CS 512 HW5

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1. stereo

(a) sparse uses less corresponding points to match stereo
dense uses more points.
sparse has large disparity and finds feature points as corresponding points
dense has small disparity, uses lots of locations.

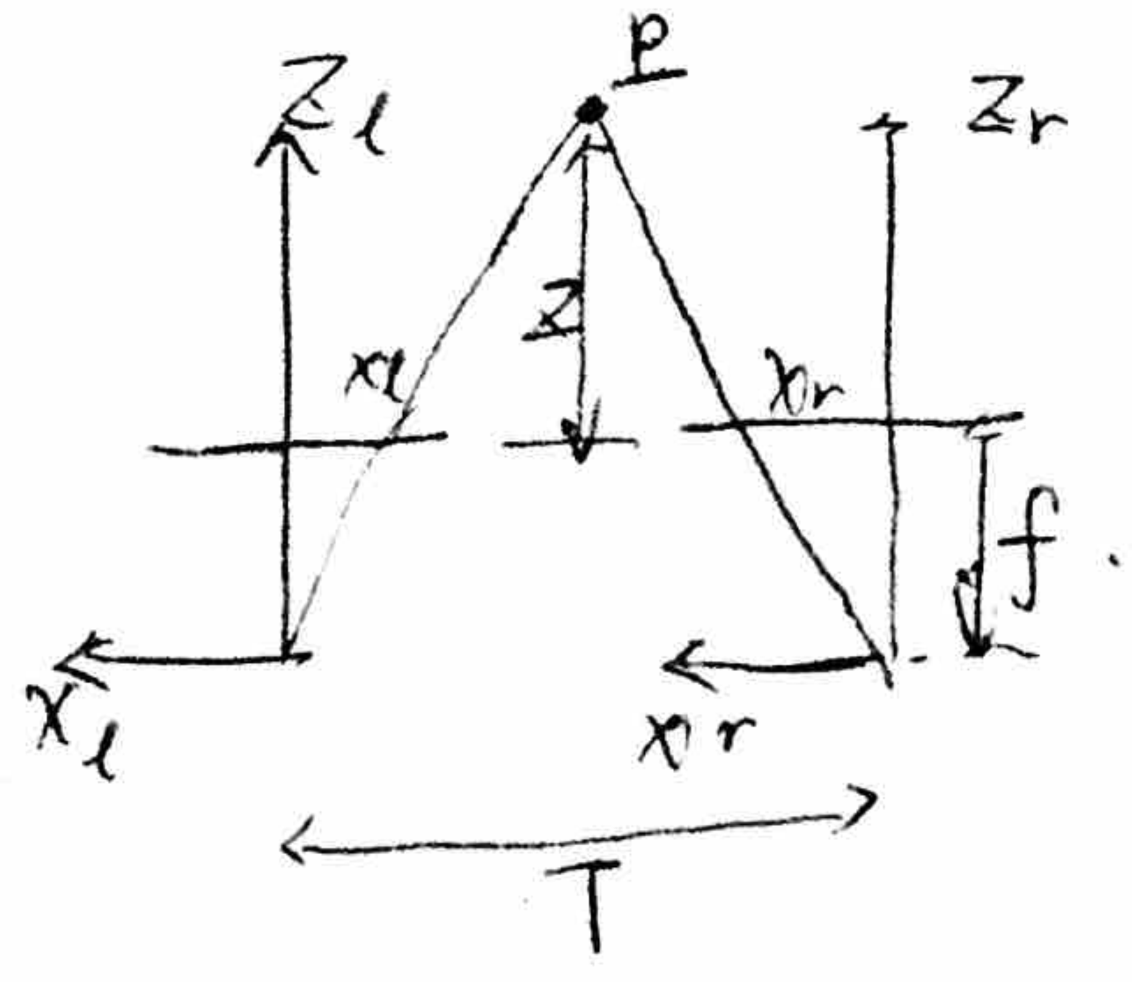
(b) correlation: $\varphi(w_1, w_2) = \sum_i w_1(x_i, y_i) \cdot w_2(x_i, y_i)$

SSD: $\varphi(w_1, w_2) = \sum_i |w_1(x_i, y_i) - w_2(x_i, y_i)|^2$

for correlation, we multiply the element. If they are correlated, the product should be high.

SSD, take the difference of two elements and square, if the value is small, the window is similar to each other.

(c)



$d = x_r - x_l = 103 - 100 = 3$

$\frac{T}{Z+f} = \frac{T-d}{Z}$

$\Rightarrow Z = f\frac{T}{d} = 10 \times \frac{100}{3} = \frac{1000}{3}$

(d) ambiguity: it may have more than one corresponding points.

