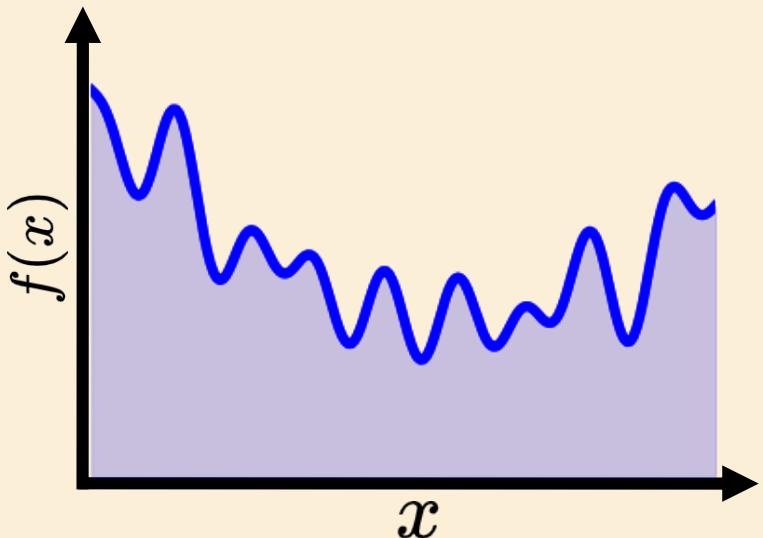


# Adaptive Multiple Control Variates for Many-Light Rendering

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Shandong University, China

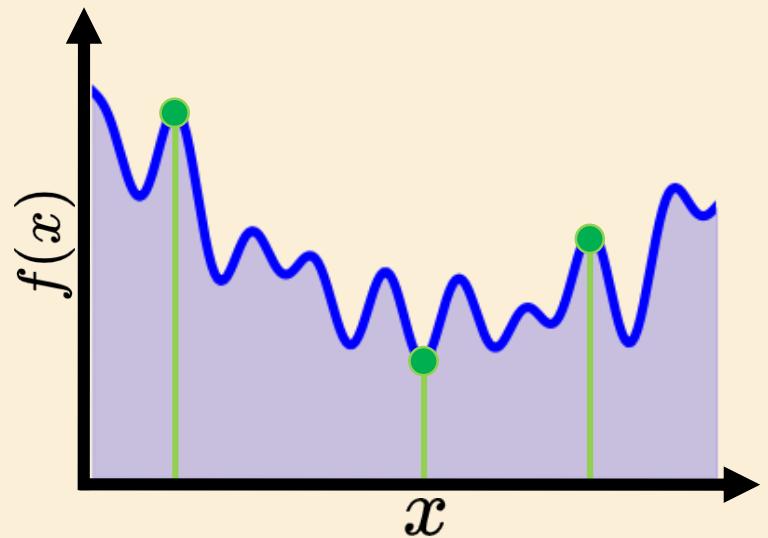
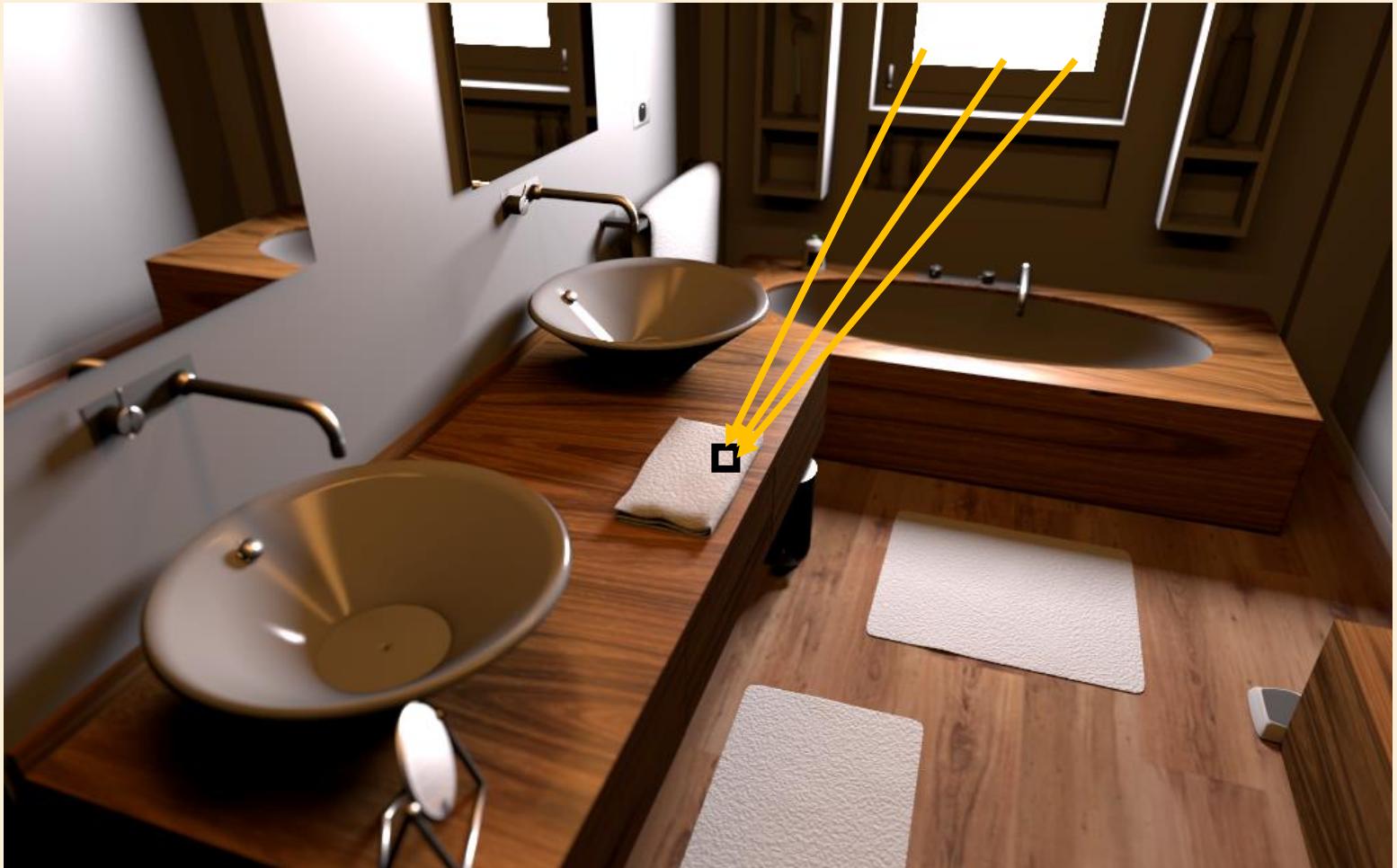


# ■ Problem Statement



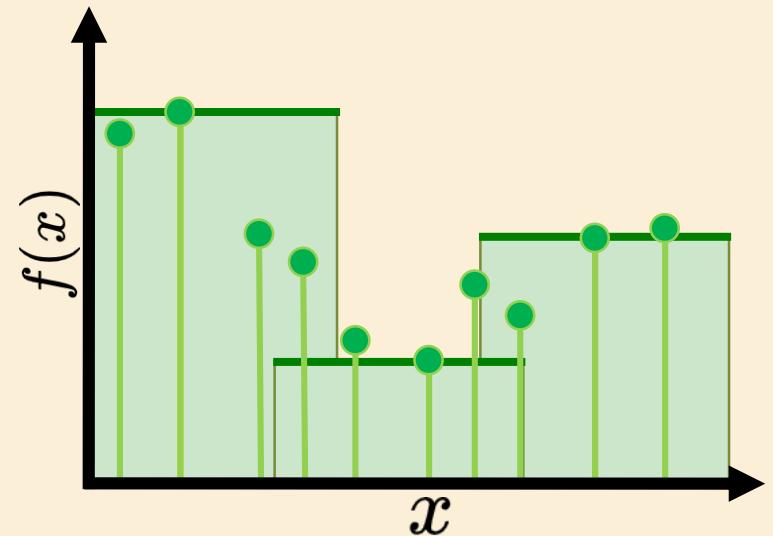
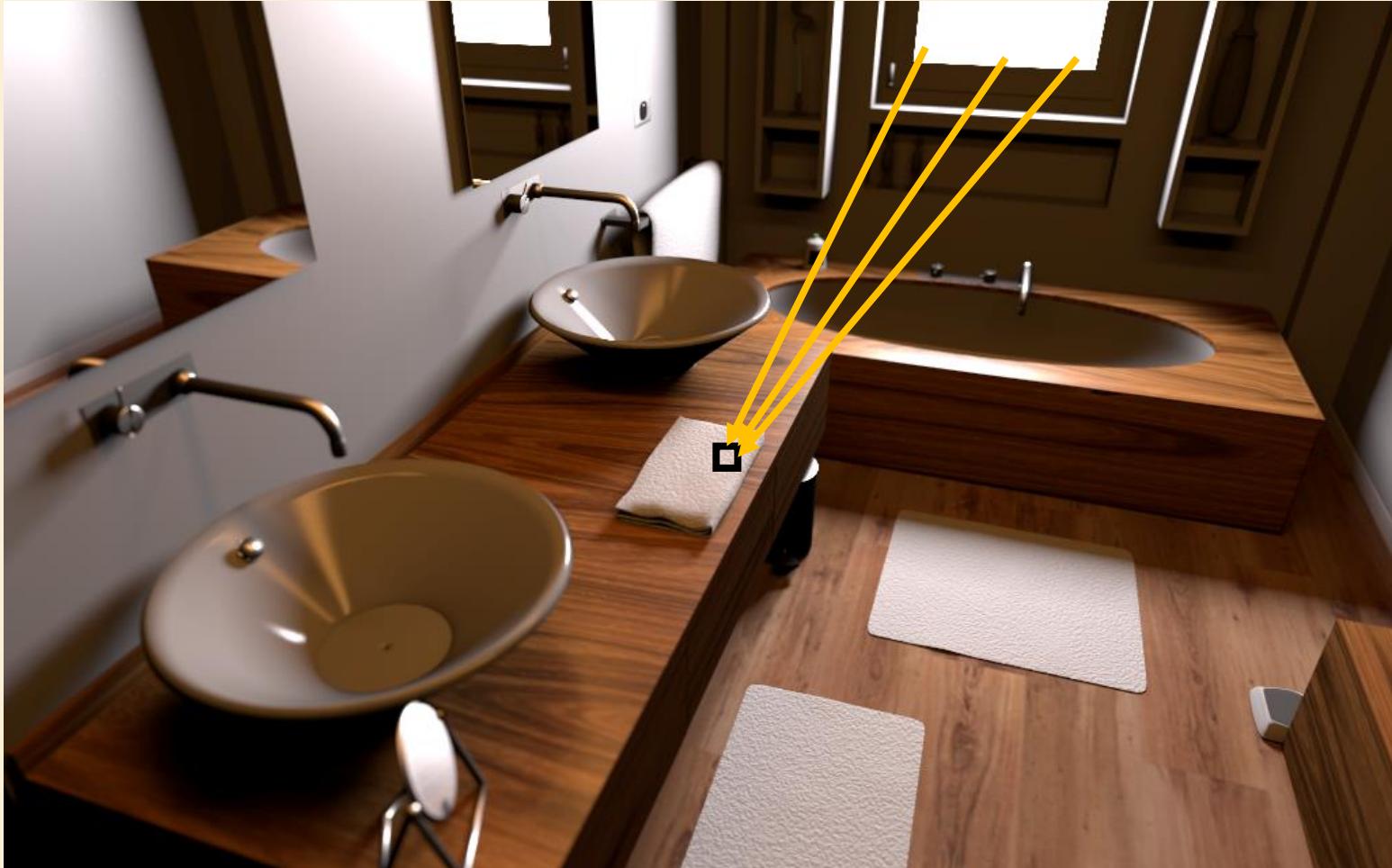
$$F = \int_{[0,1]^D} f(x) dx$$

