

# Dynamic Quadratic Decomposition-based Evolutionary Algorithm for Multi-objective Fuzzy Flexible Jobshop Scheduling

## — Supplementary File

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### I. DISCUSSION AND ANALYSIS OF RESULTS

#### A. Verification of the effectiveness of initialization strategy

We have validated the impact of different initialization methods on algorithm performance through experiments. The experimental details are presented in Table V. DQD-MOEA1, DQD-MOEA2, and DQD-MOEA3 respectively indicate that the algorithm uses only initialization rules 1, 2, and 3 to generate the initial population. The experimental results suggest that all initialization rules have an undeniable contribution to the performance of the algorithm.

#### B. Parameter analysis

DQD-MOEA comprises four steps: initialization, offspring generation, elite selection based on dynamic quadratic decomposition, and local search. In the initialization stage, three different rules generate the initial population, and thus no parameters require special settings. During offspring generation, two parents cross to produce offspring, followed by mutation operations. The cross operation is essential, thus, there is no cross probability, and the only parameter is the mutation probability. In the elite selection based on the dynamic quadratic decomposition stage, subregions with solutions are divided into two. Only the solution with the best convergence in each subregion is designated as the elite solution. It's essential to note that when there's only one solution in a subregion, it is inevitably chosen as the elite solution. Thus, performing quadratic decomposition is meaningful only when the number of solutions in the subregion exceeds 1, with no parameters needing adjustment. In other words, all subregions need to perform dynamic quadratic decomposition, so there are no

parameters that need to be adjusted at this stage. In the local search stage, a random local search operator is selected through a roulette wheel in each iteration, and no parameters need to be set. In summary, the proposed DQD-MOEA has only one parameter during execution, which is the mutation probability.

To verify the selected parameter settings, we conducted a series of experiments. Assuming unchanged termination conditions for the algorithm, we considered a wide range of population sizes, i.e.,  $\mathcal{N} \in \{20, 50, 80, 100\}$ . The HV values obtained under these parameters are presented in Table VI. DQD-MOEA-100, DQD-MOEA-20, DQD-MOEA-50, and DQD-MOEA-80 represent the algorithm using 100, 20, 50, and 80 as population sizes, respectively. The mutation probability is uniformly set at 0.8. The comparison results indicate that DQD-MOEA achieved optimal performance with a population size of 100. Therefore, for this article, the population size is set to 100. We also conducted an experiment to verify the value of the mutation probability, and the HV obtained by the algorithm under different mutation probabilities are shown in Table VII. DQD-MOEA-0.8, DQD-MOEA-0.2, DQD-MOEA-0.4, and DQD-MOEA-0.6 indicate the algorithm using 0.8, 0.2, 0.4, and 0.6 as mutation probabilities, respectively. The population size is uniformly set at 100. The comparison results show that DQD-MOEA performed best with a mutation probability of 0.8. Consequently, the mutation probability for this article is set at 0.8.

#### C. Validation of the effectiveness of different components of DQD-MOEA

We have verified the contributions of different strategies to the algorithm through experiments, and the results are presented in Table VIII. DQD-MOEA-O implies no utilization of any proposed strategies. DQD-MOEA-I indicates that only the proposed initialization strategy is employed within DQD-MOEA. DQD-MOEA-L indicates that both the proposed initialization and local search strategies are utilized within DQD-MOEA. From the table, it can be observed that the algorithm's performance is weaker than that of DQD-MOEA when only one strategy is used. This reveals that different strategies make significant contributions to the algorithm's overall performance.

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TABLE V: Comparison results of HV values obtained by DQD-MOEA under different initialization rules.

