253 HW4.

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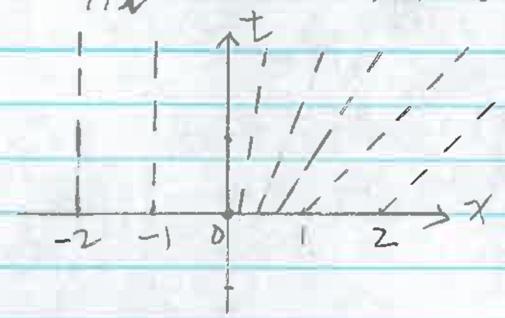
1. (a). $x(t) = x_0 + (u_0(x_0))t$

when $x < 0$ $\quad x(t) = x_0$

when $0 \le x \le 1$ $\quad x(t) = x_0 + x_0 t$ $\quad x_0 = \frac{x}{1+t}$ $\quad t = (x(t) - x_0) \cdot \frac{1}{x_0}$

when $x > 1$ $\quad x(t) = x_0 + t$

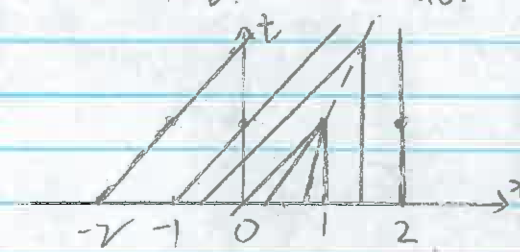


(b). $x(t) = x_0 + (u_0(x_0))t$

when $x < 0$, $\quad x(t) = x_0 + t$

when $0 \le x \le 1$ $\quad x(t) = x_0 + (1 - x_0)t$ $\quad x_0 = \frac{x(t) - t}{1-t}$ $\quad t = \frac{x(t) - x_0}{1 - x_0}$

when $x > 1$ $\quad x(t) = x_0$



(c) in (a) the characteristics don't intersect when $t > 0$

in (b) the characteristics intersect at $\{x=1, t=1\}$.

2. (a) The speed $\frac{du}{dt}$ of $x < 0$ is larger than the speed of $x > 0$. This means the initial condition on the left is passing along with time much more quickly and collide with the condition on the right.

At $t = 0$, the shock happens at $x = 0$.

(b). The density on the left of the shock is u_L

The density on the right of the shock is u_r

At time 0 the shock is at $X_T = 0$;

At time T, the shock is at $X_T = ST$.

Therefore the total amount of mass at $t=0$ is:

$$Q(0) = u_l \cdot 1 + u_r \cdot x \quad ①$$

the total amount of mass at $t=T$ is:

