

1. Noise and filtering.

$$(a) \quad SNR = \frac{E_s}{E_R} = \frac{\sigma_s^2}{\sigma_n^2} = \frac{\text{Variance of pixels in a sequence of images}}{\text{Variance in uniform area.}}$$

(b) Gaussian ~~noise~~ is a kind of noise having a probability density function equal to that of normal distribution.

Impulsive noise is a kind of noise that ~~has~~ ^{contains} impulse-like content.

Median filter handles impulsive noise better, because in impulsive noise, maximum and minimum sometimes vary a lot, which caused the average shifts too much.

(c) The value of the pixels after convolution filter is 18.

(d) Because of $\frac{d}{dx}(f * g) = (\frac{d}{dx}f) * g = f * (\frac{d}{dx}g)$,

we can first derive the filter and then use the derivative filter to convolve to the image.

(e) ① Add 0 around the images: Simply add 0's outside the boundaries of the image.

② replicate the pixels value on the boundaries.

③ Ignore the boundaries pixels of the image.

(f)

$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

the sum of all entries is 1.

Because this convolution filter is doing the average.

g) Gaussian filter is separable:

$$\begin{aligned} (I * G)(x, y) &= \sum_x \sum_y I(x, y) e^{-\frac{x^2 + y^2}{2\sigma^2}} \\ &= \sum_x I(x, y) e^{-\frac{x^2}{2\sigma^2}} \sum_y I(x, y) e^{-\frac{y^2}{2\sigma^2}} \end{aligned}$$

$$I * G = (I * G_x) * G_y$$

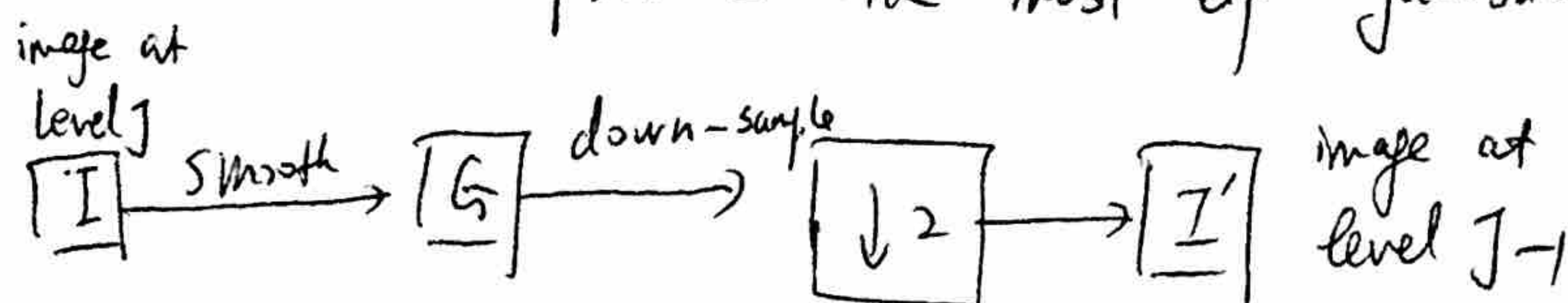
rows columns 1D convolution

(h)

$$\sigma = 2. \quad m \gg 5\sigma = 10.$$

5σ captures the most of gaussian (99.99%).

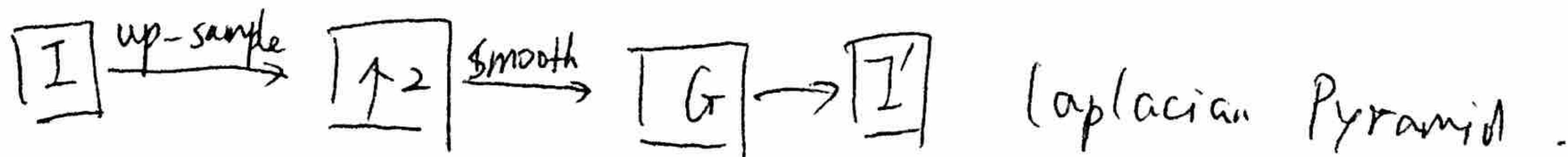
(i)



Gaussian Pyramid.

Gaussian Pyramid is to do the multi-layer analyze. Compared to a single image, we need to do down-sample to a smaller image.

(j)



Use Laplacian pyramid to up-sample to a larger image.

2. Edge detection.

(a) Use edge detection can found the location change in the image, which has a better view of how the image constructed.

Edge detection properties: depth discontinuity
discontinuity of normal
texture discontinuity
illumination discontinuity.

(b) ① Smooth: To get rid of the noise. (without affecting edges).
Use convolution filter to smooth.

② edge enhancement: to find image derivatives.

③ localization: to find 2nd derivatives.

(c) sobel and Laplacian filters to compute image gradient.

Gradient: points to the maximum change direction, which indicates the edges.

(d) ~~$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$~~ ~~$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$~~

Smooth derivation
 $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

(e) $f'(x) \cong f(x) * G'(x)$

$$\Rightarrow I_x = I * G'_x * G_y$$

$$I_y = I * G_x * G'_y$$

$$G(x) = e^{-\frac{x^2}{2\sigma^2}} \quad G'(x) = \frac{-2x}{2\sigma^2} e^{-\frac{x^2}{2\sigma^2}} = -\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

0.04	0.14	0.32	0.61	0.88	1.00	0.88	0.61	0.32	0.14	0.04
-5	-4	-3	-2	-1	0	1	2	3	4	5

$G(x)$

0.05	0.14	0.24	0.30	0.22	1	-0.22	-0.30	-0.24	-0.14	-0.05
-5	-4	-3	-2	-1	0	1	2	3	4	5

$G'(x)$

f) To compute the first derivative to find gradient max to localize the edge.

Find zero-crossings in 2nd derivative to localize the edge.

$$(\nabla^2 G \neq 1)$$

$$g) \text{ LOG} = \Delta G \frac{r^2 - 2}{r} e^{-\frac{r^2}{2}}$$

$$= (r^2 - 2) e^{-\frac{r^2}{2}}, \quad r = \sqrt{x^2 + y^2}$$

First compute ~~LOG~~ convolution with LOG, then determine: $\begin{cases} = 1 & I * \text{LOG} > 0, B \\ = 0 & I * \text{LOG} \leq 0, W \end{cases}$
Find edge and color change in transition.

ch) Canny edge detection uses direction derivative.

Condition: ① non-maximum suppression

② ~~hysteresis~~ hysteresis thresholding

ch) non-maximum is to find local max gradient magnitude in direction of gradient by comparing with their neighbours.

hysteresis thresholding is to find $D(I * G)_{ij} > T_H$ where.

Finalize the detection of edges by suppressing all the other edges that are weak and not connected to strong ones.