# ■ Light Transport Context



Reproduced from Salaün et al. (2022) – SIGGRAPH Presentation Slides

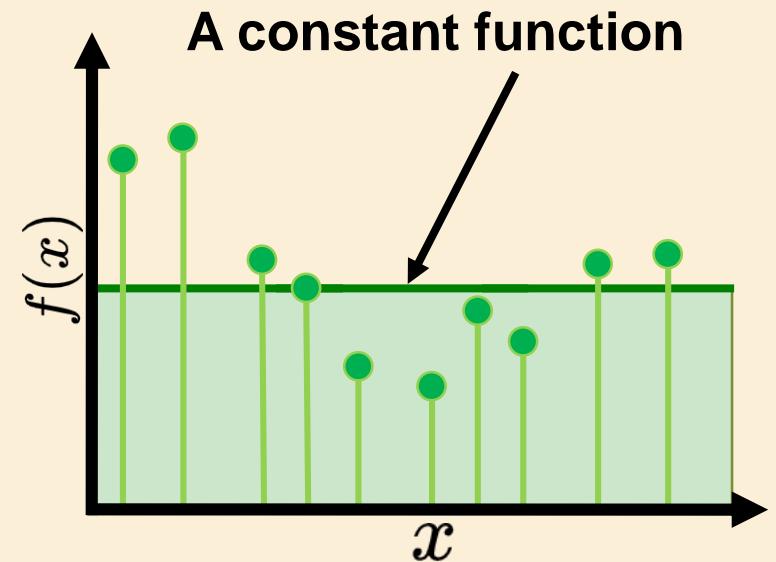
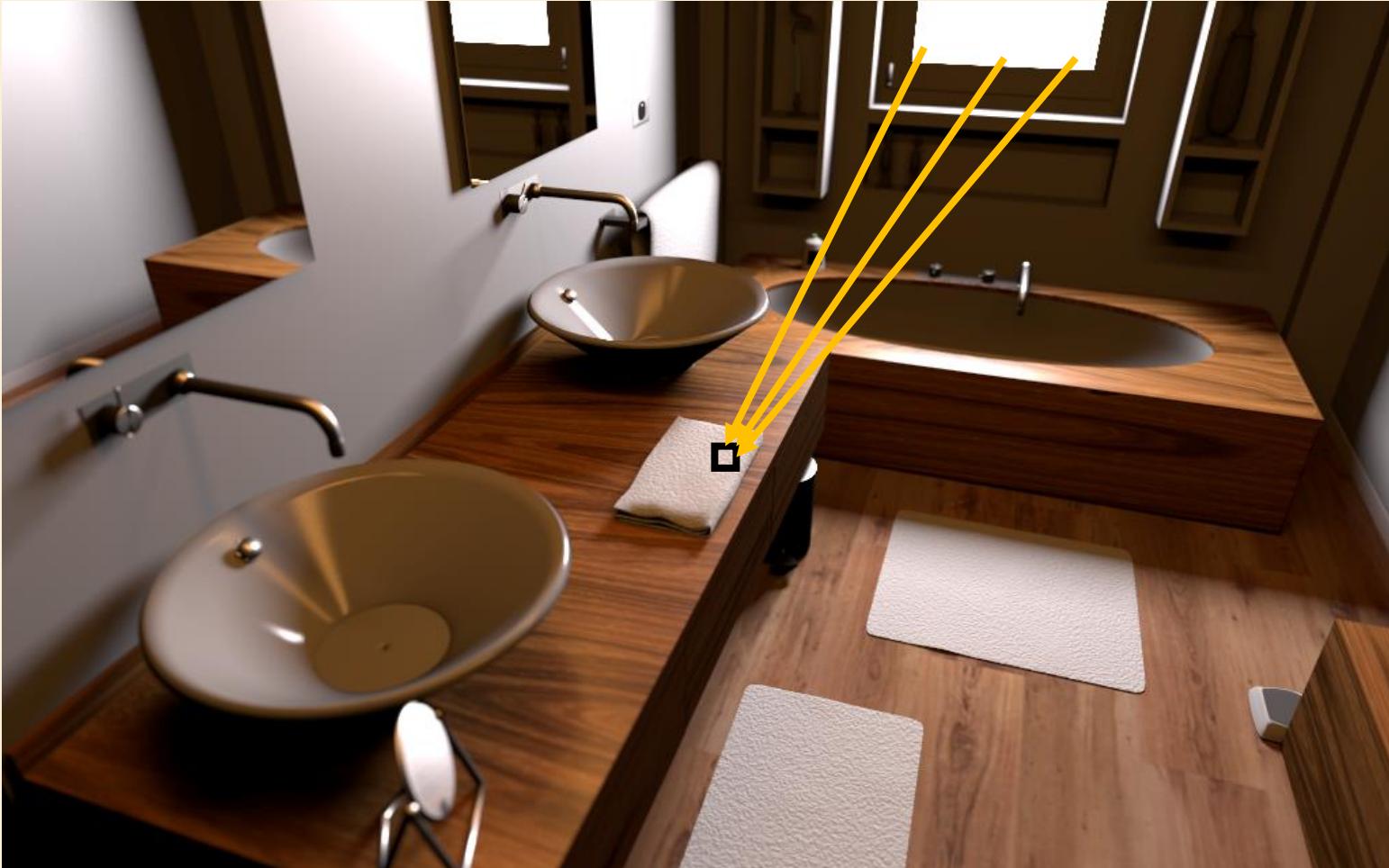
# ■ Monte Carlo



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^N f(x_i)$$

Reproduced from Salaün et al. (2022) – SIGGRAPH Presentation Slides

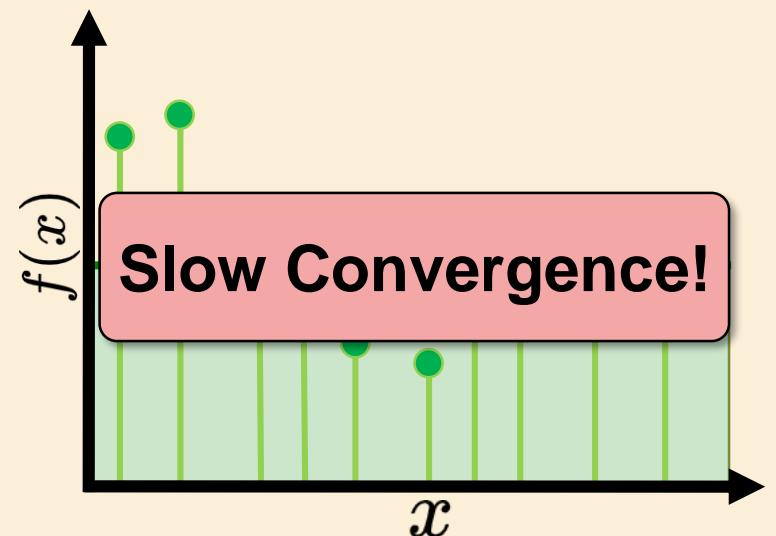
# ■ Calculus-oriented interpretation



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^N f(x_i)$$

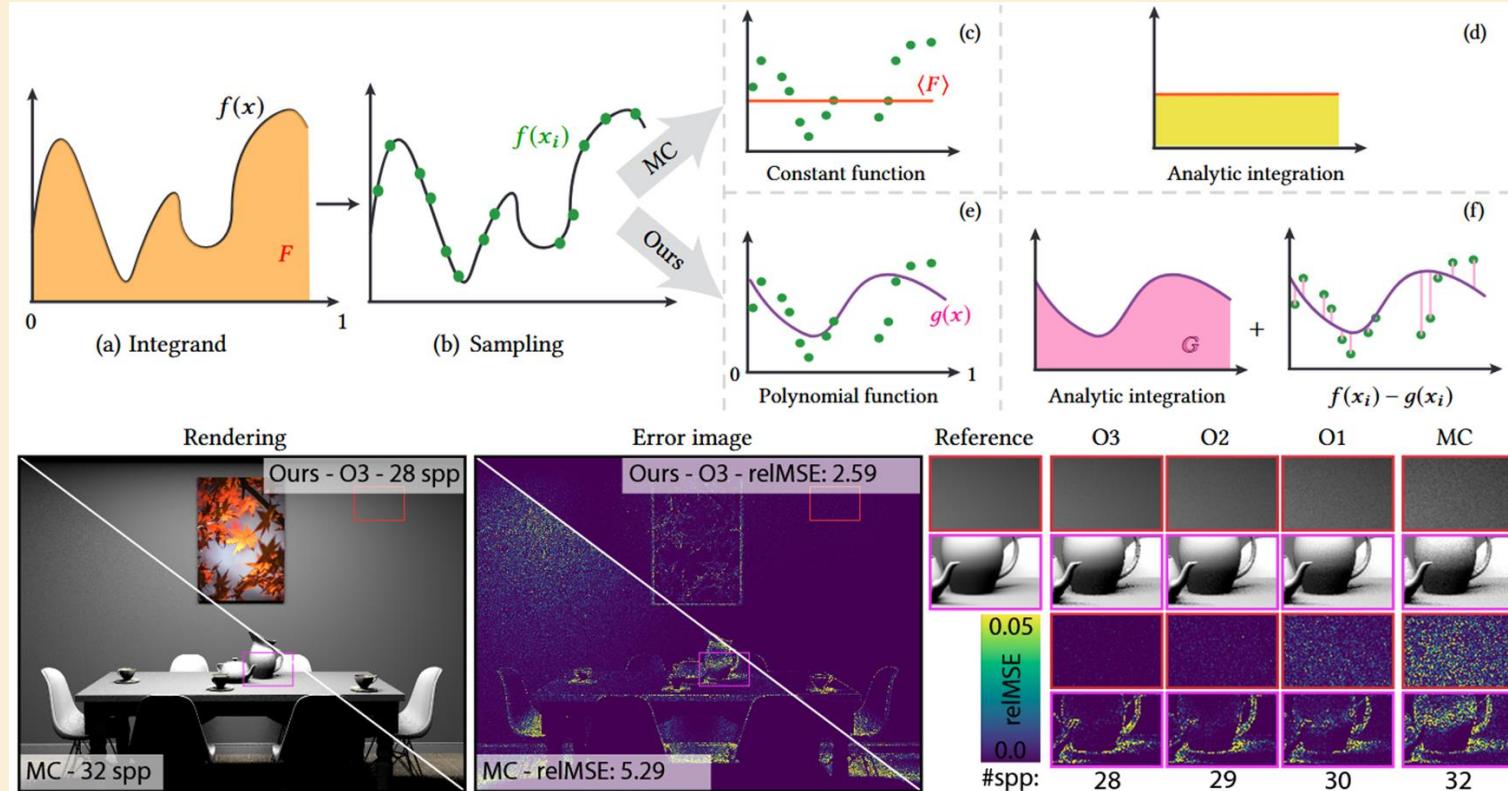
# ■ Limitations of Monte Carlo Integration

256 samples per pixel



$$\langle F \rangle = \frac{1}{N} \sum_{i=1}^N f(x_i)$$

# Regression-based Monte Carlo Integration

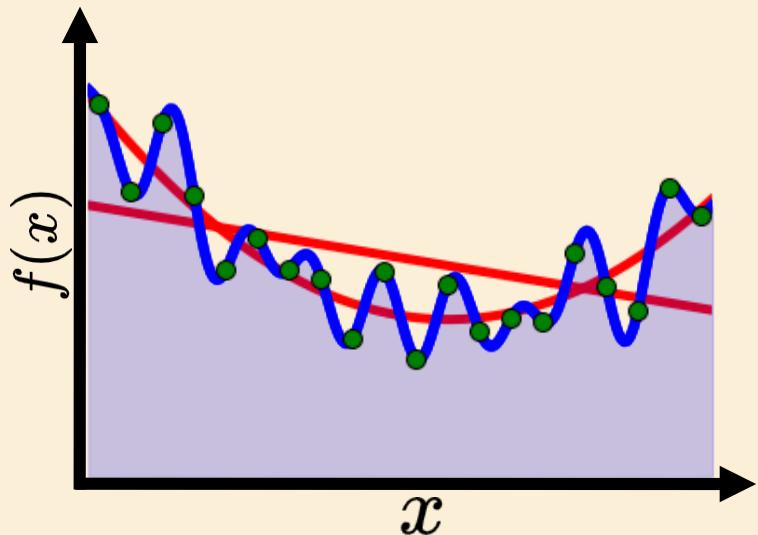


Regression

Control Variates

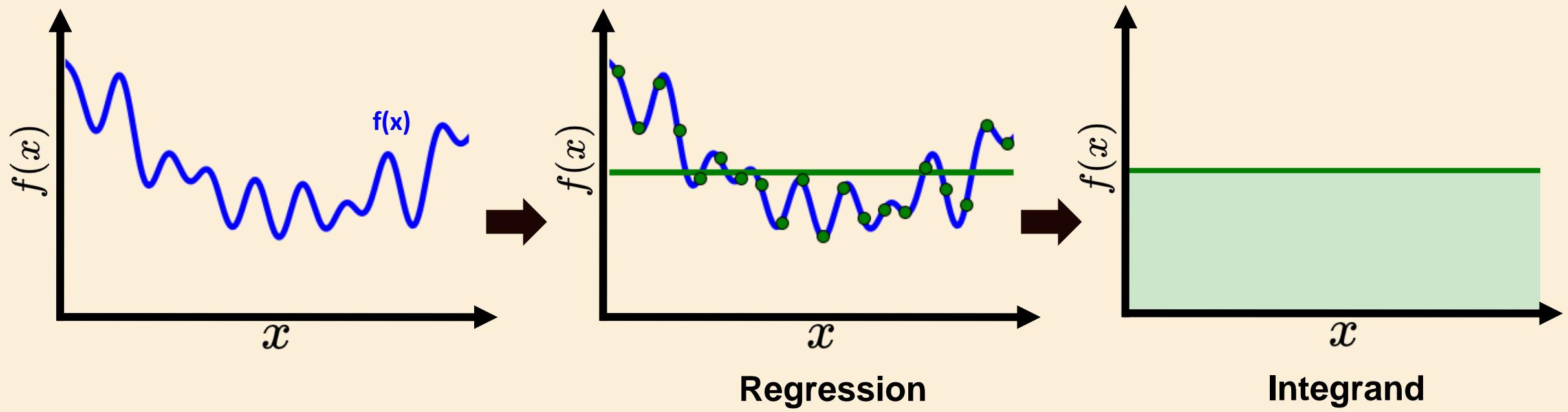
MC Integration

# ■ Regression

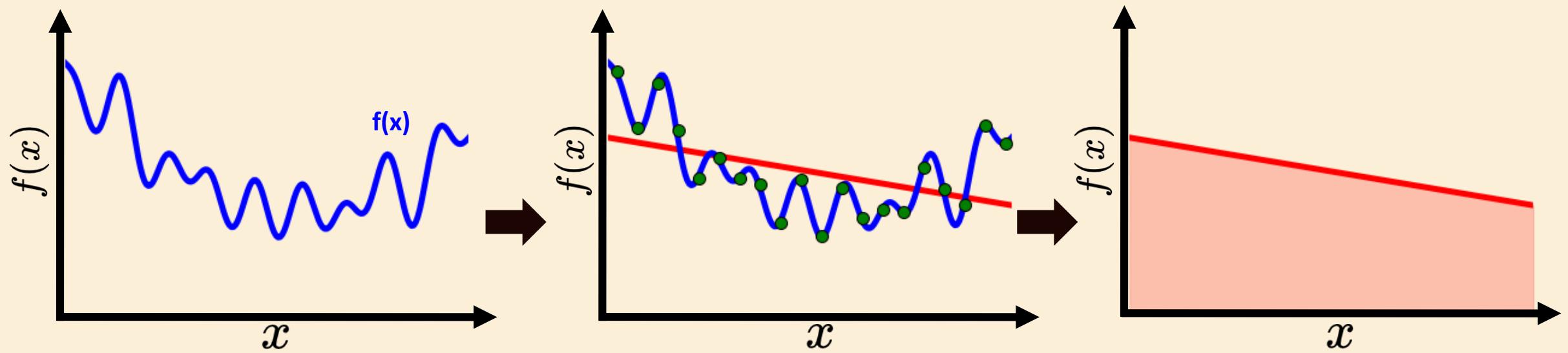


[Salaün et al. 2022] proposed using a more complex function instead of a constant function.