$$Q(T) = u_l \cdot (1+sT) + u_r \cdot (x - sT) \quad ②$$

② − ① $\quad Q(T) - Q(0) = sT(u_l - u_r).$

(c) $\quad Q(T) - Q(0) = \int_0^T \frac{u_l^2}{2} - \frac{u_r^2}{2} \, dt.$

$$= \int_0^T (u_l - u_r) \frac{u_l + u_r}{2} \, dt$$

$$= (u_l - u_r) \frac{u_l + u_r}{2} \cdot T$$

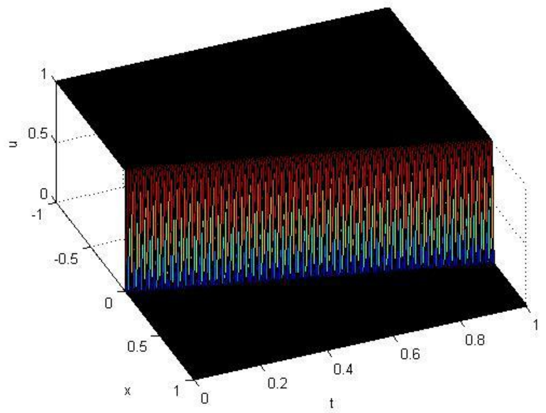
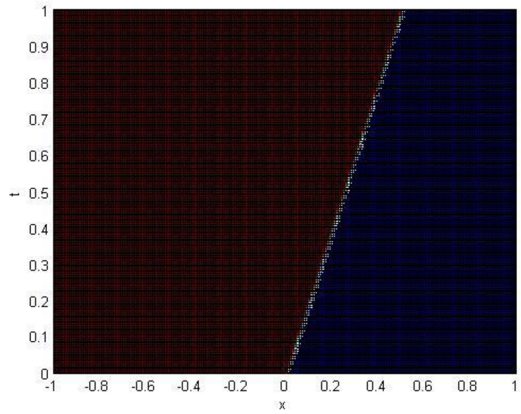
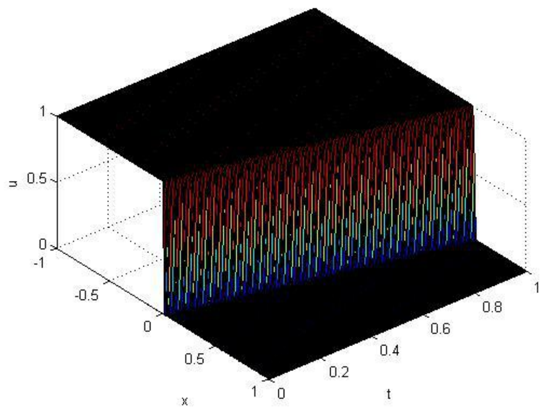
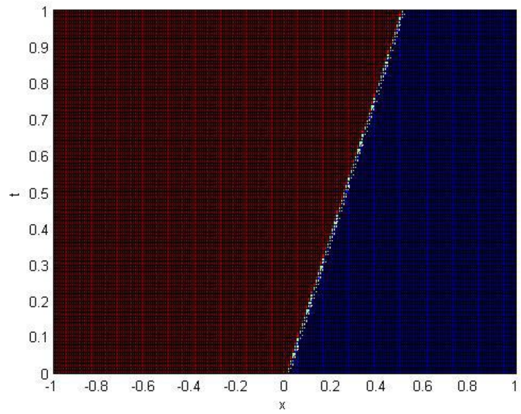
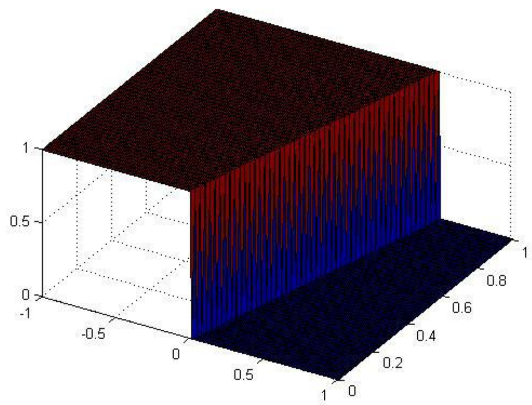
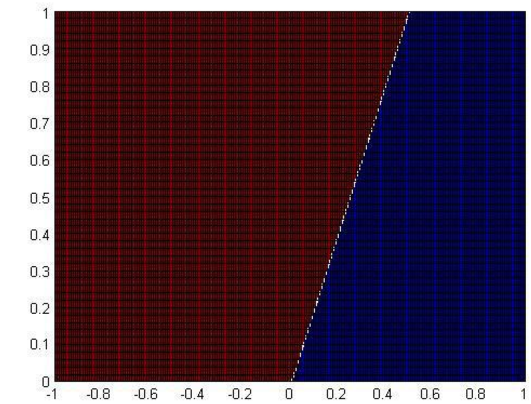
so $\quad s = \frac{u_l + u_r}{2}$

Problem 4

(a) Checking the stable condition with $u_l = 1$. $u_r = 0$.

