(e) $f(x,y) = x^2+2xy+y^2-2x$ subject to $x^2-y^2=-1$.

Introducing Lagrange's multiplier, the function now becomes,

$$\phi(x,y) = x^2+2xy+y^2-2x-\lambda(x^2-y^2+1)$$

Partially differentiating $\phi(x,y)$ wrt x & y, we get the following :-

$$\frac{\partial\phi(x,y)}{\partial x} = 2x+2y-2-2x\lambda = 0$$

or $x+y-1-x\lambda=0$

or $x(1-\lambda)+y=1$ — (i)

$$\frac{\partial\phi(x,y)}{\partial y} = 2x+2y+2y\lambda=0$$

or $x+y+\lambda y=0$

or $-y(1+\lambda)=x$ — (ii)

Putting this value of x in (i), we get

$-y(1+\lambda)(1-\lambda)+y=1$

or $-y(1-\lambda^2)+y=1$

or $\cancel{-y}+y\lambda^2\cancel{+y}=1$

or $\boxed{\lambda^2=\frac{1}{y}}$ or $\boxed{y=\frac{1}{\lambda^2}}$

or $\cancel{\lambda=\pm\frac{1}{y}}$

Putting this value of y in eqn (ii), we get :-