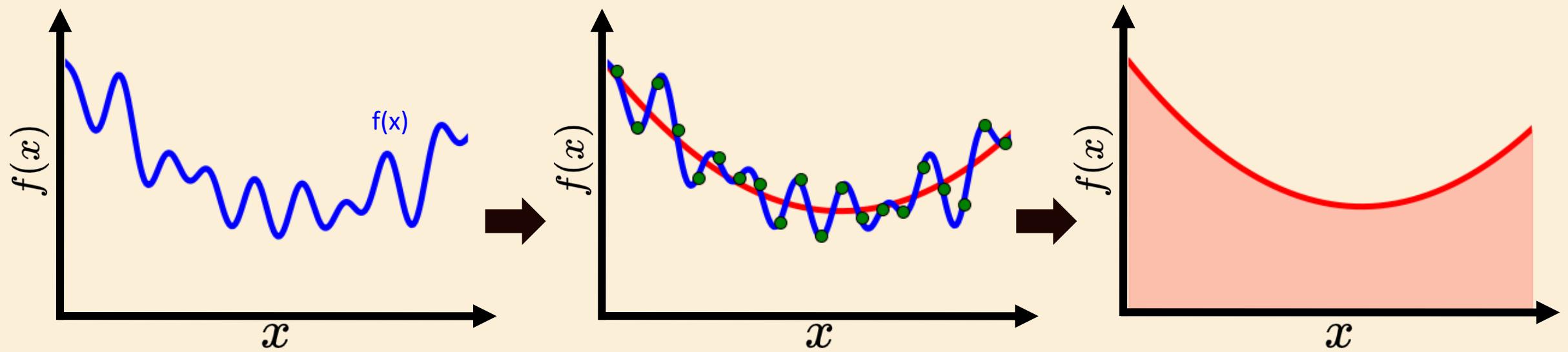
# ■ Regression



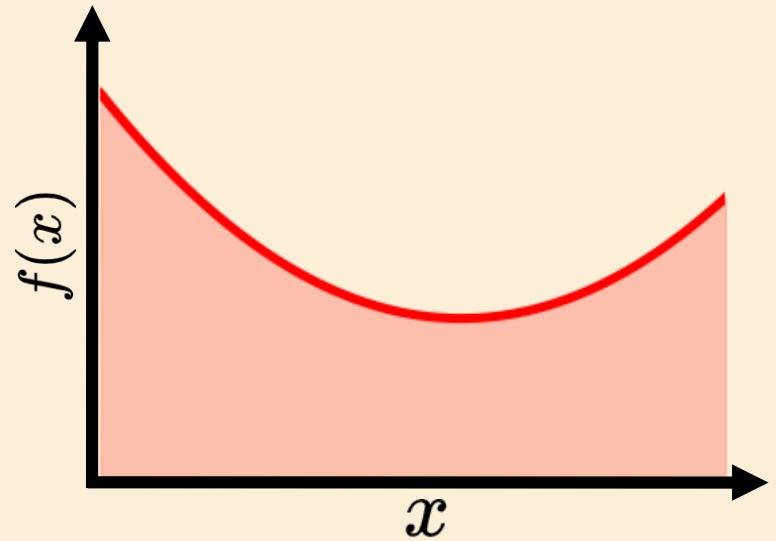
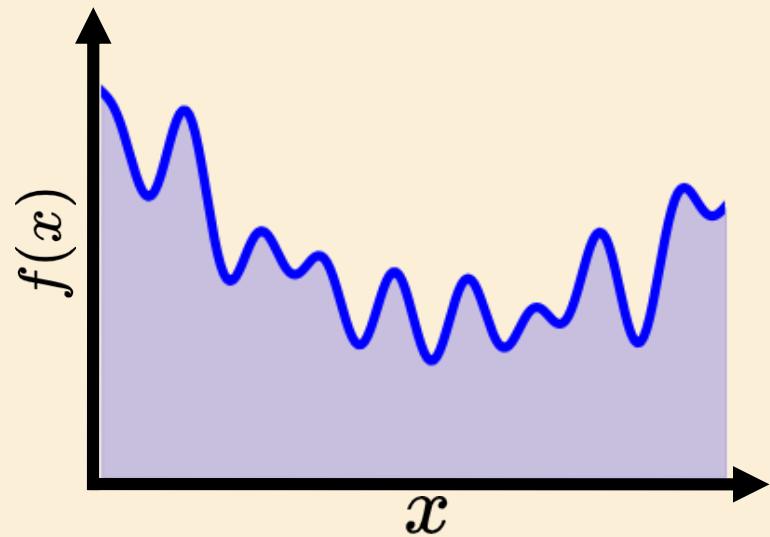
# ■ Regression



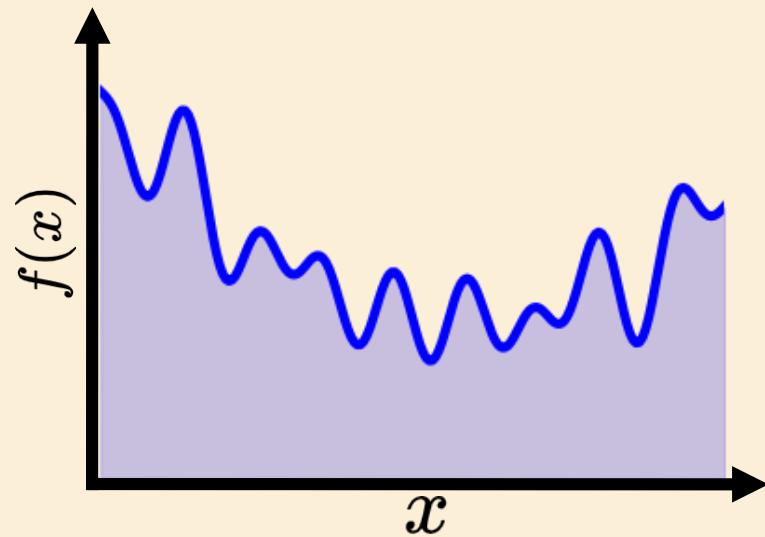
# ■ Regression



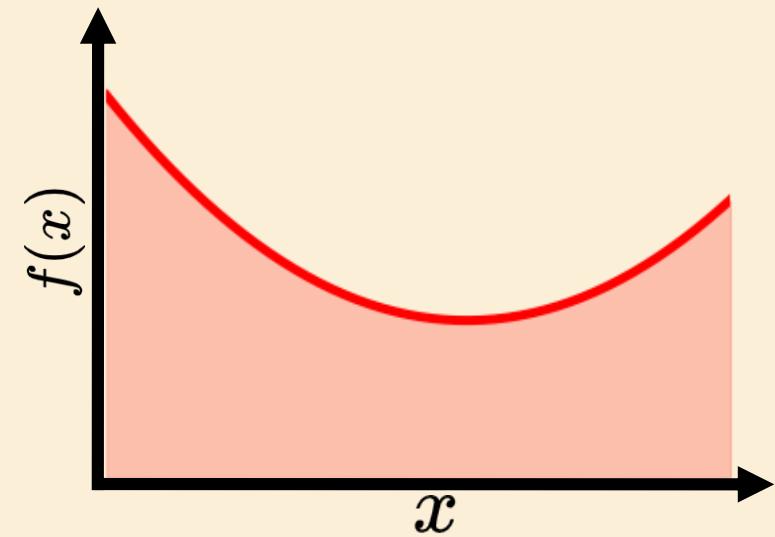
# ■ Regression Leads to Biased Result



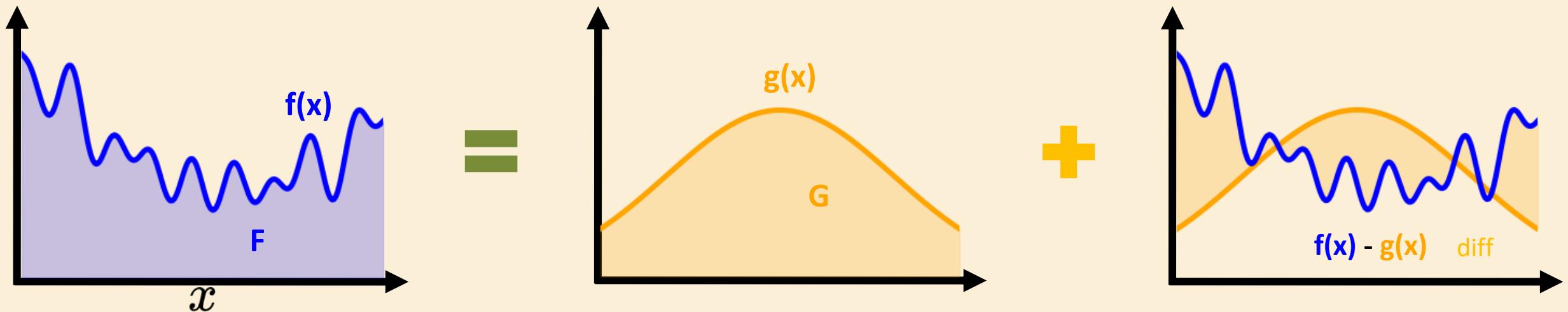
# ■ Regression Leads to Biased Result



$\neq$

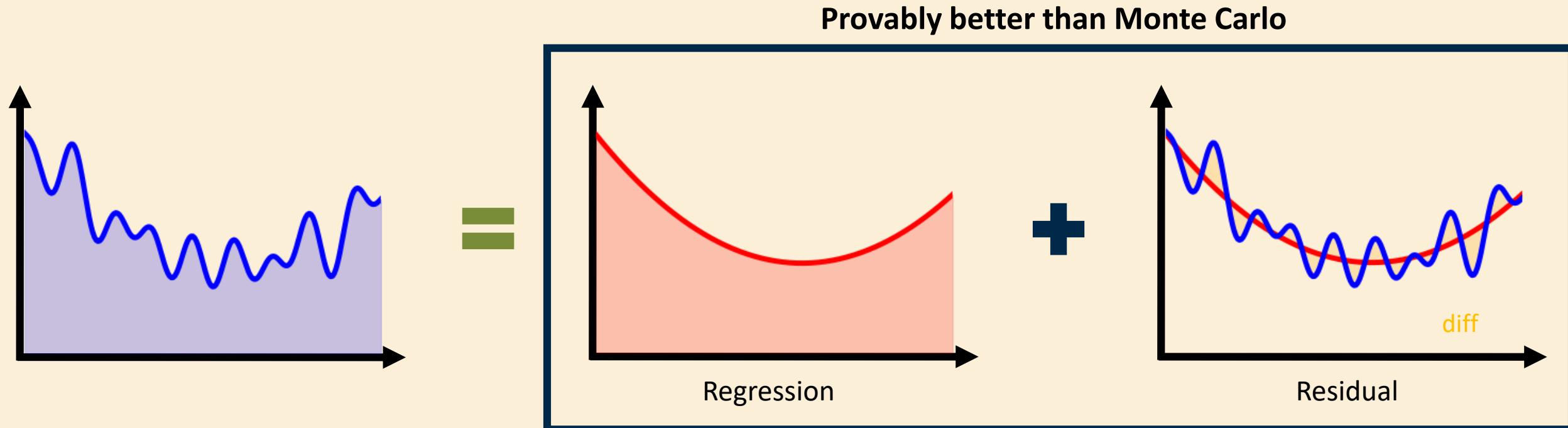


# ■ Control Variates Estimator



$$F = G + (F - G)$$

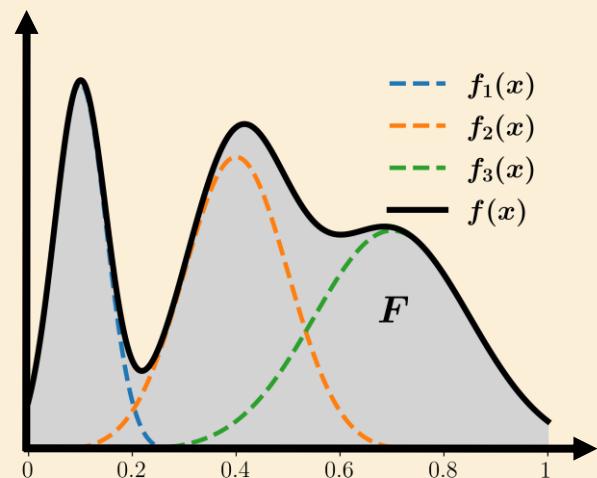
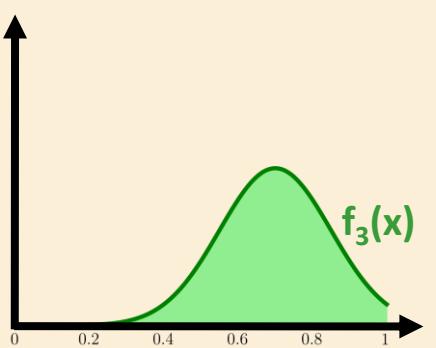
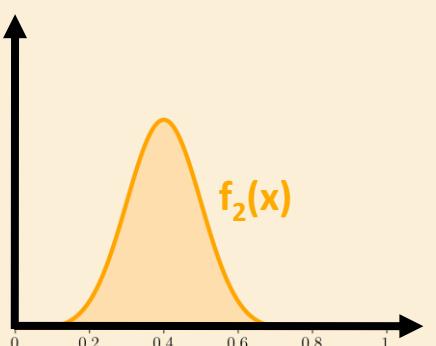
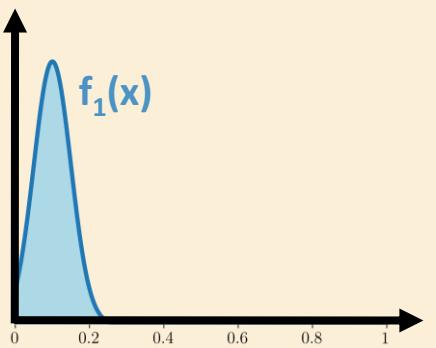
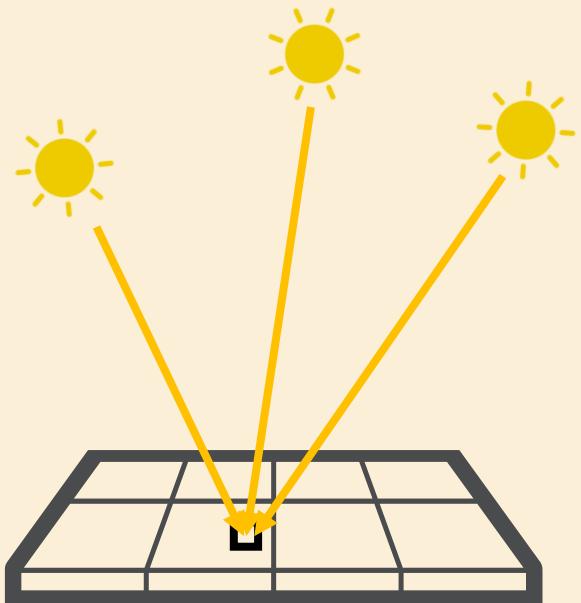
# ■ Regression-based Monte Carlo Integration