The domain is [-5, 5]. Time is [0,1]. $u_l = 1$. $u_r = 0$.

Plotting x from [-1, 1], Time from [0,1].

	3D	X-T
Hx = 0.01 Ht = 0.005		
Hx = 0.01 Ht = 0.01		
Hx = 0.01 Ht = 0.02		

When the stable condition is violated (Hx = 0.01 Ht = 0.02), we cannot see shocks appearing (only red and blue colors in 3D plot). When the stable condition is valid, we can see shocks appearing (rainbow colors in 3D plot).

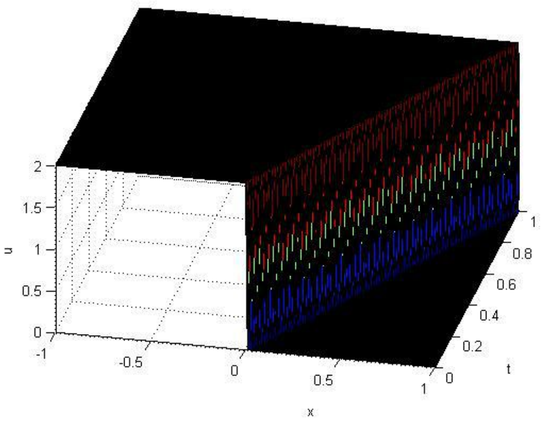
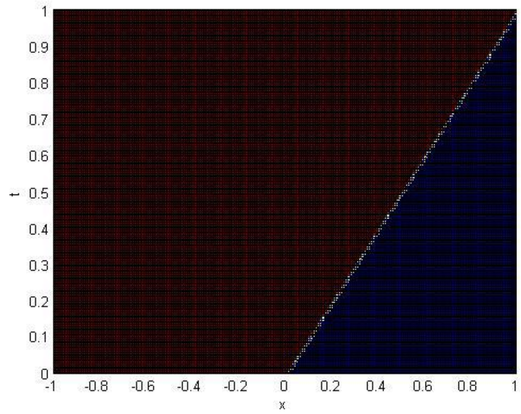
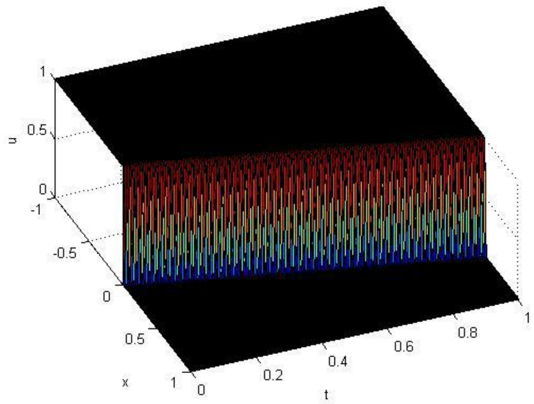
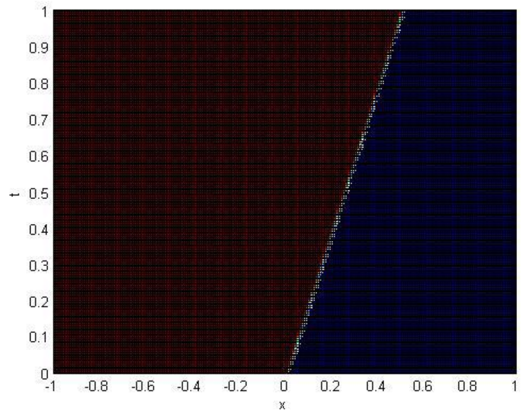
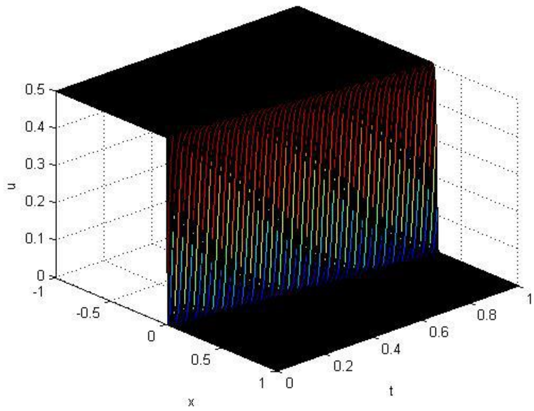
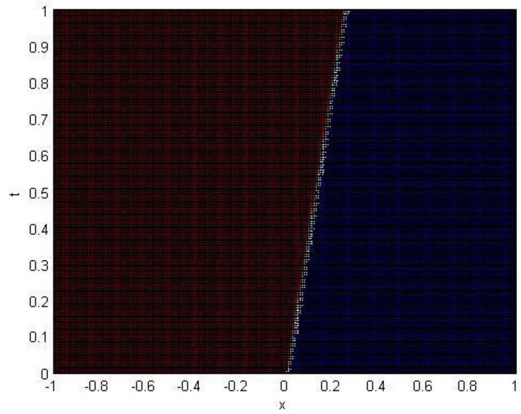
In x-t plot, we can conclude the shocking is traveling at $v = 0.5 = 0.5*(u_l + u_r)$.