	MOEA-DQD1	MOEA-DQD2	MOEA-DQD3	MOEA-DQD
data1	5.5060E-02(3.21E-03)	5.9544E-02(2.10E-03)	5.5357E-02(4.01E-03)	<b>6.1523E-02(3.15E-03)</b>
data2	5.2500E-02(7.13E-02)	8.6656E-02(5.17E-02)	8.9125E-02(6.93E-02)	<b>9.1281E-02(1.42E-03)</b>
data3	6.8064E-02(2.10E-03)	5.8202E-02(4.20E-03)	7.0689E-02(8.67E-03)	<b>7.6595E-02(9.49E-03)</b>
data4	4.0642E-02(2.00E-03)	4.5457E-02(2.57E-03)	5.3145E-02(3.96E-03)	<b>5.7379E-02(3.25E-03)</b>
data5	2.8358E-02(1.91E-02)	3.1859E-02(7.20E-02)	9.9683E-02(5.51E-02)	<b>1.1191E-01(9.81E-02)</b>
FMk01	3.7508E-02(1.21E-03)	3.1445E-02(1.68E-03)	3.6621E-02(1.92E-03)	<b>3.9085E-02(2.91E-03)</b>
FMk02	3.9638E-02(2.88E-03)	3.8868E-02(2.77E-03)	3.9681E-02(2.20E-03)	<b>4.1569E-02(1.77E-03)</b>
FMk03	5.8671E-01(2.91E-01)	5.9890E-02(1.76E-03)	5.7446E-02(1.89E-03)	<b>6.1675E-02(2.60E-03)</b>
FMk04	7.9786E-02(1.25E-03)	8.1063E-02(3.94E-04)	8.0787E-02(1.46E-03)	<b>8.1997E-02(1.19E-03)</b>
FMk05	8.8611E-03(1.98E-02)	9.0935E-02(2.03E-02)	9.5351E-03(2.13E-02)	<b>1.0126E-02(1.23E-03)</b>
FMk06	3.2126E-02(1.84E-03)	2.8719E-02(2.67E-03)	<b>3.4463E-02(2.34E-03)</b>	3.4394E-02(2.38E-03)
FMk07	3.0098E-02(1.04E-01)	3.3483E-02(1.78E-03)	4.0765E-02(7.45E-04)	<b>4.1696E-02(2.50E-03)</b>
FMk08	6.5372E-03(1.46E-02)	7.8340E-03(1.75E-02)	1.7699E-03(3.96E-03)	<b>8.1178E-03(1.82E-02)</b>
FMk09	3.1786E-02(1.24E-03)	3.5845E-02(1.35E-03)	4.2102E-02(2.14E-03)	<b>4.6464E-02(6.05E-04)</b>
FMk010	2.8877E-02(4.27E-03)	2.4617E-02(8.81E-04)	2.9567E-02(1.57E-03)	<b>3.4630E-02(1.05E-03)</b>
remanu01	2.8014E-02(6.69E-04)	2.8454E-02(6.31E-04)	2.7730E-02(1.53E-04)	<b>2.8274E-02(8.02E-04)</b>
remanu02	2.5398E-02(2.31E-03)	2.8949E-02(2.06E-03)	2.4334E-02(1.60E-03)	<b>3.0309E-02(2.59E-03)</b>
remanu03	6.1215E-03(1.37E-02)	7.2065E-03(1.61E-02)	4.7367E-03(1.06E-02)	<b>7.9578E-03(1.78E-02)</b>
remanu04	5.4256E-02(2.27E-03)	4.8606E-02(3.32E-03)	5.0288E-02(1.19E-02)	<b>5.6203E-02(1.94E-02)</b>
remanu05	7.3525E-03(1.64E-02)	1.0114E-02(2.26E-02)	6.8361E-03(1.53E-02)	<b>1.9562E-02(2.69E-02)</b>
remanu06	6.9455E-02(4.34E-03)	9.7228E-02(2.12E-03)	6.6856E-02(2.46E-03)	<b>9.5646E-02(2.46E-03)</b>
remanu07	7.2313E-03(1.62E-02)	1.0267E-02(3.69E-03)	9.3791E-03(1.41E-02)	<b>2.4571E-02(3.40E-02)</b>
remanu08	4.9044E-02(5.26E-03)	1.0933E-01(2.49E-03)	5.7721E-02(2.84E-03)	<b>1.1876E-01(2.20E-03)</b>

TABLE VI: Comparison results of HV values obtained by DQD-MOEA under different population sizes.