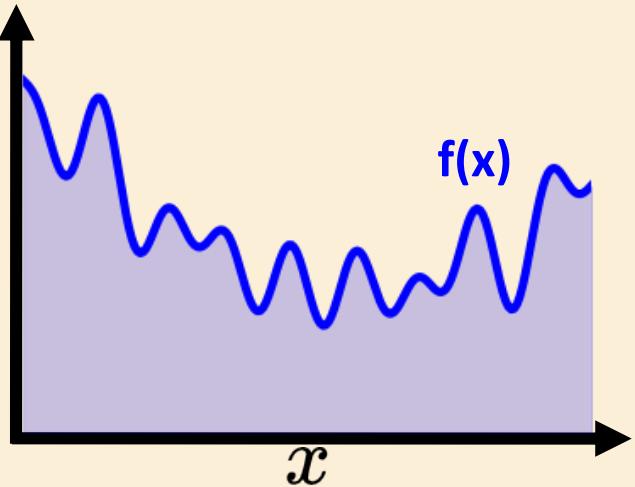
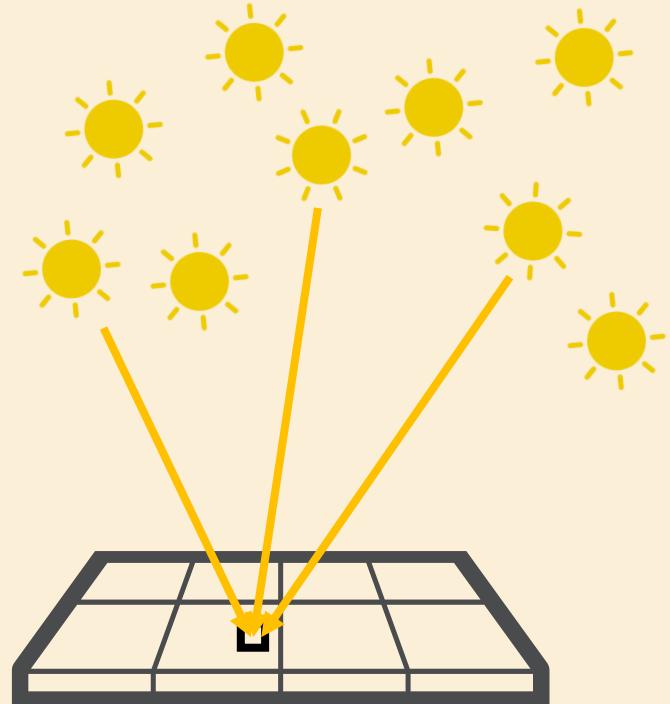
## ■ Challenges

- The least-squares regression solution is not exact for any finite number of samples.
- The integrand can be highly discontinuous due to complex scene configurations (especially in scenes with many lights or complex visibility).

# An Example Scene



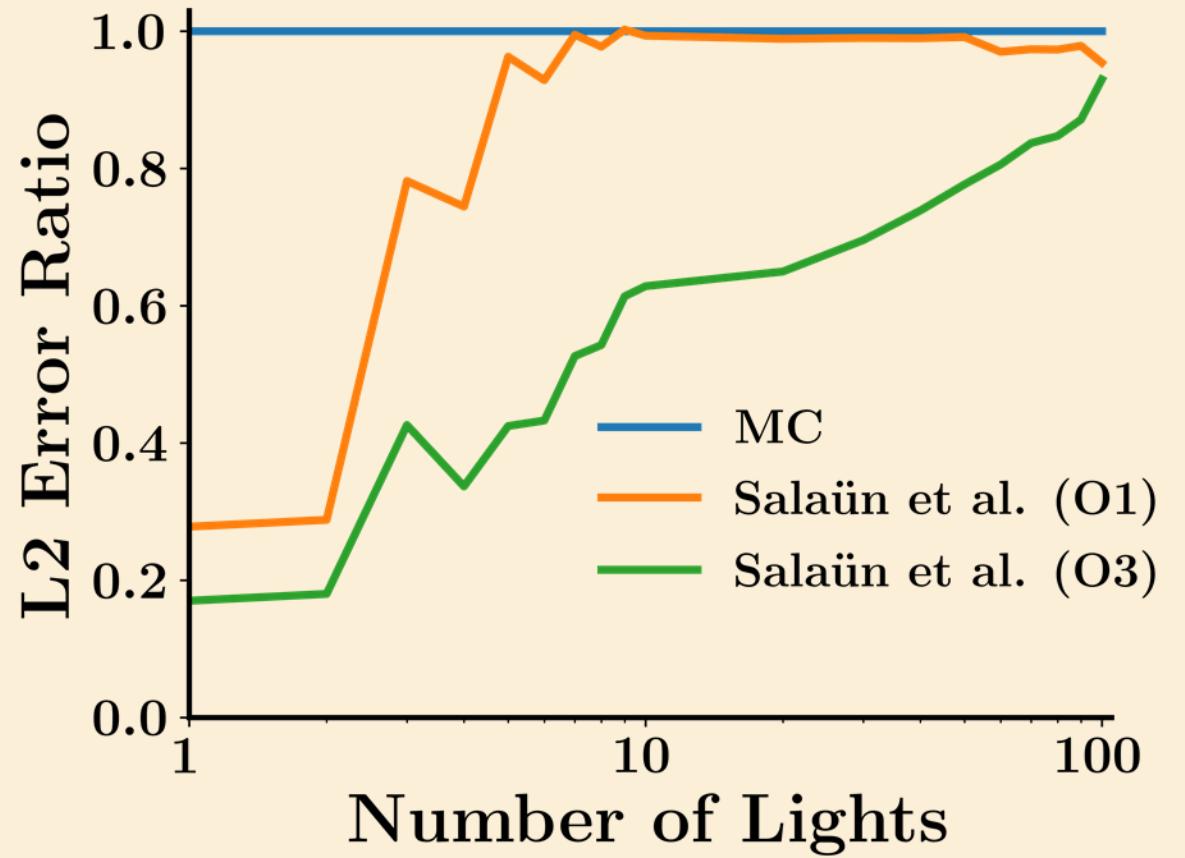
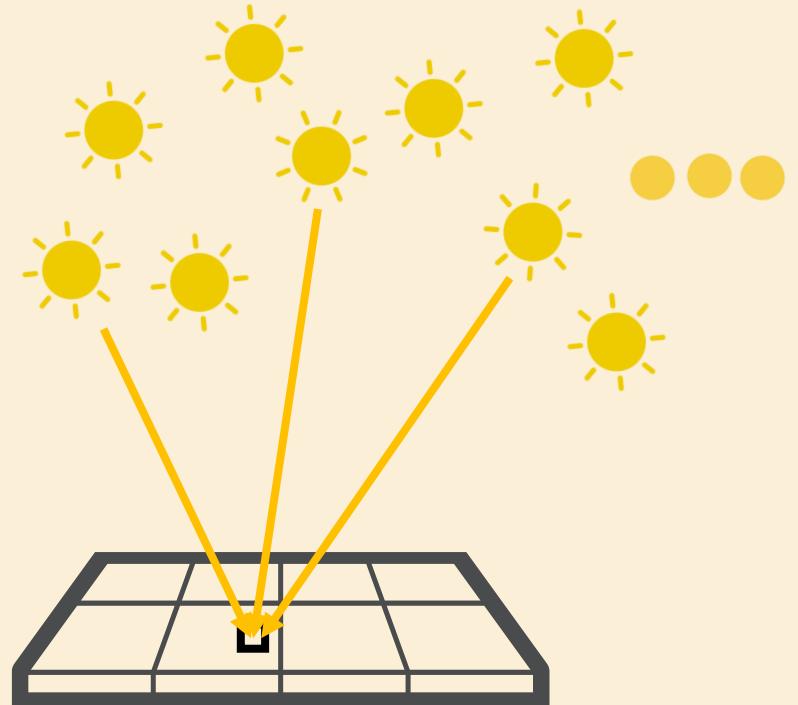
## An Example Scene



High-frequency!

Hard for regression!

# ■ Many-light Discontinuity



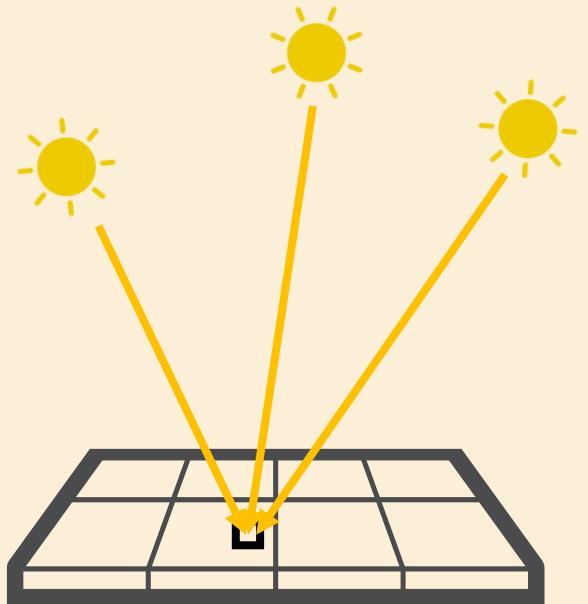
②

①

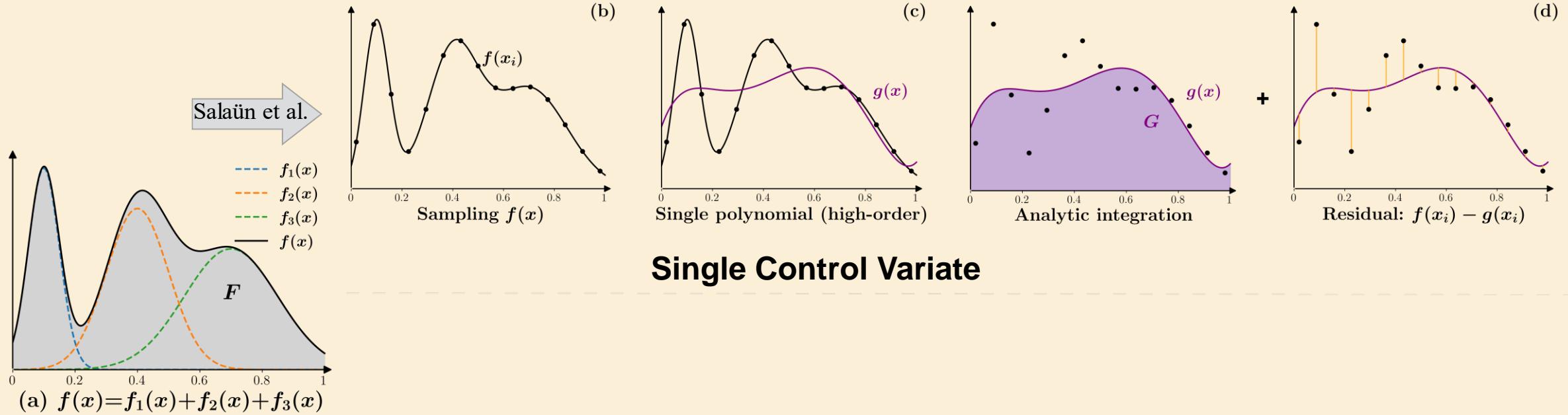
## Adaptive Multiple Control Variates

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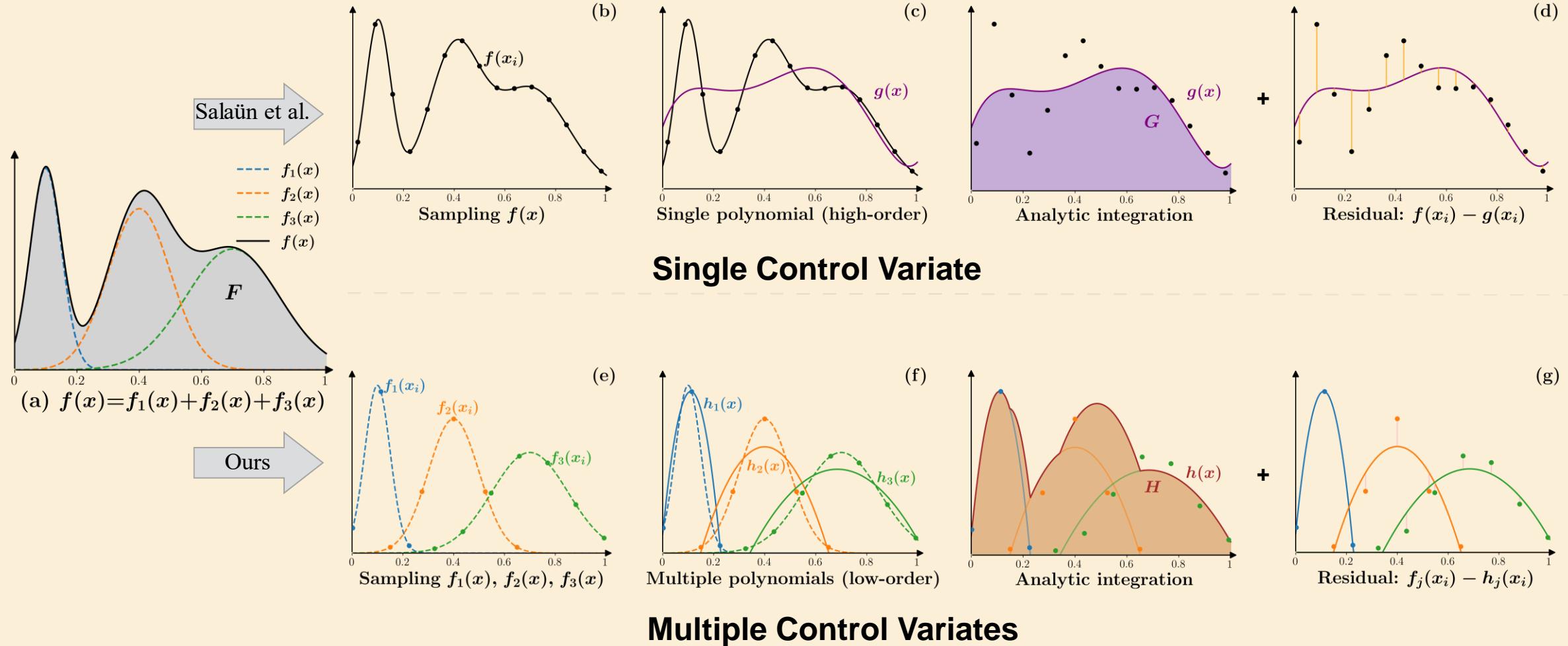
## ■ An Example Scene



