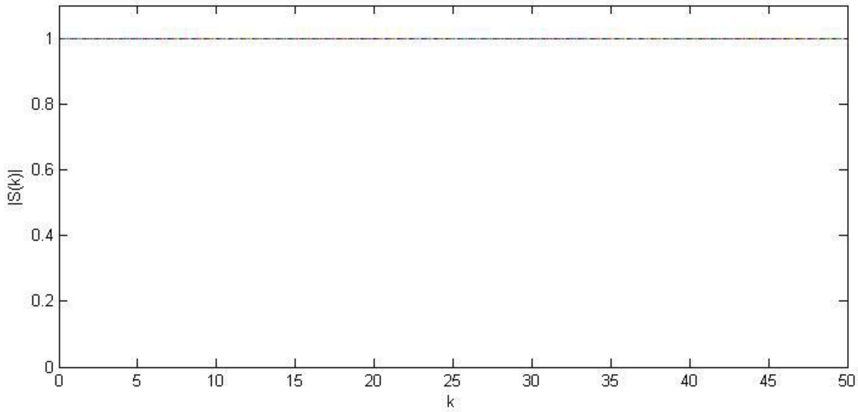


Figure 2-a2-1

For dispersion, again one can set a=1, since we can re-scale x and t to achieve that. That means V_{ph} is only related to h_t and h_x.

We first set h_t=0.01 to see what's going on. As h_x goes to 0, V_{ph} goes to 1. (Figure 2-a2-2)

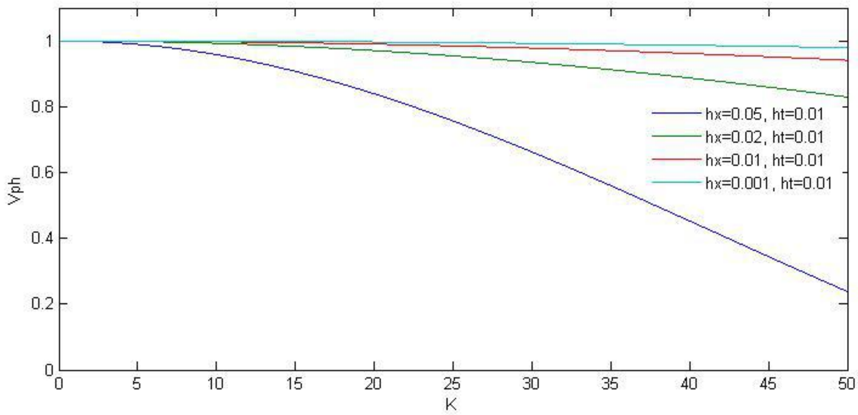


Figure 2-a2-2

We than set h_x=0.01 to see what's going on. As h_t goes to 0, V_{ph} goes to 1. (Figure 2-a2-3)

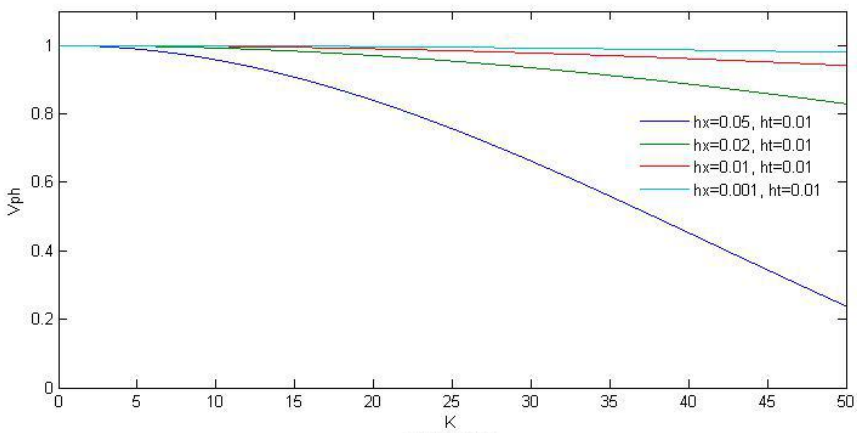


Figure 2-2a-3

Finally, we set $\lambda=0.25(h_t=h_x)$ to see what's going on. As h_x goes to 0, V_{ph} goes to 1. (Figure 2-a2-4)