	DQD-MOEA-100	DQD-MOEA-20	DQD-MOEA-50	DQD-MOEA-80
data1	<b>1.2485E-01(3.87E-03)</b>	5.6959E-02(5.49E-03)	6.7076E-02(8.29E-03)	1.0204E-01(6.97E-03)
data2	<b>8.1338E-02(2.57E-03)</b>	5.3335E-02(3.26E-02)	4.5589E-02(2.11E-03)	6.6248E-02(3.95E-03)
data3	<b>9.1322E-02(9.84E-03)</b>	4.8198E-02(5.15E-03)	4.9281E-02(4.04E-03)	7.7515E-02(7.21E-03)
data4	<b>3.4743E-02(3.38E-03)</b>	1.5476E-02(4.18E-03)	1.8411E-02(2.77E-03)	2.6828E-02(1.90E-03)
data5	<b>4.5887E-02(6.94E-02)</b>	1.1714E-02(2.16E-02)	1.6183E-02(3.37E-02)	1.1356E-02(2.09E-03)
FMk01	<b>5.4345E-02(3.54E-03)</b>	2.8931E-02(3.25E-03)	3.5123E-02(3.49E-03)	4.6457E-02(3.57E-03)
FMk02	<b>4.4651E-02(1.31E-03)</b>	1.9067E-02(1.71E-03)	2.4313E-02(1.57E-03)	3.4347E-02(2.01E-03)
FMk03	<b>6.9465E-02(4.07E-03)</b>	8.4209E-02(1.67E-01)	4.2145E-02(4.29E-03)	6.3067E-02(2.75E-03)
FMk04	<b>1.6951E-01(4.39E-03)</b>	7.8329E-02(9.40E-03)	1.0209E-01(7.13E-03)	1.5684E-01(3.64E-03)
FMk05	<b>7.5837E-02(7.51E-02)</b>	2.6718E-02(1.95E-03)	2.7938E-02(1.87E-03)	6.5110E-02(6.71E-02)
FMk06	<b>1.0299E-01(5.13E-03)</b>	4.8035E-02(5.17E-03)	5.6781E-02(5.56E-03)	8.2123E-02(3.66E-03)
FMk07	<b>5.7418E-02(3.57E-03)</b>	2.9379E-02(2.99E-03)	3.4846E-02(2.57E-03)	5.1203E-02(1.93E-03)
FMk08	<b>8.0257E-02(1.85E-01)</b>	4.5337E-02(1.11E-01)	1.3398E-02(1.44E-03)	1.9371E-02(2.11E-03)
FMk09	<b>3.7384E-02(2.20E-03)</b>	1.6744E-02(1.35E-03)	1.9537E-02(3.10E-03)	3.2745E-02(2.91E-03)
FMk010	<b>4.3570E-02(2.11E-03)</b>	1.9056E-02(2.60E-03)	1.8336E-02(3.99E-03)	3.6306E-02(2.44E-03)
remanu01	<b>3.3372E-02(1.23E-03)</b>	1.8699E-02(2.49E-03)	2.1143E-02(1.80E-03)	2.9136E-02(2.30E-03)
remanu02	<b>4.1876E-02(1.96E-03)</b>	1.7272E-02(3.01E-03)	5.1934E-02(8.27E-02)	3.7047E-02(2.08E-03)
remanu03	<b>3.0115E-02(4.36E-02)</b>	6.3478E-03(1.90E-02)	6.0666E-03(2.27E-03)	1.1680E-02(2.16E-03)
remanu04	<b>6.4837E-02(2.14E-03)</b>	3.5394E-02(3.29E-03)	4.1879E-02(2.75E-03)	5.9441E-02(1.52E-03)
remanu05	<b>7.3950E-03(1.57E-02)</b>	6.6935E-04(2.12E-03)	4.8304E-03(1.02E-02)	3.5824E-03(1.13E-02)
remanu06	<b>6.9110E-02(1.88E-03)</b>	3.3446E-02(3.30E-03)	3.9469E-02(3.18E-03)	6.0226E-02(3.61E-03)
remanu07	<b>4.6248E-02(6.68E-02)</b>	1.0423E-02(3.02E-02)	7.8398E-03(1.52E-03)	3.1002E-02(5.61E-02)
remanu08	7.3810E-02(4.38E-03)	3.2267E-02(3.67E-03)	3.9928E-02(5.96E-03)	<b>8.1603E-02(1.90E-03)</b>

TABLE VII: Comparison results of HV values obtained by DQD-MOEA under different mutation probabilities.