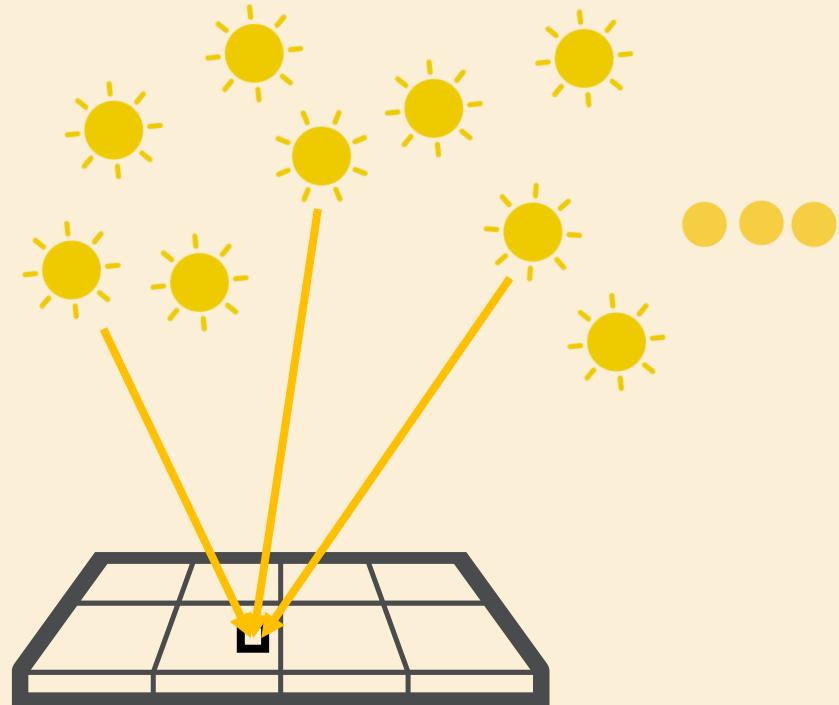
# ■ Multiple Control Variates vs. Single Control Variate



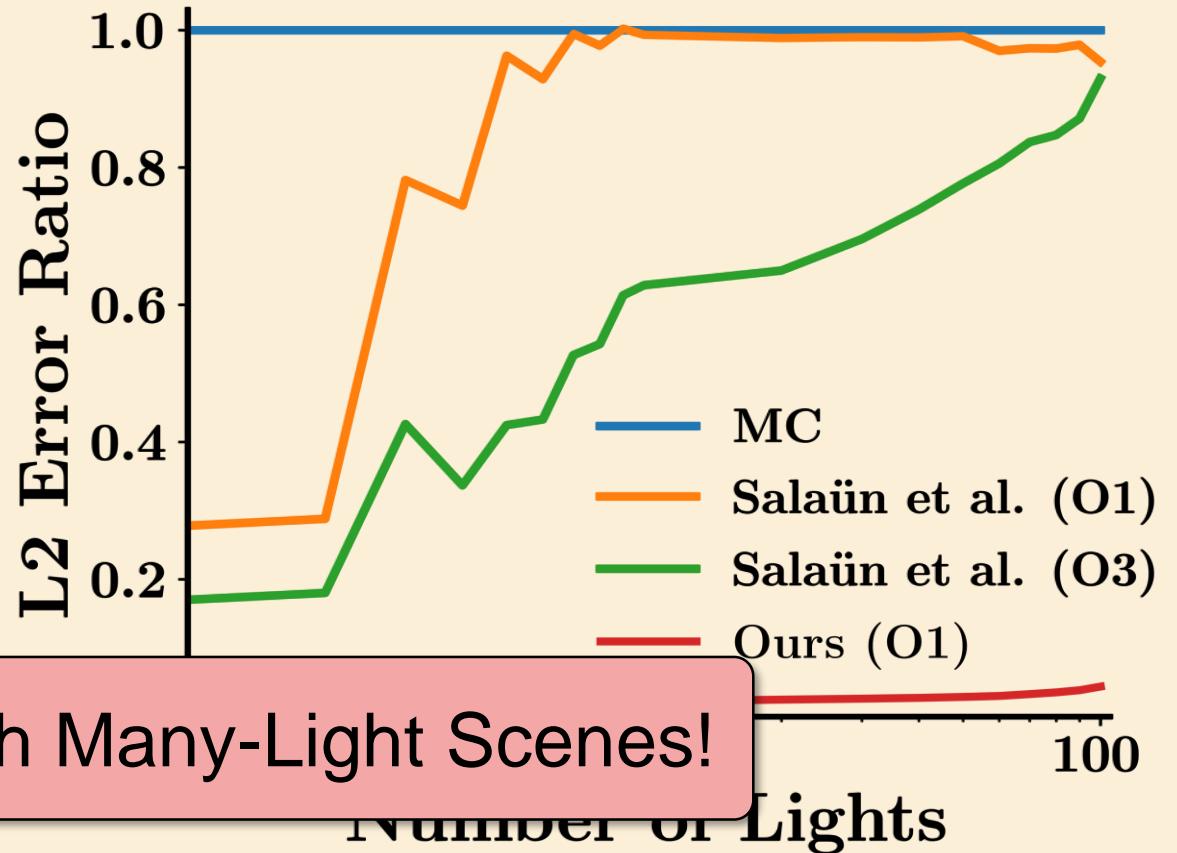
# ■ Multiple Control Variates vs. Single Control Variate



# ■ Results of Multiple Control Variates

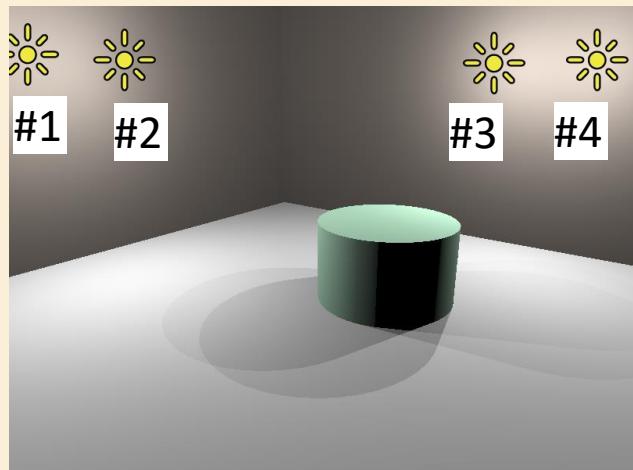


Scalability with Many-Light Scenes!

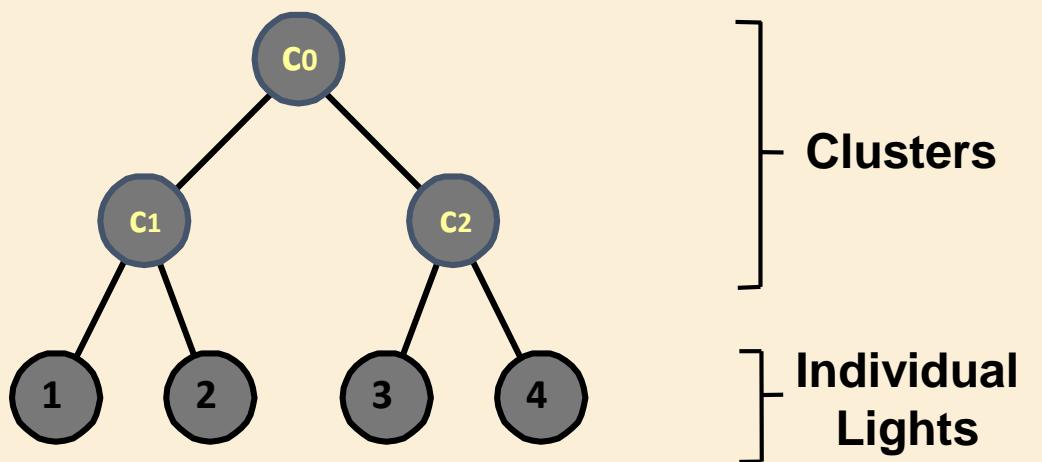


# ■ Lightcuts

## Simple Example

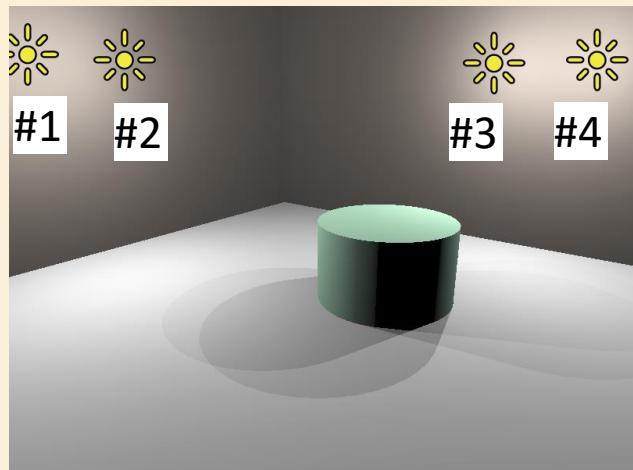


## Light Tree

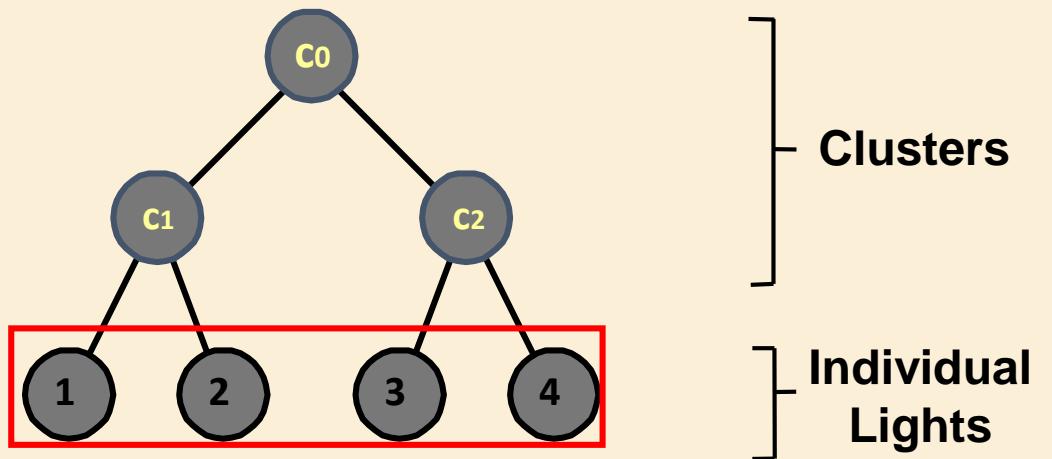


# ■ Lightcuts

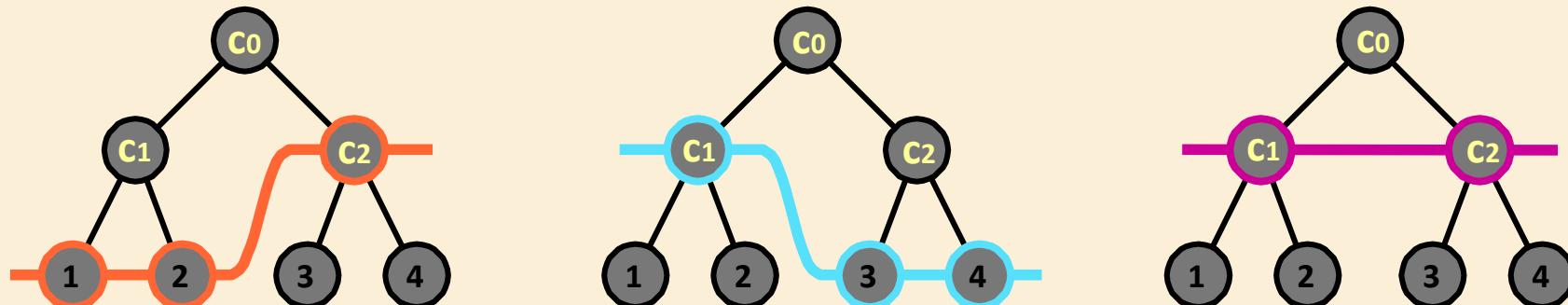
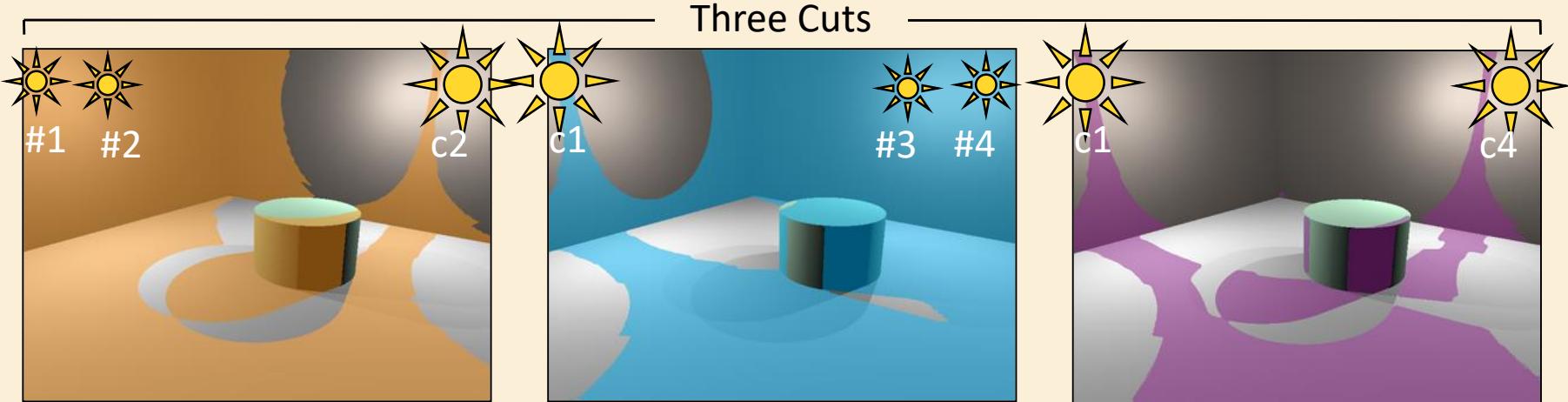
## Simple Example



