CS 512 HW6

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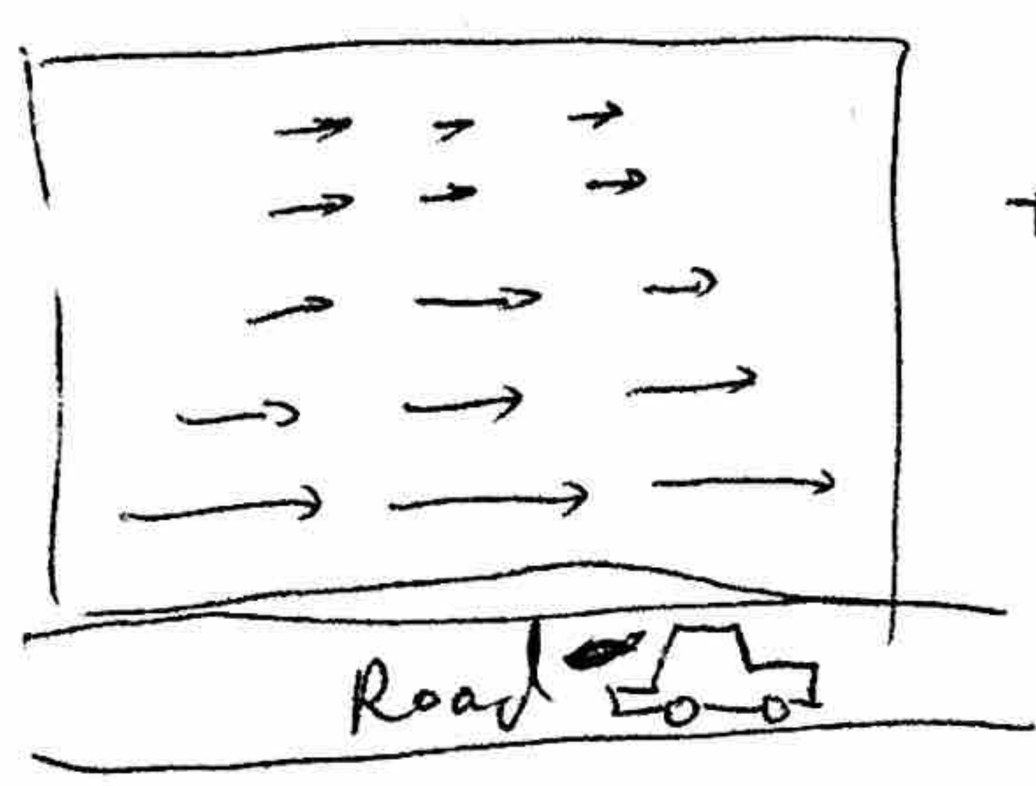
1. Motion

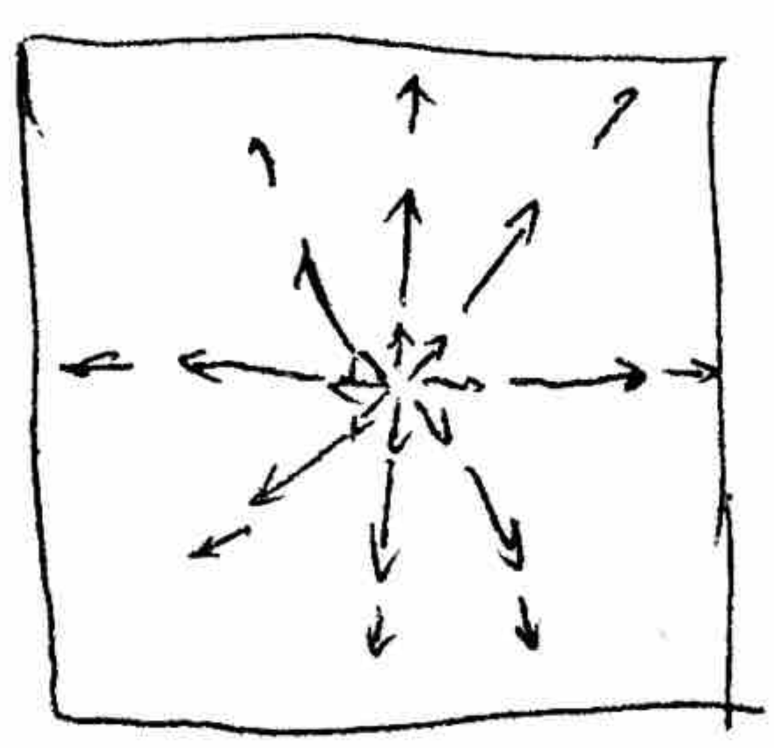
(a) 3D motion vectors are in the real world, then we can project them into 2D motion vectors, the observed 2D motion vectors are called optical flow. Yes. it is possible 3D motion doesn't produce optical flow.