Problem 4

(b) Investigating the speed of shock.

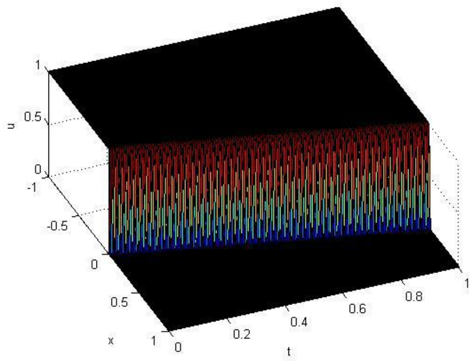
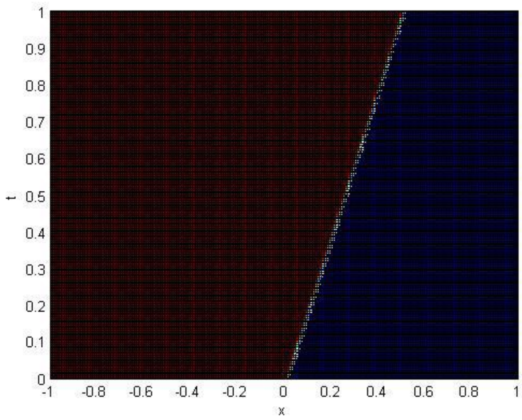
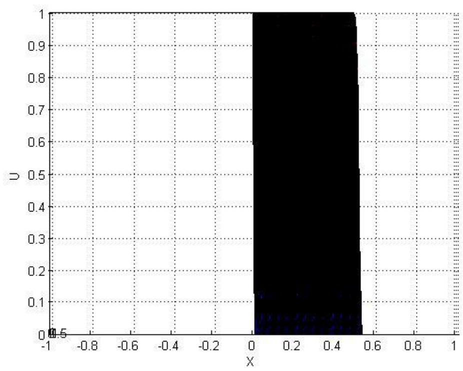
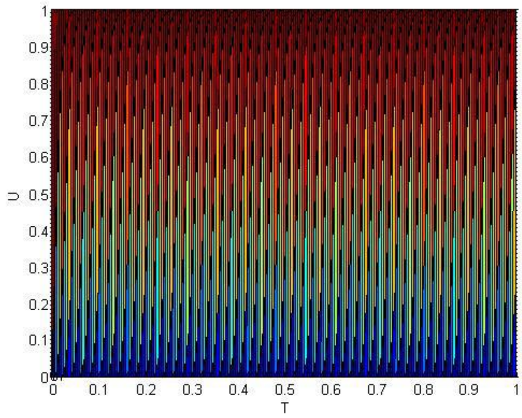