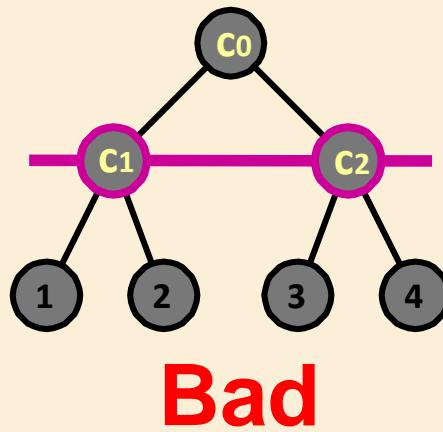
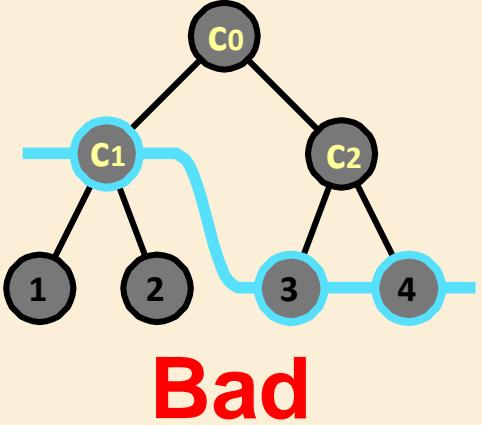
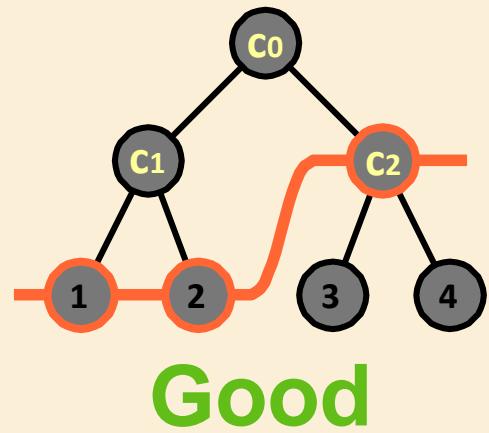
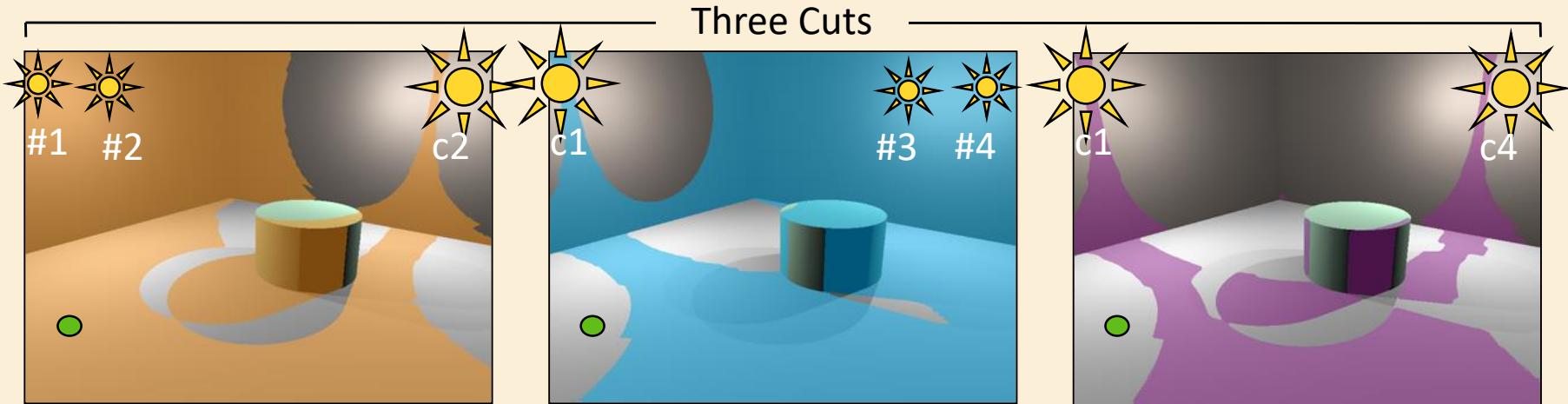
## Light Tree



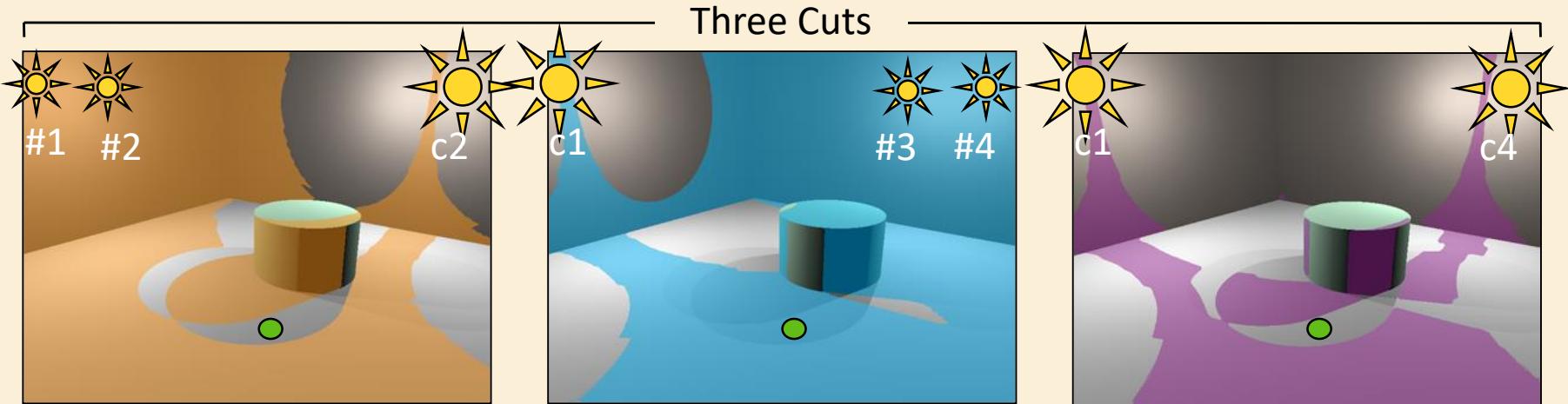
# ■ Lightcuts



# ■ Lightcuts



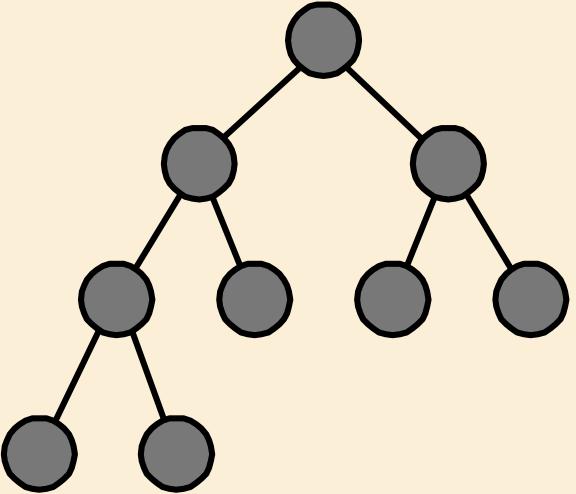
# ■ Lightcuts



Spatially adaptive control variates!

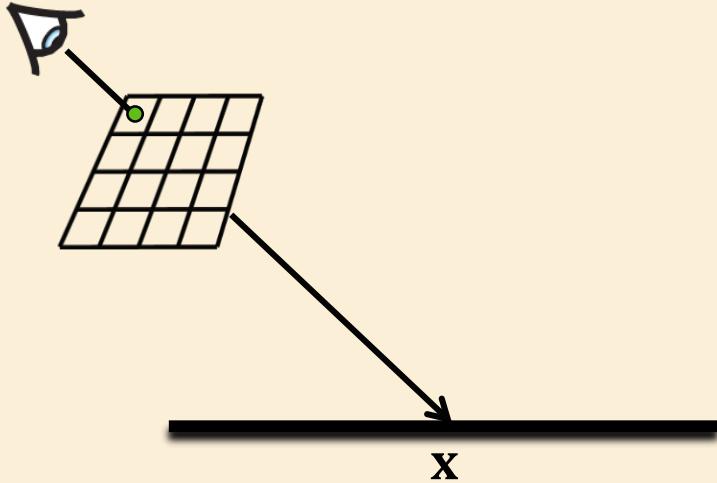
## ■ Multiple Control Variates Construction Based on Lightcuts

- For all lights, we first construct a light BVH tree.



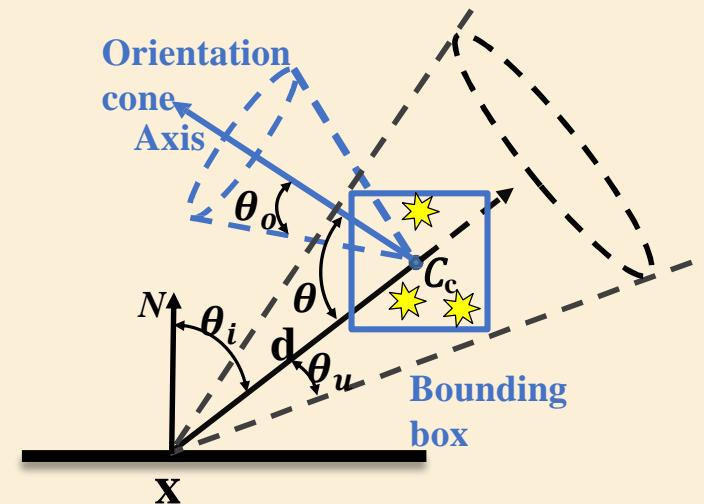
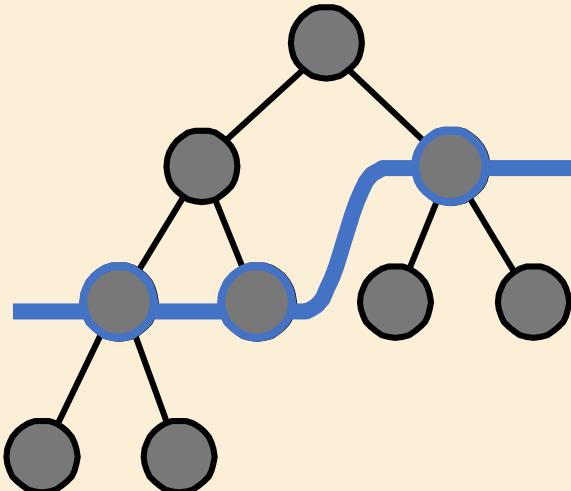
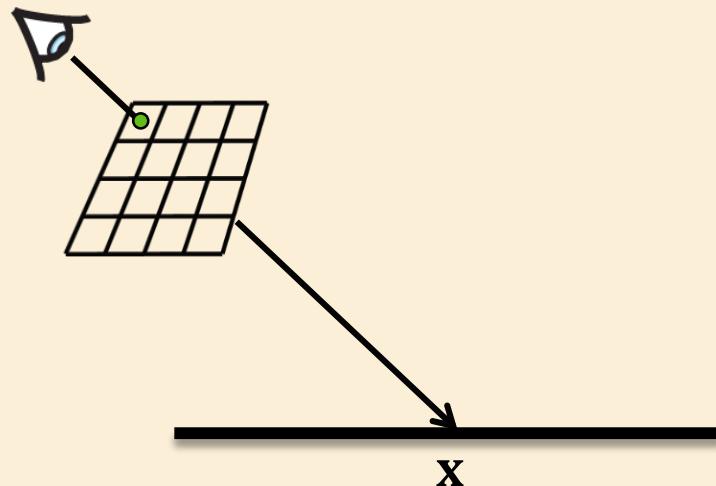
## ■ Multiple Control Variates Construction Based on Lightcuts

- For all lights, we first construct a light BVH tree.
- For each pixel, we select a representative point and shoot a camera ray to obtain the corresponding shading point  $x$ .



# ■ Multiple Control Variates Construction Based on Lightcuts

- For all lights, we first construct a light BVH tree.
- For each pixel, we select a representative point and shoot a camera ray to obtain the corresponding shading point  $\mathbf{x}$ .
- For a shading point, we construct a light cut and compute the corresponding weight.

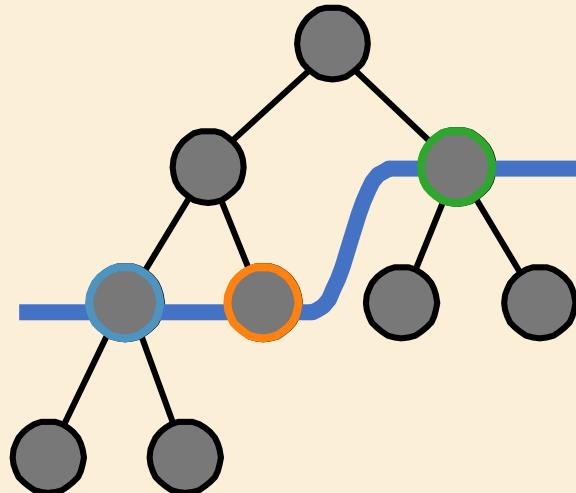


Conty Estevez, A., & Kulla, C. (2018). Importance sampling of many lights with adaptive tree splitting.  
Proceedings of the ACM on Computer Graphics and Interactive Techniques, 1(2), 1-17.