(e) $M_{left \leftarrow right} = M_{left \leftarrow world} M_{world \leftarrow right}$

$= R_l^T T_l^{-1} T_r R_r$

$= \begin{bmatrix} R_l^T R_r & R_l^T (T_r - T_l) \\ 0 & 1 \end{bmatrix}$

$R = R_l^T R_r$

$T = R_l^T (T_r - T_l)$

2. Epipolar geometry

(a) Epipolar plane: the plane connected camera centers and world point

Epipolar line: intersection of the epipolar plane with images

Epipoles: intersection of all epipolar lines.

(b) $E = R^T [T]_x$

Epipolar constraint: $P_r^T E P_\ell = 0$.

(c) $F = k_r^{*T} E k_\ell^{*-1}$

$P_r^T F P_\ell = 0$

(contain $[T]_x$, which is Rank 2

(d) Rank 2. because they are skew symmetric matrix.

(e) Given P_ℓ on left image, the corresponding right epipolar line is $F P_\ell$

(f) Given P_r on right image, the corresponding left epipolar line is $F^T P_r$.

(g) Instead of fully calibrate left and right camera, weak calibration only find F matrix from 8-points. correspondence

(h) $\overset{x}{(100}, \overset{y}{200)} \longleftrightarrow \overset{x'}{(50}, \overset{y'}{100)}$

$$\begin{bmatrix} xx' & xy' & x & yx' & yy' & y & x' & y' & 1 \\ & & & & \vdots & & & & \end{bmatrix} \begin{bmatrix} f_{11} \\ \vdots \\ f_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5000 & 10000 & 100 & 10000 & 20000 & 200 & 50 & 100 & 1 \\ & & & & \vdots & & & & \end{bmatrix} \text{— — —}$$

(i) $q_i = \dfrac{p_i - \mu_p}{\sigma_p}$ $\quad q_i' = \dfrac{p_i' - \mu_p'}{\sigma_p'}$

Need to normalize points before computing F ~~since~~ to make the 8-point algo more stable and valid.

$$\underline{F = M'^T F' M}$$

(j) Since epipolar constrain $P_r^T F P_l = 0$

given $P_l \Rightarrow e_r^T F P_l = 0 \Rightarrow e_r^T F = 0$

$\Rightarrow F^T e_r = 0$

$F^T = UDV^T \Rightarrow F = (UDV^T)^T = VDU^T$

$\Rightarrow$ right epipolar is the last col of U.

$\Longleftrightarrow$ e_r is left ~~left~~ null space of F

Similarly, e_l is right null space of F.

3. Reconstruction

(a) ① move from image to camera coords

② align right with left

③ align both with baseline

④ make the image co-planar.

After retify, the two images looks like axis-aligned stereo

(b) ① reconstruction if all parameters are known.

$\longrightarrow$ absulute reconstruction

② if only internal parameters are known.

$\longrightarrow$ Euclidian reconstruction.

③ no parameters are known.

$\longrightarrow$ reconstruction up to unknown 3D projective map.

(c) $[\,p_l \quad -Rp_r \quad p_l \times Rp_r\,] \begin{bmatrix} a \\ b \\ c \end{bmatrix} = T.$

(d) $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = [\,p_l \quad -Rp_r \quad p_l \times Rp_r\,]^{-1} T$

$P = \frac{1}{2}(ap_l + bRp_r + T)$

(e) only k_l^*, k_r^* are known,

R, T are unknown.

$\hat{T} = \begin{bmatrix} \hat{T}_x \\ \hat{T}_y \\ \hat{T}_z \end{bmatrix}$, $\hat{T}_x = \sqrt{1 - \alpha_{11}}$

$\hat{T}_y = \frac{-\alpha_{12}}{\hat{T}_x}$

$\hat{T}_z = \frac{-\alpha_{13}}{\hat{T}_x}$

$R = [\,r_1 \quad r_2 \quad r_3\,]$

$r_1 = -w_1 + w_2 \times w_3$

$r_2 = -w_2 + w_3 \times w_1$

$r_3 = -w_3 + w_1 \times w_2.$

f) normalize $E \longrightarrow \hat{E}$ (baseline 1)

$$E = R^T [T]_\times$$

$$E^T E = (R^T [T]_\times)^T R^T [T]_\times$$

$$= [T]_\times^T [T]_\times$$

$$E^T E = \begin{bmatrix} T_y^2 + T_z^2 & -T_x T_y & -T_x T_z \\ & T_z^2 + T_x^2 & \\ & & T_x^2 + T_y^2 \end{bmatrix}$$

$$tr[E^T E] = 2T_x^2 + 2T_y^2 + 2T_z^2 = 2(T_x^2 + T_y^2 + T_z^2) = 2\|T\|$$

$$\hat{E} = \frac{E}{\|T\|} = \frac{2E}{2\|T\|} = \frac{2E}{tr[E^T E]}$$

(g) 4 options for sign ambiguity:

(++, +-, -+, --)

choose the one with all positive z coords