Peeking Blackjack

Stanford CS221 Fall 2014-2015

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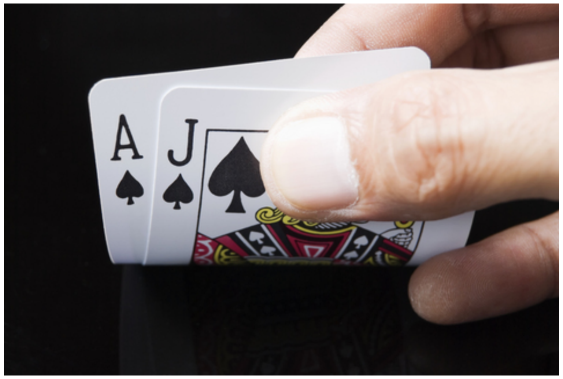
Note: grader.py only provides basic tests. Passing grader.py does not by any means guarantee full points.



The search algorithms explored in the previous assignment work great when you know exactly the results of your actions. Unfortunately, the real world is not so predictable. One of the key aspects of an effective AI is the ability to reason in the face of uncertainty.

Markov decision processes (MDPs) can be used to formalize uncertain situations where the goal is to maximize some kind of reward. In this homework, you will implement the algorithms that can be used to automatically construct an optimal policy for any such uncertain situation. You will then formalize a modified version of Blackjack as an MDP, and apply your algorithm to come up with an optimal policy.

Problem 1: Value Iteration Warmup

In this problem, you will perform the value iteration updates manually on a very basic game just to solidify your intuitions about solving MDPs. The set of possible states in this game is {-2, -1, 0, 1, 2}. You start at state 0, and if you reach either -2 or 2, the game ends. At each state, you can take one of two actions: {-1, +1}.

If you're in state s and choose -1:

- You have an 80% chance of reaching the state $s - 1$.
- You have a 20% chance of reaching the state $s + 1$.

If you're in state s and choose +1:

- You have a 30% chance of reaching the state $s + 1$.
- You have a 70% chance of reaching the state $s - 1$.

If your action results in transitioning to state -2, then you receive a reward of 20. If your action results in transitioning to state 2, then your reward is 100. Otherwise, your reward is -5. Assume the discount factor γ is 1.

a. Give the value of $V_{\text{opt}}(s)$ for each state s after running two iterations of value iteration. What is the resulting policy?

Problem 2: Implementing Value Iteration

Now, you will implement value iteration for finding the optimal policy automatically on on any MDP. Later, we'll create the specific MDP for Blackjack.

a. Implement the value iteration algorithm as discussed in lecture to compute the values $V_{\text{opt}}(s)$ of the optimal policy as well as the optimal policy π_{opt} itself. Fill out the `solve()` function in class `ValueIteration`.

b. If we add noise to the transitions of an MDP, does the optimal value get worse? Specifically, consider an MDP with reward function $\text{Reward}(s, a, s')$, state space States, and transition function $T(s, a, s')$. We define a new MDP which is identical to the original, except for its transition function,

$T'(s, a, s')$, which is defined as

$$T'(s, a, s') = \frac{T(s, a, s') + \alpha}{\sum_{s' \in \text{States}} \left[T(s, a, s') + \alpha\right]}$$

for some $\alpha > 0$. Let V_1 be the optimal value function for the original MDP, and V_2 the optimal value function for the MDP with added uniform noise. Is it always the case that $V_1(s_{\text{start}}) \geq V_2(s_{\text{start}})$? If so, prove it in `writeup.pdf` and put `return None` for each of the code blocks. Otherwise, construct a counterexample by filling out `CounterexampleMDP` and `counterexampleAlpha()`.

Problem 3: Peeking Blackjack

Now that we have written general-purpose MDP algorithms, let's use them to play (a modified version of) Blackjack. For this problem, you will be creating an MDP to describe a modified version of Blackjack.

For our version of Blackjack, the deck can contain an arbitrary collection of cards with different values, each with a given multiplicity. For example, a standard deck would have card values $\{1, 2, \ldots, 13\}$ and multiplicity 4. However, you could also have a deck with card values $\{1, 5, 20\}$, or any other set of numbers. The deck is shuffled (each permutation of the cards is equally likely).

The game occurs in a sequence of rounds. Each round, the player either (i) takes a card from the top of the deck (costing nothing), (ii) peeks at the top card (costing `peekCost`, in which case the next round, that card will be drawn), or (iii) quits the game. Note that it is not possible to peek twice; if the player peeks twice in a row, then `succAndProbReward()` should return `[]`.

The game continues until one of the following conditions becomes true:

- The player quits, in which case her reward is the sum of the cards in her hand.
- The player takes a card, and this leaves her with a sum that is greater than the threshold, in which case her reward is 0.
- The deck runs out of cards, in which case it is as if she quits, and she gets a reward which is the sum of the cards in her hand.

As an example, assume the deck has card values $\{1, 2, 3\}$, with multiplicity 1. Let's say the threshold is 4. Initally, the player has no cards, so her total is 0. At this point, she can peek, take, or quit. If she takes, the three possible successor states are
$\{$(1, `None`, (0, 1, 1)), (2, `None`, (1, 0, 1)), (3, `None`, (1, 1, 0))$\}$. She will receive 0 reward for reaching any these states.
If she instead peeks, the three possible successor states are $\{$(0, 0, (1, 1, 1)), (0, 1, (1, 1, 1)), (0, 2, (1, 1, 1))$\}$ (0, 1, and 2 refer to the index of the peeked card). Note that she has to pay the `peekCost` to reach these states.