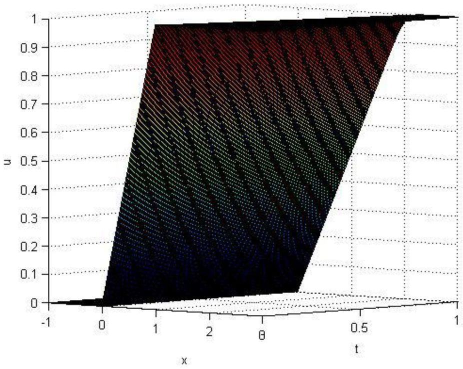
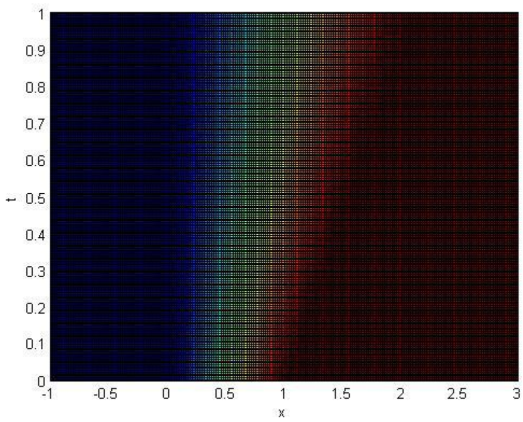
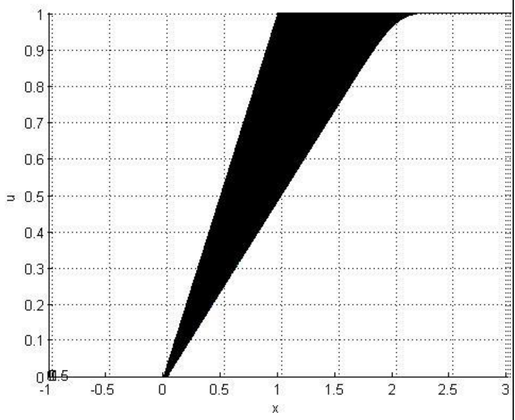
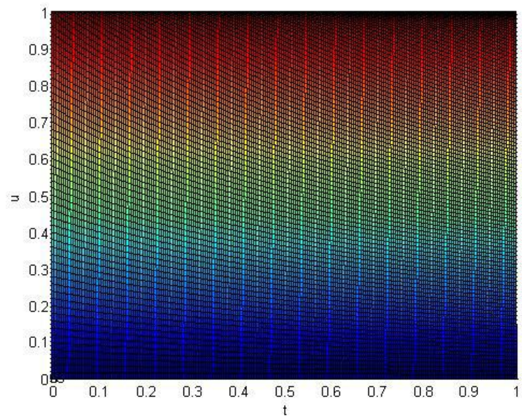
The domain is [-5, 5]. Time is [0,1]. $h_x = 0.01$. $h_t = 0.005$.

