Supporting Information

Xu *et al.* 10.1073/pnas.0806082105

SI Text

In this supporting information (SI) we list all of the equations of the continuous and discrete dynamical systems used to generate the flow and map data used in the article. The embedding parameters for each time series are also specified. The time series are derived from the x component in each system to build the corresponding complex network.

We also present the results of some calculations, omitted from the published article to ensure the conciseness of the main message.

Flow Data. For the flow data, the embedding dimension is set to be $D_e = 10$ and the time delay τ is chosen where the mutual information (1) reaches its first minimum.

Chaotic Lorenz system (2). $\sigma = 10$, $r = 28$, $b = 8/3$. The sampling interval $T = 0.05$ and the time delay $\tau = 3$.

$$\begin{cases} \dot{x} &= \sigma(y - x) \\ \dot{y} &= x(r - z) - y \\ \dot{z} &= xy - bz \end{cases} \quad [1]$$

Rossler system (3). $a = 0.1, b = 0.1$. The sampling interval $T = 0.1$ and the time delay $\tau = 10$.

$$\begin{cases} \dot{x} &= -(y + z) \\ \dot{y} &= x + ay \\ \dot{z} &= b + (x - c)z \end{cases} \quad [2]$$

Periodic Rossler systems.

Period 2: $c = 6$.
Period 3: $c = 12$.
Period 4: $c = 8.5$.
Period 8: $c = 8.7$.

Chaotic Rossler systems. $c = 9$ and $c = 18$.

Chaotic Chua circuit (4). $a = 9.4$, $b = 16$, $m_0 = -8/7$, $m_1 = -5/7$, $f_x = m_1 x + 0.5(m_0 - m_1)(|x + 1| - |x - 1|)$. The sampling interval $T = 0.05$ and the time delay $\tau = 10$.

$$\begin{cases} \dot{x} &= a(y - x - f_x) \\ \dot{y} &= x - y + z \\ \dot{z} &= -by \end{cases} \quad [3]$$

Chaotic Mackey–Glass delay system (5). $a = 0.2, b = 0.1, d = 17$. The sampling interval $T = 0.25$ and the time delay $\tau = 40$.

$$\dot{x}(t) = \frac{ax(t - d)}{1 + 10(t - d)^{10}} - bx(t) \quad [4]$$

Noisy periodic flow data:

$$x(t) = \sin(3t/25) + n(t) \quad [5]$$

Here, the time delay is $\tau = 14$ and $n(t)$ is white Gaussian noise.

Map Data. For map data, the time delay τ is 1 and the embedding dimension d_e is 5.

Chaotic logistic map (6). $r = 4$.

$$x_{n+1} = rx_n(1 - x_n) \quad [6]$$

Chaotic Henon map (7). $a = 1.4$, $b = 0.3$.

$$\begin{cases} x_{n+1} &= y_n + 1 - ax_n^2 \\ y_{n+1} &= bx_n \end{cases} \quad [7]$$

Chaotic Ikeda map (8). $t_n = 0.4 - 6/(1 + x_n^2 + y_n^2)$, $\mu = 0.7$.

$$\begin{cases} x_{n+1} &= 1 + \mu(x_n \cos t_n - y_n \sin t_n) \\ y_{n+1} &= \mu(x_n \sin t_n + y_n \cos t_n) \end{cases} \quad [8]$$

Hyperchaotic generalized Henon map (9). $a = 1.9$, $b = 0.03$.

$$\begin{cases} x_{n+1} &= a - y_n^2 - bz_n \\ y_{n+1} &= x_n \\ z_{n+1} &= y_n \end{cases} \quad [9]$$

Hyperchaotic folded-tower map (10). $a = 3.8$, $b = 0.2$.

$$\begin{cases} x_{n+1} &= ax_n(1 - x_n) - 0.05(y_n + 0.35)\,(1 - 2z_n) \\ y_{n+1} &= 0.1((y_n + 0.35)\,(1 + 2z_n) - 1)\,(1 - 1.9x_n) \\ z_{n+1} &= 3.78z_n(1 - z_n) + by_n \end{cases}$$

[10]

The effect of correlated noise Fig. S1 repeats the calculation from Fig. 2*C* of the text with correlated noise. We have repeated our analysis with a correlated noise process, $s_n = 0.8s_{n-1} - 0.5s_{n-2} + 0.6s_{n-3} + \varepsilon_n$ [where $\varepsilon_n \sim N(0, 1)$], and found our results to be indistinguishable.

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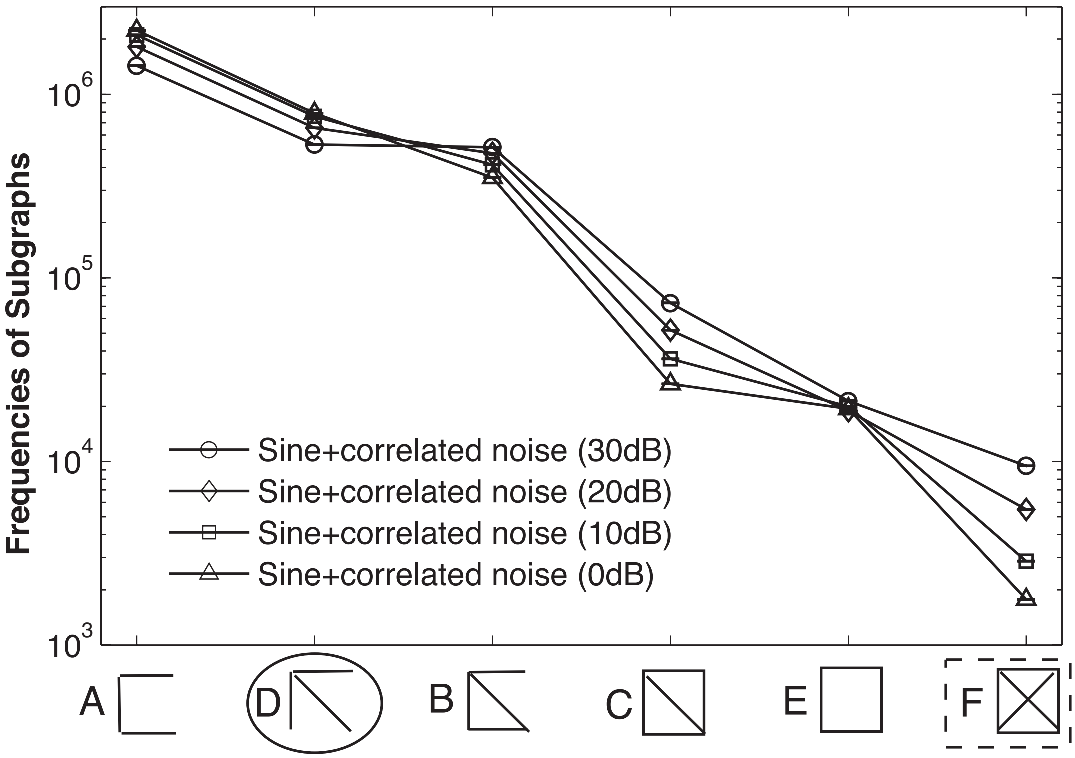

Fig. S1. Subgraph ranks for periodic signal plus correlated noise [$s_n = 0.8s_{n-1} - 0.5s_{n-2} + 0.6s_{n-3} + \varepsilon_n$, where $\varepsilon_n \sim N(0, 1)$]. Note that the relative subgraph ranks are the same as Fig. 2*C*; there are only some small differences in the frequency with which subgraph E is expressed (this subgraph appears to be insensitive to the magnitude of noise when the noise is correlated).