In your code, you should signify the end of the game by setting your deck to `None`. For example, let's say her current state is (3, `None`, (1, 1, 0)). If she quits, the successor state will be (3, `None`, `None`). If she takes, the successor states are (3 + 1, `None`, (0, 1, 0)) or (3 + 2, `None`, `None`). Note that in the second successor state, the deck is set to `None` to signify the game ended with a bust. You should also set the deck to `None` if the deck runs out of cards.

a. Implement the game of Blackjack as an MDP by filling out the `succAndProbReward()` function of class `BlackjackMDP`. To help out out, we have already given you `startState()`.

b. Let's say you're running a casino, and you're trying to design a deck to make people peek a lot. Assuming a fixed threshold of 20, and a peek cost of 1, your job is to design a deck where for at least 10% of states, the optimal policy is to peek. Fill out the function `peekingMDP()` to return an instance of `BlackjackMDP` where the optimal action is to peek in at least 10% of states.

Problem 4: Learning to play Blackjack

So far, we've seen how MDP algorithms can take an MDP which describes the full dynamics of the game and return an optimal policy. But suppose you go into a casino, and no one tells you the rewards or the transitions. We will see how reinforcement learning can allow you to play the game and learn the rules at the same time!

a. You will first implement a generic Q-learning algorithm `QLearningAlgorithm`, which is an instance of an `RLAlgorithm`. As discussed in class, reinforcement learning algorithms are capable of executing a policy while simultaneously improving their policy. Look in `simulate()`, in `util.py` to see how the `RLAlgorithm` will be used. In short, your algorithm will be run in a simulation of the MDP, and will alternately be asked for an action to perform in a given state, and then be informed of the result of that action, so that it may learn better actions to perform in the future.

Recall that Q-learning attempts to learn a Q function for an MDP, and uses the Q function to construct its policy. To improve generalization, instead of learning the Q function directly, we represent states and actions using a feature representation. That is, we have a `featureExtractor()` function that maps from a (state, action) pair to a feature vector. We then learn a weight vector that maps from this feature representation to an approximate Q value. You can see this in action in the `getQ()` function. This function computes a dot product of the current weight vector with the feature values extracted from a given (state, action) pair.

Note that we represent a feature vector as a list of (object, double) pairs. This represents the same information as a dict, in that each object (feature) is mapped to a double (value). This sparse list representation is more efficient than using a dict when most of the feature values are 0. Because we only ever use our feature vector in dot products with or addition to the weight vector, we can represent the feature vector as a list and the weight vector as a dict. We can then always iterate over the feature vector and look up corresponding values in the weight vector dict.

At every step, Q-learning will select an action according to an ϵ-greedy policy. That is, with probability ϵ, it will select an action uniformly at random, and with probability $1 - \epsilon$, it will select action

$$\pi(s) = \arg\max_{a \in \text{Actions}(s)} Q(s, a)$$

where Q is its current estimate of the Q function for the MDP. We've implemented this action selection step for you in `QLearningAlgorithm.getAction()`.

After action selection, the simulation will call `QLearningAlgorithm.incorporateFeedback()` so that the Q-learning algorithm can improve its estimate of Q. The function `incorporateFeedback` will be called with parameters (s, a, r, s'), where s and a are the state and action that the algorithm chose in the previous `getAction` step, and r and s' are the reward that was received and the state to which the MDP transitioned. In the `incorporateFeedback` function, you first compute the residual

$$r = \left[\text{Reward}(s, a, s') + \gamma \max_{a' \in \text{Actions}(s')} Q^{(t)}(s', a')\right] - Q^{(t)}(s, a)$$

This should then be used to update the weight vector representing our Q function:

$$w^{(t+1)} = w^{(t)} + \eta \cdot r \cdot \phi(s, a)$$

Here ϕ is the feature extractor `self.featureExtractor` and η is the step size `getStepSize()`. Implement `QLearningAlgorithm.incorporateFeedback()`.

b. Call `simulate` using your algorithm and the `identityFeatureExtractor()` on the MDP `smallMDP`, with 30000 trials. Compare the policy learned in this case to the policy learned by value iteration. Don't forget to set the explorationProb of your Q-learning algorithm to 0 after learning the policy. How do the two policies compare (i.e., for how many states do they produce a different action)? Now run `simulate()` on `largeMDP`. How does the policy learned in this case compare to the policy learned by value iteration? What went wrong?

c. To address the problems explored in the previous exercise, we incorporate domain knowledge to improve generalization. This way, the algorithm can use what it learned about some states to improve its prediction performance on other states. Implement `blackjackFeatureExtractor`. Using this feature extractor, you should be able to get pretty close to the optimum on the `largeMDP`.

d. Now let's explore the way in which value iteration responds to a change in the rules of the MDP. Run value iteration on `originalMDP` to compute an optimal policy. Then apply your policy to `newThresholdMDP` by calling `simulate` with `FixedRLAlgorithm`, instantiated using your computed policy. What reward do you get? What happens if you run Q learning on `newThresholdMDP` instead? Explain.