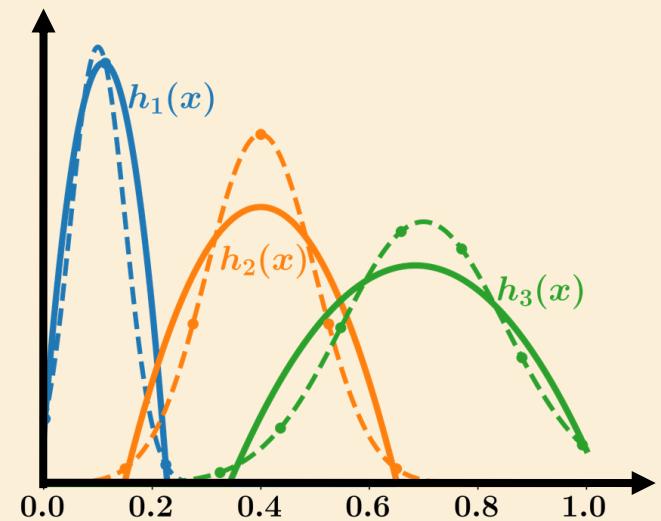
$$I_C(\mathbf{x}) = \frac{f_{\mathbf{x}} |\cos \theta'_i| \cos(\theta')}{d^2}$$

# ■ Multiple Control Variates Construction Based on Lightcuts

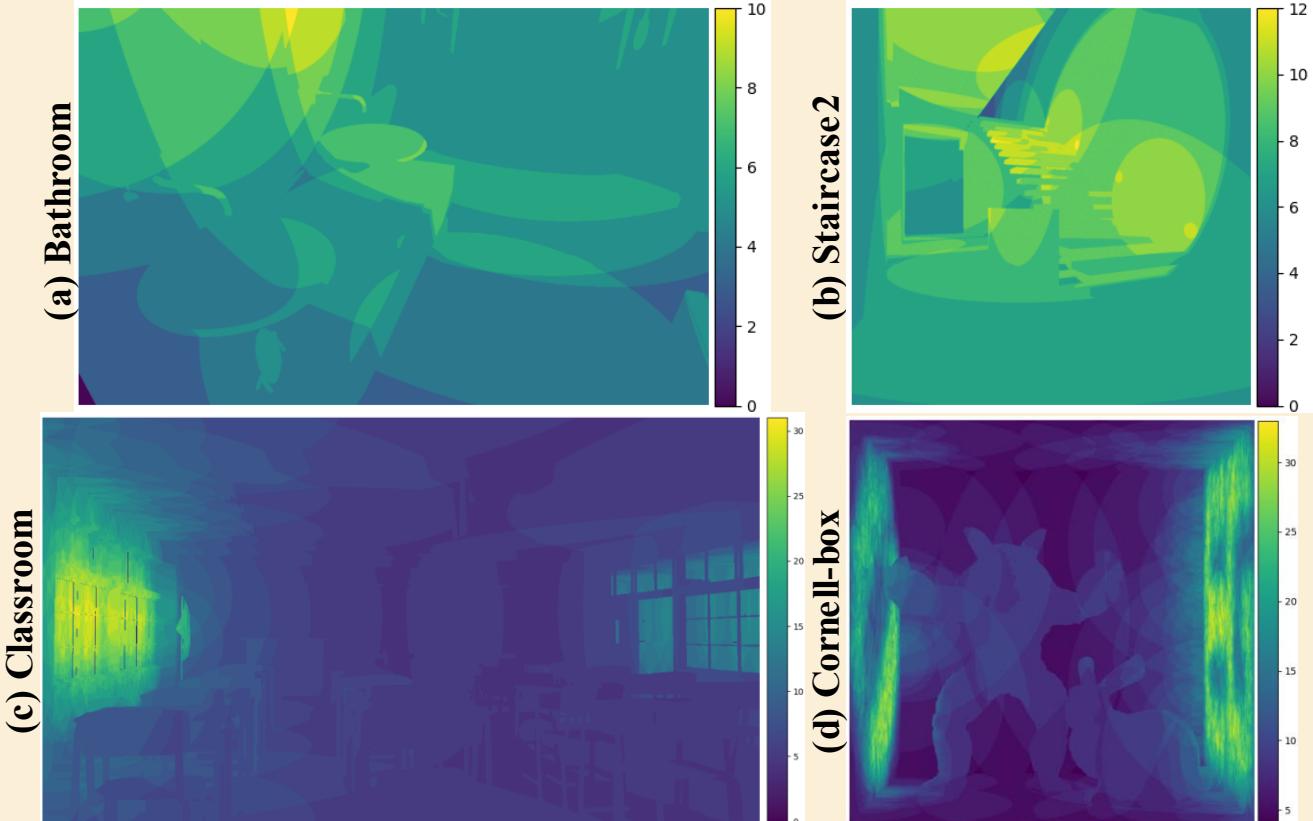
- For all lights, we first construct a light BVH tree.
- For each pixel, we select a representative point and shoot a camera ray to obtain the corresponding shading point  $\mathbf{x}$ .
- For a shading point, we construct a light cut and compute the corresponding weight.
- For each node in the cut, we create a low-order polynomial as a control variate.



SGD



# ■ Visualization of the Number of Control Variates

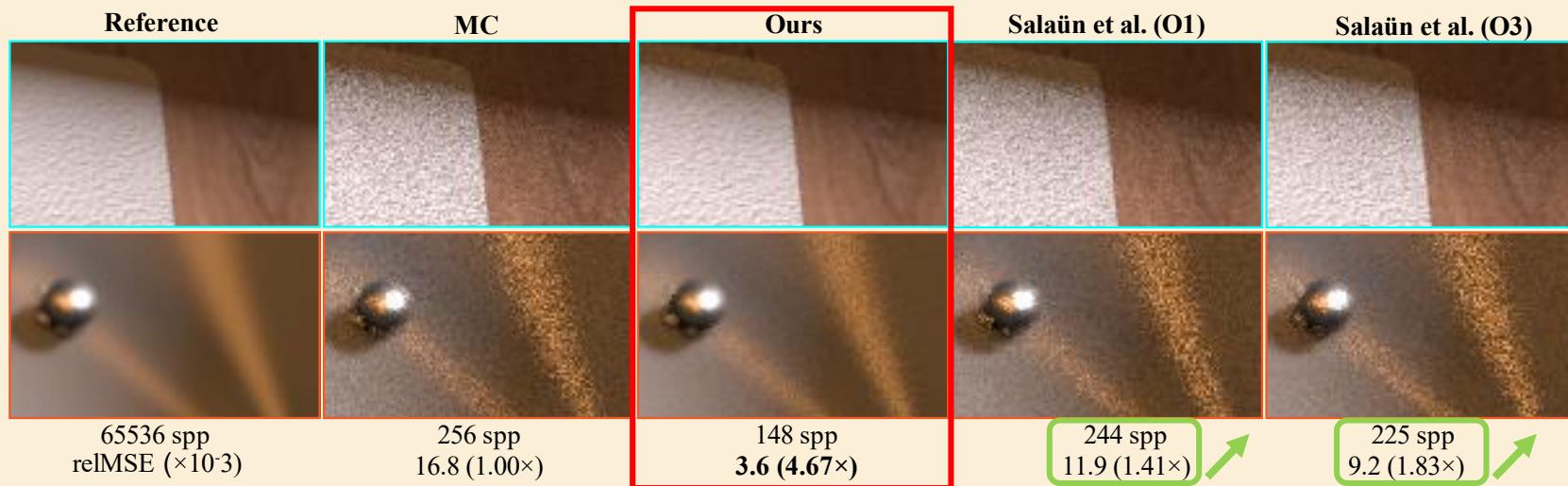
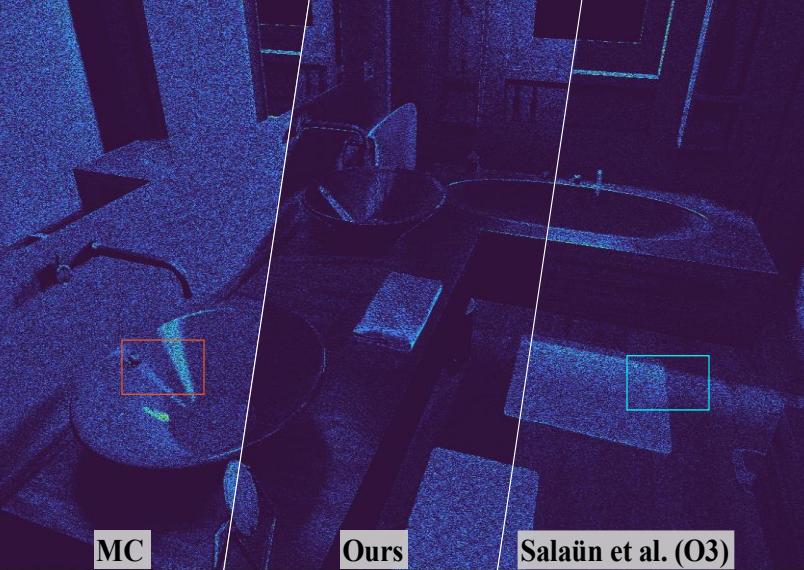
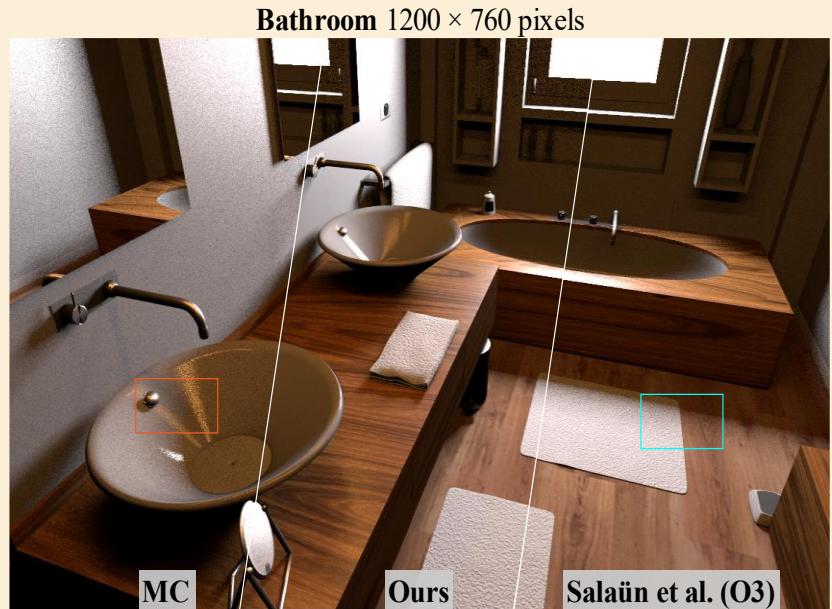


| Scene       | # lights | avg. cut size | # control variates |
|-------------|----------|---------------|--------------------|
| Staircase2  | 21       | 7.9           | 0 - 12             |
| Bathroom    | 32       | 5.1           | 0 - 10             |
| Classroom   | 162      | 7.6           | 0 - 31             |
| Cornell-box | 1494     | 10.28         | 4 - 33             |

# **Results (Equal Time Comparison)**

---

# ■ Comparison to Salaün et al. [SGH\*22]



Bathroom scene has 32 mesh lights.

**Ours**

0-10 (avg. 5.1) control variates  
(polynomial with order 1)

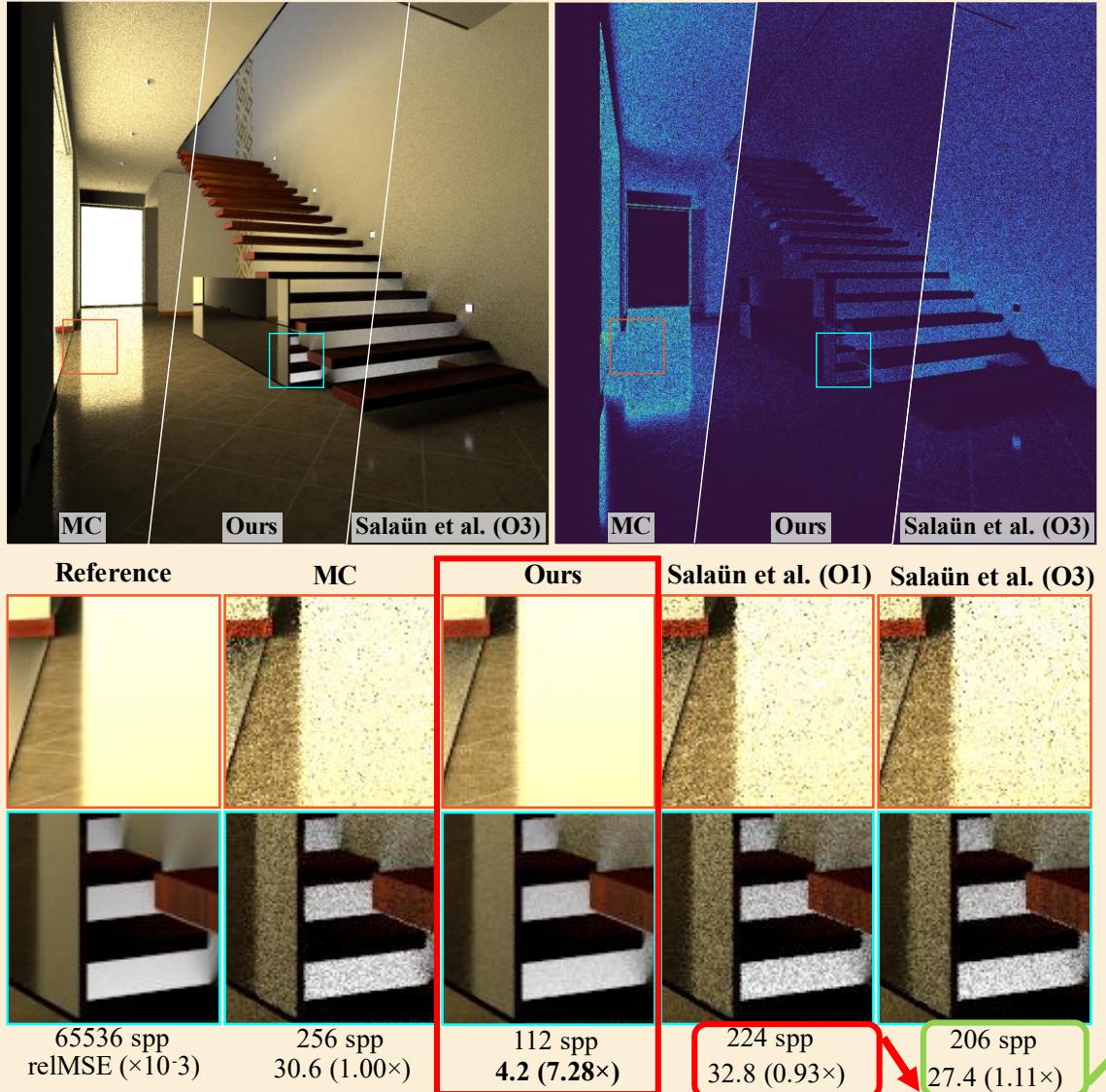
**vs.**

**[SGH\*22]**

Single control variate  
(polynomials with order 1 & 3)