(b)



the projected motion vector close to the car are larger.

(c)



the larger projected motion closer to the plane are larger.

(d) $V = \frac{f}{Z^2}(\bar{V} Z - V_z P)$

$V_x = \frac{f}{Z^2}(V_x Z - V_z X)$

$V_y = \frac{f}{Z^2}(V_y Z - V_z Y)$

$V_z = 0$

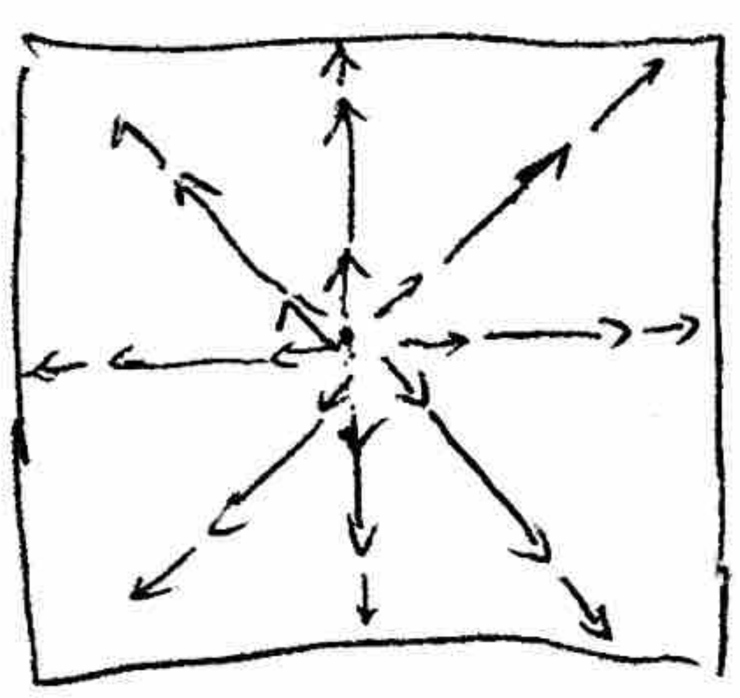
(e) $V_x = w_x Y - w_y Z - r_x$

$V_y = -w_z X + w_x Z - r_y$

$V_z = w_y X - w_x Y - r_z$

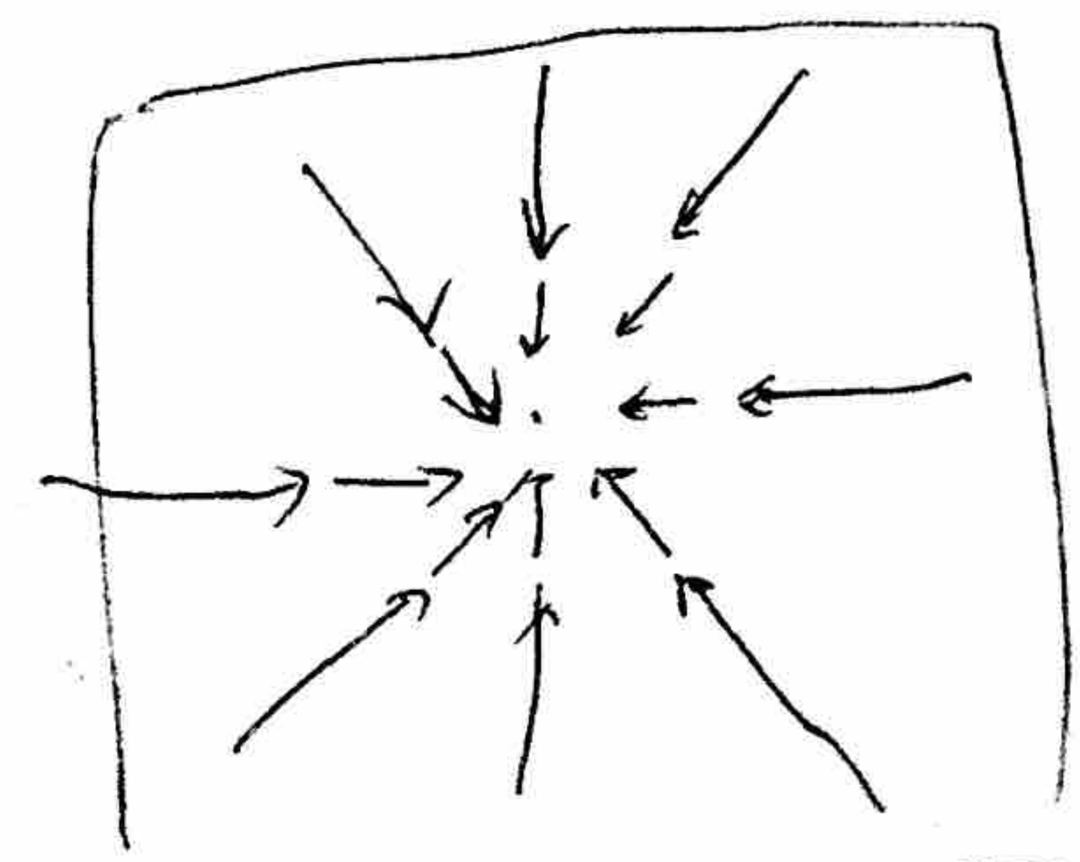
(f)