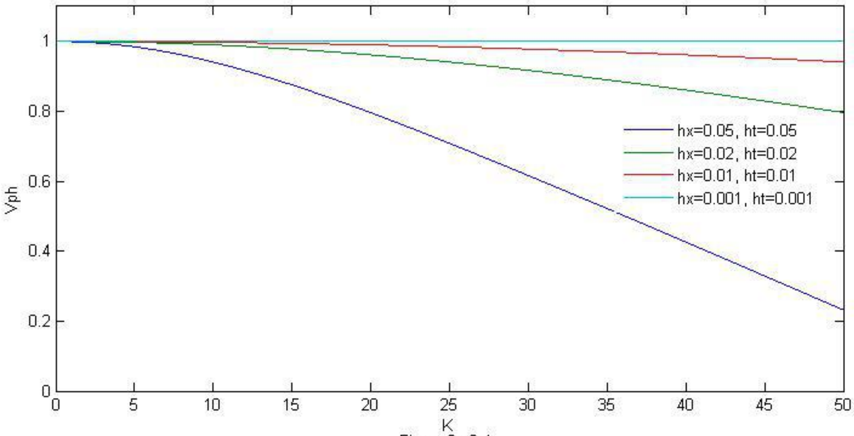
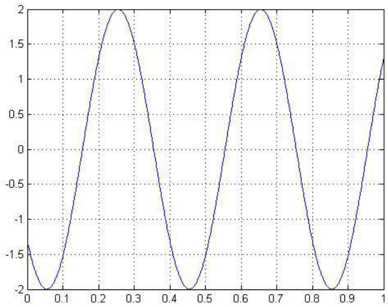
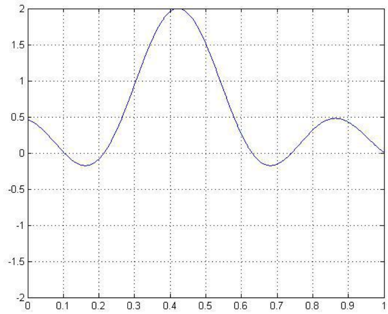
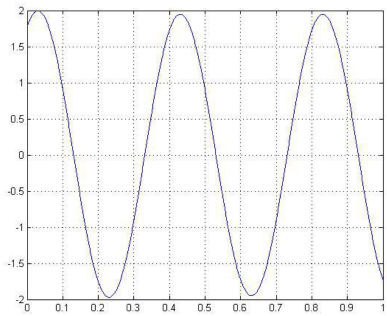
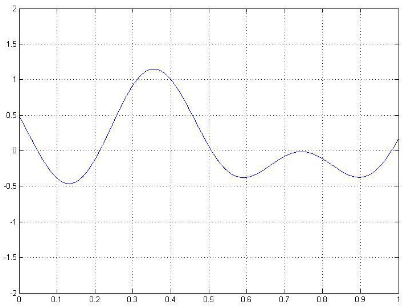
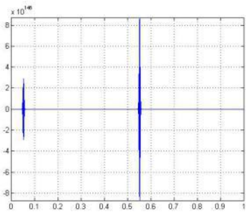
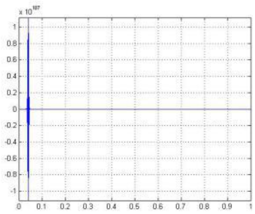
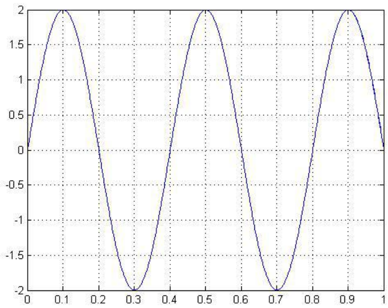
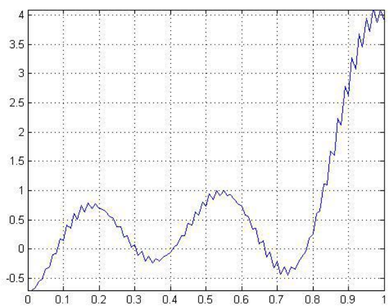


Figure 2-a2-4

Our conclusion here is for C-N, the dispersion always exists. To control it, we can try to make both h_x and h_t smaller. However, for functions decomposed into Fourier series with distinct small K_m and large K_n(i.e $K_n\to\infty$), we cannot avoid dispersion.

2-b Observation of Dissipation and Dispersion

	Dissipation	Dispersion
1st Upwind CFL λ=1	 2Cos(5πx) No dissipation	 0.5(cos(15x)+cos(x)+cos(5x)+cos(10x)) No Dispersion
1st Upwind CFL λ=0.5	 2Cos(5πx) Dissipation: The max is smaller than 2 on the right.	 0.5(cos(5πx)+cos(0.5πx)-cos(πx)-cos(3πx)); Dispersion: Small Wiggles Observed
1st Upwind non-CFL λ=10	Unstable  2Cos(5πx)	Unstable  0.5(cos(15x)+cos(x)+cos(5x)+cos(10x))
Crank-Nicholson	 2Cos(5πx) No dissipation	 0.5(cos(15x)+cos(x)+cos(5x)+cos(10x)) Dispersion