	DQD-MOEA-0.8	DQD-MOEA-0.2	DQD-MOEA-0.4	DQD-MOEA-0.6
data1	<b>1.2721E-01(5.14E-03)</b>	5.6958E-02(7.51E-03)	8.8387E-02(7.41E-03)	1.0200E-01(6.86E-03)
data2	<b>8.2108E-02(3.24E-03)</b>	5.2667E-02(3.28E-02)	7.9266E-02(4.60E-02)	<b>8.9501E-02(5.35E-02)</b>
data3	<b>9.0635E-02(1.14E-02)</b>	4.7686E-02(3.66E-03)	7.0521E-02(6.70E-03)	8.1199E-02(4.92E-03)
data4	<b>3.5270E-02(3.13E-03)</b>	1.4985E-02(3.59E-03)	2.3188E-02(5.29E-03)	2.6493E-02(6.11E-03)
data5	<b>4.8258E-02(7.09E-02)</b>	1.2184E-02(2.08E-02)	1.8730E-02(3.23E-02)	2.1626E-02(3.67E-02)
FMk01	<b>5.5496E-02(2.70E-03)</b>	2.8544E-02(4.19E-03)	4.5854E-02(5.06E-03)	5.0556E-02(5.04E-03)
FMk02	<b>4.3719E-02(2.02E-03)</b>	1.9173E-02(1.73E-03)	3.0164E-02(2.01E-03)	3.3382E-02(2.37E-03)
FMk03	6.9506E-02(4.57E-03)	3.7175E-02(1.46E-01)	6.2690E-02(2.40E-02)	<b>7.4329E-02(2.77E-02)</b>
FMk04	<b>1.6893E-01(6.01E-03)</b>	7.5966E-02(9.97E-03)	1.2342E-01(1.35E-02)	1.3597E-01(1.62E-02)
FMk05	<b>7.6513E-02(8.00E-02)</b>	2.6240E-02(1.95E-03)	4.3105E-02(3.82E-03)	4.7326E-02(3.74E-03)
FMk06	<b>1.0371E-01(6.33E-03)</b>	4.8836E-02(6.93E-03)	7.6429E-02(7.08E-03)	8.4768E-02(8.65E-03)
FMk07	<b>5.6913E-02(2.89E-03)</b>	2.9284E-02(1.68E-03)	4.6029E-02(3.39E-03)	5.1465E-02(3.46E-03)
FMk08	<b>7.9946E-02(1.84E-01)</b>	4.4600E-02(1.10E-01)	7.1214E-02(1.74E-01)	7.7443E-02(1.88E-01)
FMk09	<b>3.7204E-02(1.22E-03)</b>	1.6417E-02(2.00E-03)	2.5759E-02(2.74E-03)	2.8890E-02(3.02E-03)
FMk010	<b>4.3052E-02(2.54E-03)</b>	1.9243E-02(2.99E-03)	2.9512E-02(3.20E-03)	3.3090E-02(4.06E-03)
remanu01	<b>3.2832E-02(1.27E-03)</b>	1.8567E-02(3.25E-03)	2.7516E-02(3.29E-03)	3.2056E-02(4.25E-03)
remanu02	<b>4.1828E-02(2.74E-03)</b>	1.7045E-02(3.46E-03)	2.4938E-02(3.55E-03)	2.9806E-02(4.62E-03)
remanu03	<b>3.0140E-02(4.38E-02)</b>	6.4474E-03(1.93E-02)	9.6811E-03(2.90E-02)	1.1830E-02(3.54E-02)
remanu04	<b>6.5263E-02(2.37E-03)</b>	3.5198E-02(4.05E-03)	5.1611E-02(4.96E-03)	6.0730E-02(5.08E-03)
remanu05	<b>7.0929E-03(1.50E-02)</b>	5.6993E-04(1.80E-03)	9.3823E-04(2.97E-03)	6.0725E-03(3.39E-03)
remanu06	<b>7.0342E-02(1.77E-03)</b>	3.4048E-02(3.72E-03)	4.9752E-02(4.39E-03)	5.8150E-02(5.52E-03)
remanu07	4.6066E-02(6.68E-02)	1.0478E-02(3.04E-02)	3.5686E-02(4.56E-02)	<b>4.8056E-02(5.24E-02)</b>
remanu08	<b>7.4621E-02(4.74E-03)</b>	3.3329E-02(3.18E-03)	4.7353E-02(5.13E-03)	5.5852E-02(5.88E-03)

TABLE VIII: Comparison results of HV values obtained by DQD-MOEA under different strategies.