# ■ Comparison to Salaün et al. [SGH\*22]

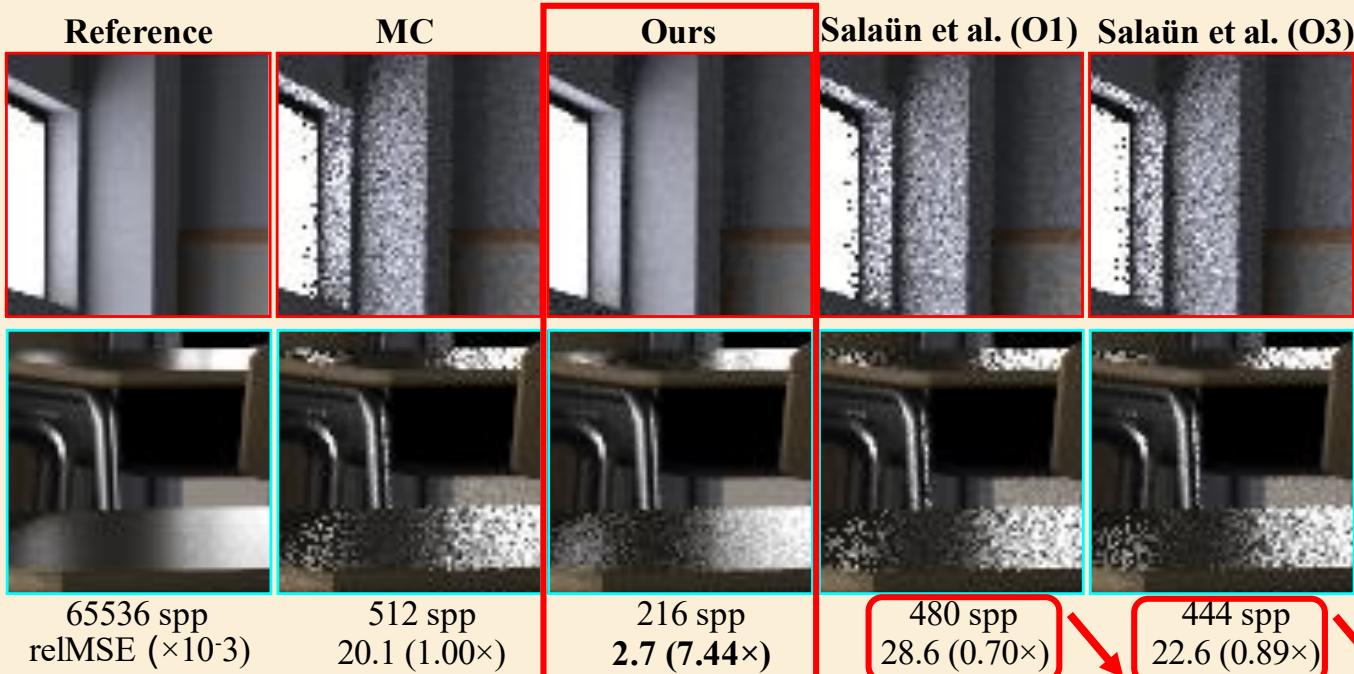
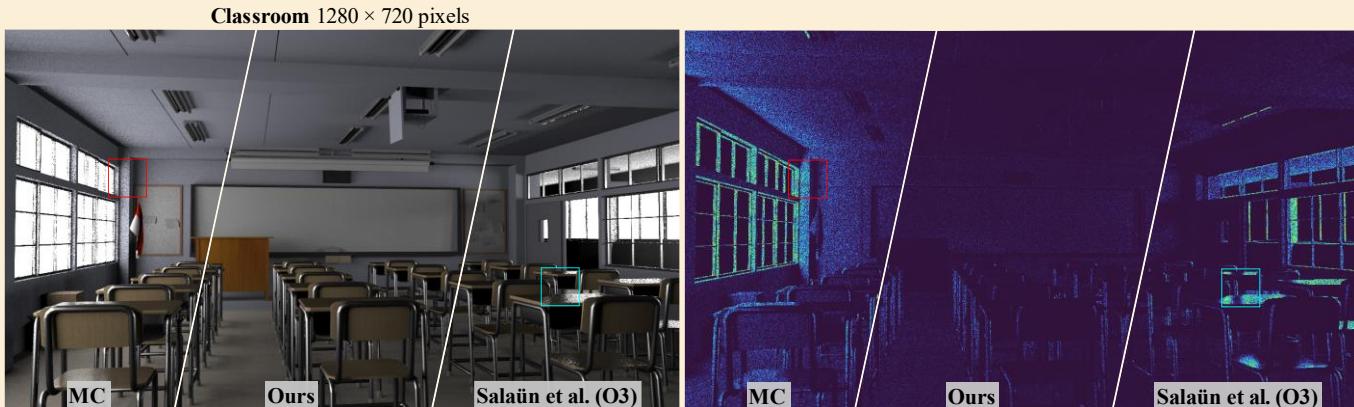
Staircase2 1024 × 1024 pixels



Staircase2 scene has 21 mesh lights.

**Ours**  
 0-12 (avg. 7.9) control variates  
 (polynomial with order 1)  
 vs.  
**[SGH\*22]**  
 Single control variate  
 (polynomials with order 1 & 3)

# ■ Comparison to Salaün et al. [SGH\*22]



Classroom scene has 162 mesh lights.

## Ours

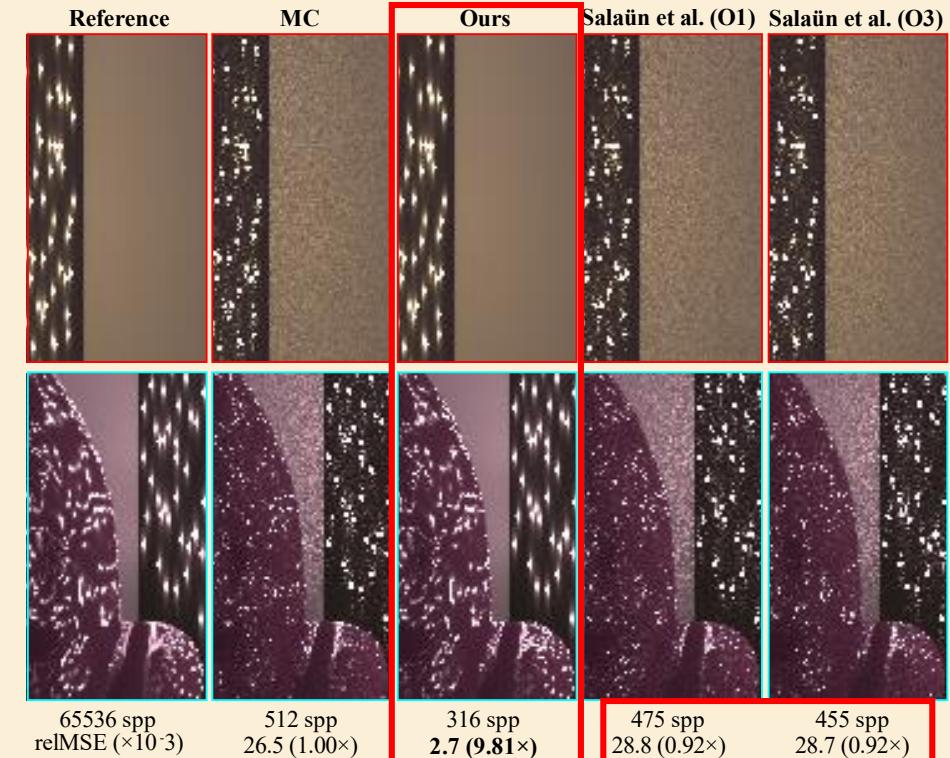
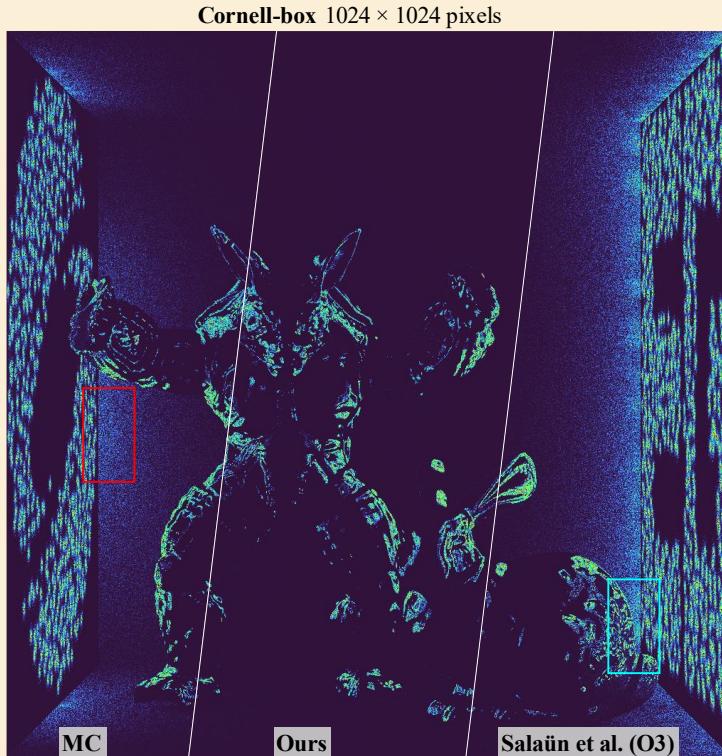
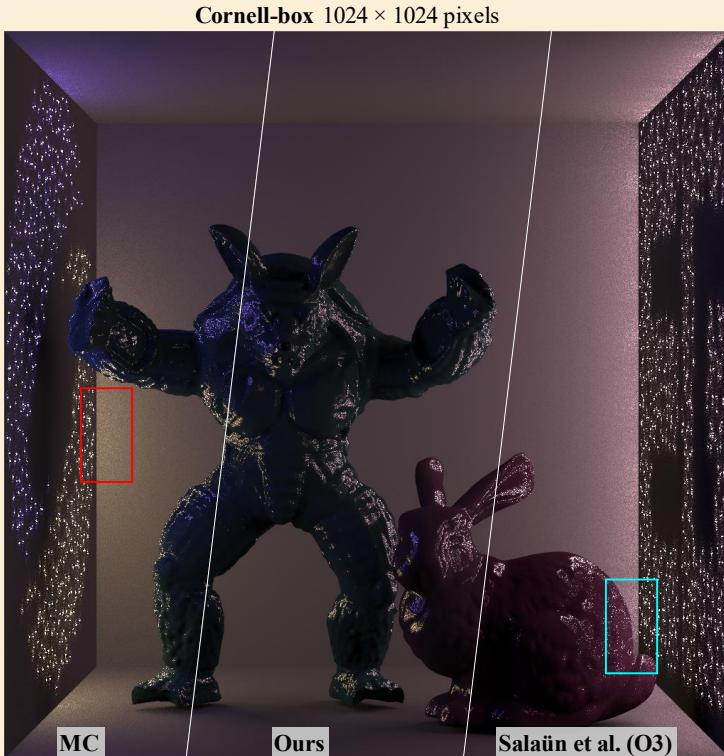
0-31 (avg. 7.6) control variates  
(polynomial with order 1)

vs.

## [SGH\*22]

Single control variate  
(polynomials with order 1 & 3)

# ■ Comparison to Salaün et al. [SGH\*22]



Cornell-box scene has 1494 lights.

## Ours

4-33 (avg. 10.28) control variates (polynomial with order 1)

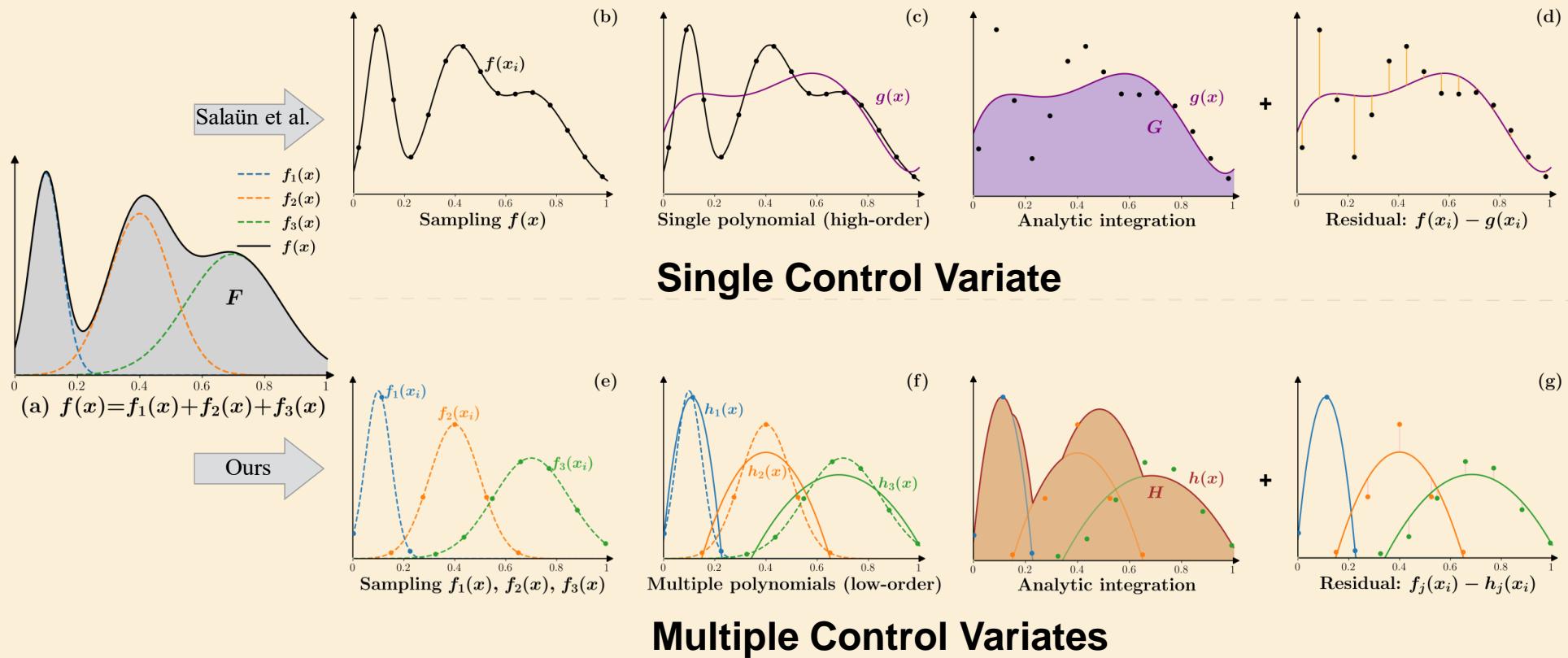
vs.

[SGH\*22]

Single control variate (polynomials with order 1 & 3)

# Conclusion

- We introduce an adaptive multiple control variates framework to improve the Monte Carlo integration for many-light rendering.



## ■ Limitations and Future Work

### Limitations

- Limited to specific many-light rendering.
- Visibility-related discontinuities are not yet handled.

### Future work

- Extend to support other rendering applications.
- Incorporate visibility-aware strategies.

## Acknowledgements

- We would like to thank the reviewers for the valuable comments.
- We would like to thank the following for scenes used in our experiments:  
nacimus (Bathroom), Veach (Veach-mis), NewSee2I035 (Staircase2),  
JayArtist (Living-room2), NovaZeeke (Classroom), Wig42 (Staircase1).
- We would like to thank Corentin Salaün for sharing the code and slides.

# Thank you!

## Adaptive Multiple Control Variates for Many-Light Rendering

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