253 Sheng Xu 1205525 HW1 P1

1(a) ① $\frac{V_{k,l+1}-V_{k,l}}{h_t}$

$= \frac{1}{h_t}[V_t h_t] + \frac{1}{h_t} O_\alpha(h_t^2) = V_t + O_1(h_t)$

$\frac{V_{k+1,l}-V_{k-1,l}}{2h_x}$

$= \frac{1}{2h_x}[V_{k,l} + V_x h_x + \frac{1}{2!}h_x^2 + O_\alpha(h_x^3)]$

$- \frac{1}{2h_x}[V_{k,l} - V_x h_x + \frac{1}{2!}h_x^2 + O_\beta(h_x^3)]$

$= V_x + O_1(h_x^2)$

$O_1 = O_\alpha + O_\beta$

$O_1' = a \cdot O_1$ so $E_{xt} = V_t + O_2(h_t) + aV_x + O_1'(h_x^2)$

$= O_2(h_t) + O_1'(h_x^2) = O(h_t, h_x^2)$

Since $V_t + aV_x = 0$

stability ② $V_{k,l+1} = \frac{aht}{2hx} V_{k-1,l} + V_{k,l} - \frac{aht}{2hx} V_{k+1,l}$

So $e_{l+1}^{(j)} \exp(ijkhx) = \lambda e_l^{(j)} \exp[ij(k-1)hx] + e_l^{(j)} \exp[ijkhx] - \lambda e_l^{(j)} \exp[ij(k+1)hx]$

$\lambda = \frac{aht}{2x}$

So $S(j) = \lambda \exp(-ijhx) + 1 - \lambda \exp(ijhx)$

$= 1 - i2\lambda \sin\theta$ $(\theta = jhx)$

So $|S(j)| > 1$ when $\theta \neq k\pi$, $\lambda > 0$, unstable