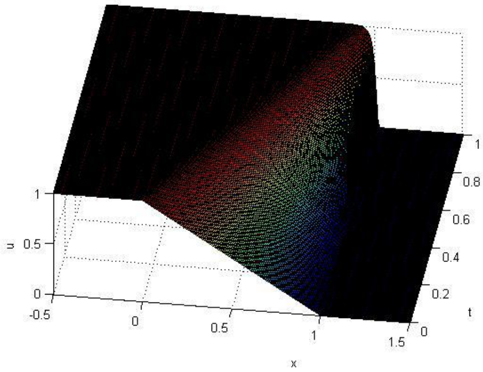
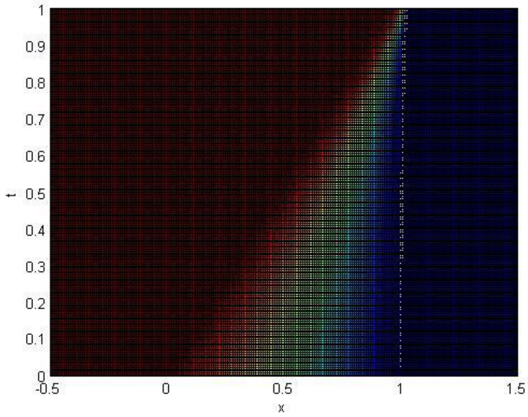
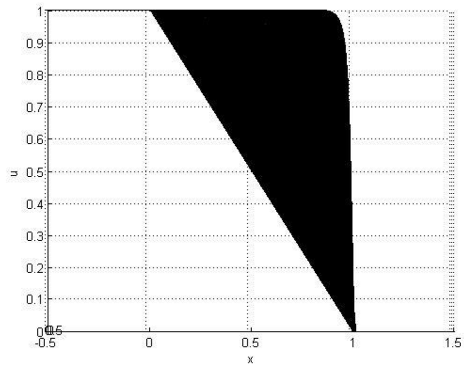
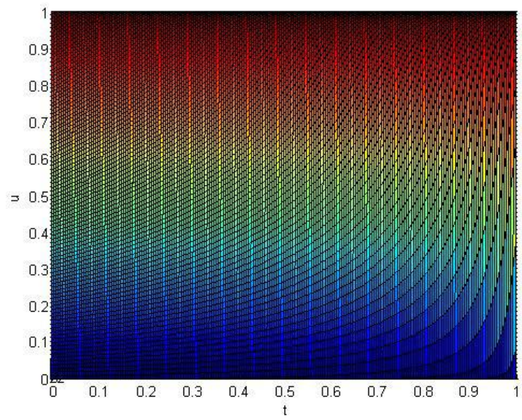
Plotting x from [-1, 1], Time from [0,1].

	3D	X-T
$u_l = 2$ $u_r = 0$		 $v = 1 = 0.5*(u_l + u_r)$.
$u_l = 1$ $u_r = 0$		 $v = 0.5 = 0.5*(u_l + u_r)$.
$u_l = 0.5$ $u_r = 0$		 $v = 0.25 = 0.5*(u_l + u_r)$.

This shows $v = 0.5*(u_l + u_r)$.

Problem 4: Plots of all 3 different u_0. T in [0,1] (u: Initial Condition; V: Speed of Shock)

u_1= 0 (x<0) 1 (x>0) V= 0.5	 3D	 X-T
	 U-X	 U-T
u_2= 0 (x<0) X [0,1] 1 (x>1) No Shock	 3D	 X-T
	 U-X	 U-T

<table>
<tr><td rowspan="2">u_3= 1 (x<0)
1-X [0,1]
0 (x>1)</td><td>
3D</td><td>
X-T (Inverse Fan)</td></tr>
<tr><td>
U-X</td><td>
U-T</td></tr>
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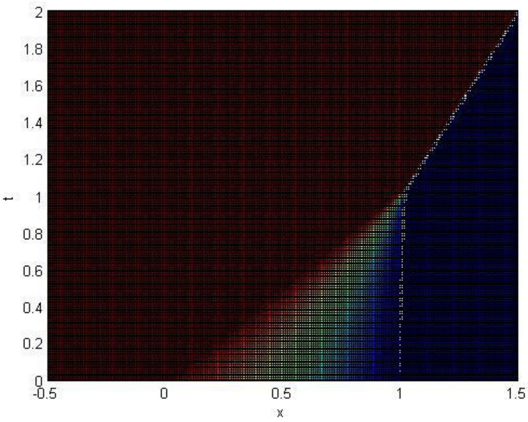
For u_3 of Question 1(b), we plot another x-t picture until T=2:



The shock appears at t=1, the speed is 0.5. The x-axis of shock (depending on time t) is x=0.5*(t+1).