when $r_z > 0$



focus of expansion

$r_z < 0$

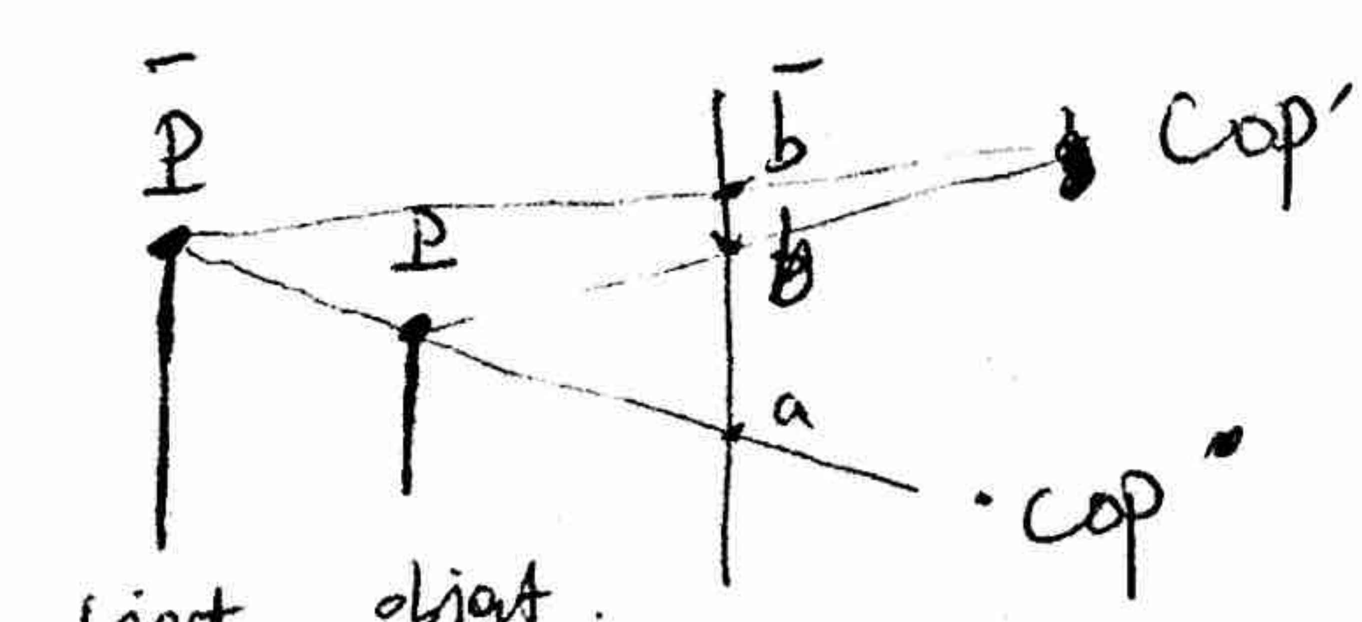


focus of contraction

(g) instantaneous epipoles

$$\begin{cases} x_0 = \dfrac{r_x}{r_z} f \\ y_0 = \dfrac{r_y}{r_z} f \end{cases}$$

(h)



when cop at cop, P and $\bar{P}$ both projected on the same point a on image.

when cop goes to cop'. P and $\bar{P}$ projected to b and $\bar{b}$. this is the motion parallax.

