Hence, we get 4 set of solutions.

$$\left.\boxed{\begin{array}{c} x=0,\ y=2 \\ x=0,\ y=-2 \\ x=2,\ y=0 \\ x=-2,\ y=0 \end{array}}\right\} \text{Ans.}$$

All the 4 solutions satisfy the constraint $x^2+y^2=4$.

Also $f(0,2)$ and $f(0,-2) = -4 \Leftarrow$ minima

and $f(2,0)$ and $f(-2,0) = 4 \Leftarrow$ maxima.

Hence, the above 4 sets of x and y are the extrema of the function.

(b) $f(x,y) = x^2y - \log x$ subject to $x+2y=0$

Introducing Lagrange's multiplier, we get a new function,

$$\phi(x,y) = x^2y - \log x - \lambda(x+2y)$$

Now, differentiating partially w.r.t x and y, we get,

$$\frac{\partial\phi(x,y)}{\partial x} = 0 \text{ or } \frac{\partial}{\partial x}\left(x^2y - \log x - \lambda(x+2y)\right) = 0$$

$$\text{or } 2xy - \frac{1}{x} - \lambda = 0$$

$$\text{or } 2x^2y - 1 - \lambda x = 0. \quad \text{— (i)}$$

$$\frac{\partial\phi(x,y)}{\partial y} = 0 \text{ or } \frac{\partial}{\partial y}\left(x^2y - \log x - \lambda(x+2y)\right) = 0$$

$$\text{or } x^2 - 2\lambda = 0$$

$$\text{or } x = \pm\sqrt{2\lambda}$$