MATH 226, HOMEWORK 2, DUE OCT. 16, 2015

For problems that include programming, please include the code and all outputted figures and tables. Please label these clearly and refer to them appropriately in your answers to the questions.

(1) Derive the Peano kernel for the midpoint rule on interval $[a, b]$ and show the error for the midpoint rule is

$$E(f) = \frac{(b-a)^3}{24} f''(\xi),$$

where $\xi \in [a, b]$.

(2) Prove that $(n+1)$-point Gauss quadrature is the only $(n+1)$-point quadrature rule with degree of precision $2n+1$.

(3) Suppose that an numerical quadrature $I_h(f)$ has the following asymptotic expansion

$$I(f) - I_h(f) = c_1 h^{r_1} + c_2 h^{r_2} + c_3 h^{r_3} + \cdots$$

Here $0 < r_1 < r_2 < r_3 < \cdots$ and c_i are independent of h. Assume that we have computed $I_h(f)$, $I_{h/2}(f)$, and $I_{h/4}(f)$. Show how Richardson extrapolation can be used to the maximum extent to combine these values to get a higher order approximation to $I(f)$. What is the order of the new approximation?

(4) Write a code that uses adaptive composite Simpson's rule to approximate the integral

$$\int_1^\pi x^2 \sin x \mathrm{d}x.$$

Given the tolerance $\varepsilon = 10^{-8}$, report the approximation value.