Relative motion

$$\Delta V_x = (x - x_0)\, r_z \left(\frac{1}{Z} - \frac{1}{\bar{Z}}\right)$$

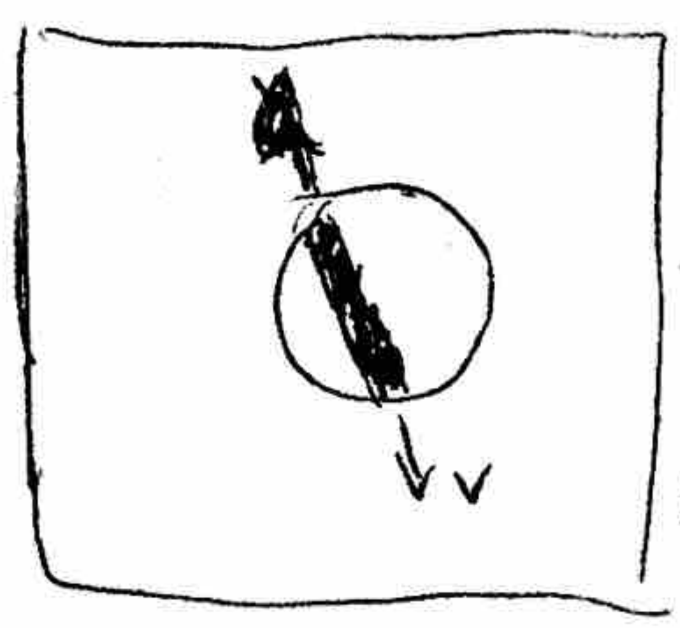
$$\Delta V_y = (y - y_0)\, r_y \left(\frac{1}{Z} - \frac{1}{\bar{Z}}\right).$$

2. Optical Flow

(a) $\frac{d}{dt} I(x(t), y(t), t) = 0$ Assume image brightness of object is constant

$\Rightarrow \nabla I \cdot V = -I_t$ OFCE

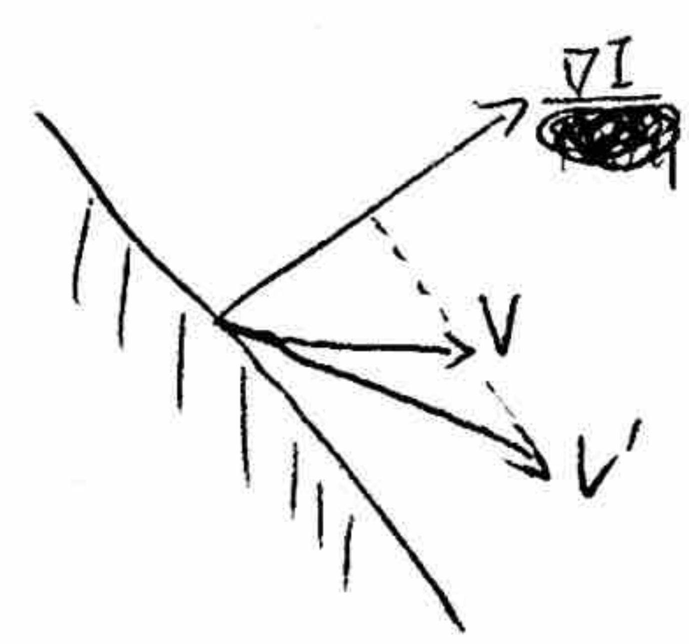
(b)