	MOEA-DQD-O	MOEA-DQD-I	MOEA-DQD-L	MOEA-DQD
data1	8.2707E-02(4.42E-03)	9.7789E-02(5.19E-03)	1.1569E-01(8.76E-03)	<b>1.4478E-01(6.68E-03)</b>
data2	8.7700E-02(3.14E-02)	1.0400E-01(3.83E-02)	1.2246E-01(4.37E-02)	<b>1.1783E-01(2.70E-03)</b>
data3	5.3345E-02(2.70E-03)	6.4058E-02(3.50E-03)	7.7143E-02(6.67E-03)	<b>8.6632E-02(9.19E-03)</b>
data4	1.7698E-02(4.13E-03)	2.0878E-02(4.95E-03)	2.5320E-02(6.96E-03)	<b>3.3555E-02(2.64E-03)</b>
data5	1.4245E-02(2.41E-02)	1.6985E-02(2.89E-02)	1.9794E-02(3.39E-02)	<b>4.4789E-02(6.48E-02)</b>
FMk01	3.5936E-02(3.63E-03)	4.2838E-02(3.77E-03)	5.0434E-02(5.13E-03)	<b>5.6896E-02(3.64E-03)</b>
FMk02	2.2263E-02(1.62E-03)	2.6223E-02(1.82E-03)	3.0889E-02(1.43E-03)	<b>4.2506E-02(1.79E-03)</b>
FMk03	9.7289E-02(1.91E-01)	1.1163E-01(2.14E-01)	1.3586E-01(2.57E-01)	<b>6.5462E-02(3.37E-03)</b>
FMk04	9.1081E-02(1.12E-02)	1.0655E-01(1.26E-02)	1.2894E-01(1.45E-02)	<b>1.6350E-01(4.69E-03)</b>
FMk05	3.1566E-02(2.59E-03)	3.7109E-02(2.86E-03)	4.3718E-02(3.56E-03)	<b>7.1546E-02(7.21E-02)</b>
FMk06	6.7698E-02(8.31E-03)	7.9367E-02(8.39E-03)	9.4968E-02(1.02E-02)	<b>1.1638E-01(5.03E-03)</b>
FMk07	3.3838E-02(2.14E-03)	4.0069E-02(2.49E-03)	4.8788E-02(3.59E-03)	<b>5.4873E-02(2.90E-03)</b>
FMk08	5.4008E-02(1.32E-01)	6.1859E-02(1.50E-01)	7.3646E-02(1.77E-01)	<b>7.9156E-02(1.82E-01)</b>
FMk09	1.9406E-02(2.40E-03)	2.2507E-02(2.30E-03)	2.7215E-02(3.42E-03)	<b>3.5880E-02(1.60E-03)</b>
FMk010	2.2673E-02(3.21E-03)	2.6786E-02(3.13E-03)	3.2319E-02(3.93E-03)	<b>4.2442E-02(2.11E-03)</b>
remanu01	2.1655E-02(2.63E-03)	2.5728E-02(3.13E-03)	3.0384E-02(3.01E-03)	<b>3.2892E-02(1.00E-03)</b>
remanu02	2.0280E-02(3.60E-03)	2.3722E-02(3.66E-03)	2.8652E-02(5.06E-03)	<b>4.1794E-02(2.60E-03)</b>
remanu03	1.5036E-02(2.61E-02)	1.8316E-02(3.25E-02)	2.1975E-02(3.85E-02)	<b>4.1372E-02(4.68E-02)</b>
remanu04	4.0977E-02(3.51E-03)	4.8392E-02(3.54E-03)	5.8108E-02(6.18E-03)	<b>6.4844E-02(2.39E-03)</b>
remanu05	7.7578E-04(2.45E-03)	9.0603E-04(2.87E-03)	1.0407E-03(3.29E-03)	<b>7.3029E-03(1.55E-02)</b>
remanu06	3.9808E-02(4.05E-03)	4.7123E-02(4.33E-03)	5.6573E-02(4.49E-03)	<b>6.9501E-02(2.09E-03)</b>
remanu07	1.2513E-02(3.61E-02)	1.4584E-02(4.21E-02)	1.6470E-02(4.73E-02)	<b>4.5849E-02(6.39E-02)</b>
remanu08	3.8419E-02(4.67E-03)	4.4866E-02(4.56E-03)	5.2764E-02(6.24E-03)	<b>7.4029E-02(4.88E-03)</b>