You cannot tell the motion of the stick if it moves in direction of v.

Only the motion in directon of spatial gradient can be estimated.

(c) From OFCE we have $\frac{\nabla I}{\|\nabla I\|} \cdot V = -\frac{I_t}{\|\nabla I\|}$



we have two motion vector V and V', their projection on ∇I are the same, which is similar to aperture problem.

(d) $E(V) = \sum_{(x,y) \in patch} (\nabla I(x,y) \cdot V + I_t)^2$

$V^* = \underset{V}{\operatorname{argmin}}\, E(V)$

$\nabla E(V) = 0$

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} \sum -I_x I_t \\ \sum -I_y I_t \end{bmatrix}$$ Solve for (x_t, y_t)

weighted block estimation means to give each block a wight, w. and the objective function turns to: $E(V) = \sum_{(x,y)} w (I_x x_t + I_y y_t + I_t)^2$

$$w(x,y) = \frac{1}{\|(x,y) - (x_c, y_c)\| + 1}$$

$$\begin{bmatrix} \sum w I_x^2 & \sum w I_x I_y \\ \sum w I_x I_y & \sum w I_y^2 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} -\sum w I_x I_t \\ -\sum w I_y I_t \end{bmatrix}$$

e) Instead local motion being constant, it is described by a affine map, which is more accurate.

$$E(a) = \sum_{(x,y)\in Patch} (I_x x_t + I_y y_t + I_t)^2$$

$$= \sum (I_x(a_1 + a_2 x + a_3 y) + I_y(a_4 + a_5 x + a_6 y) + I_t)^2$$

$$\begin{bmatrix} \sum I_x^2 & \sum I_x^2 x & \sum I_x^2 y & \sum I_x I_y & \sum I_x I_y & \sum I_x I_y y \\ & & \vdots & & & \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_6 \end{bmatrix} = \begin{bmatrix} -\sum I_x I_t \\ -\sum I_x I_t x \\ \vdots \end{bmatrix}$$

f) H-S: $E(v(x,y,t)) = \iint_{image} E_{of}^2(v(x,y,t)) + \alpha^2 E_s^2(v(x,y,t))$

$$E_{of}^2 = (I_x \cdot u + I_y \cdot v + I_t)^2$$

$$E_s^2 = \|\nabla u\|^2 + \|\nabla v\|^2 = u_x^2 + u_y^2 + v_x^2 + v_y^2$$

Its advantage is to add the regularization.

Difficulties: when compute u and v, we need $\bar{u}$ and $\bar{v}$, which are also related to u, v..

(g) ① start a initial guess of u, v.

② compute $\bar{u}$, $\bar{v}$.

③ compute updated u, V. $\Rightarrow$ $\begin{cases} u^{n+1} = \bar{u}^n - \dfrac{(I_x \bar{u}^n + I_y \bar{v}^n + I_t) I_x}{I_x^2 + I_y^2 + \alpha^2} \\ v^{n+1} = \bar{v}^n - \dfrac{(I_x \bar{u}^n + I_y \bar{v}^n + I_t) I_y}{I_x^2 + I_y^2 + \alpha^2} \end{cases}$

④ repeat ② ③.

Stop when:

$$\max(\text{mab}(u^{n+1} - u^n), \text{mab}(v^{n+1} - v^n)) < \tau$$

We can use L-K method to guess the initial u, v, values (or affine method)