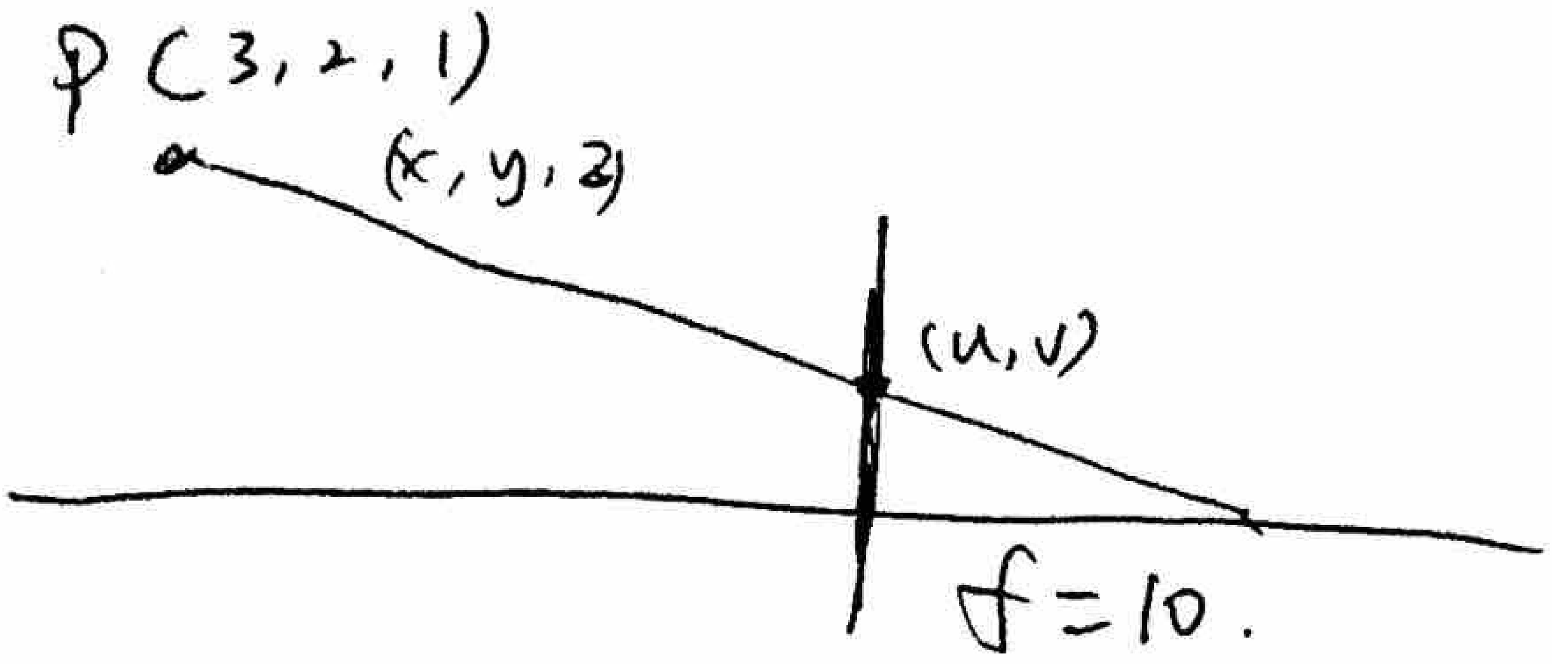
1. (a)



(30, 20)

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z}\begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

$$= \begin{bmatrix} 30 \\ 20 \end{bmatrix}.$$

(30, 20)

$u = \frac{xf}{z} = 30$

$v = \frac{yf}{z} = 20,$

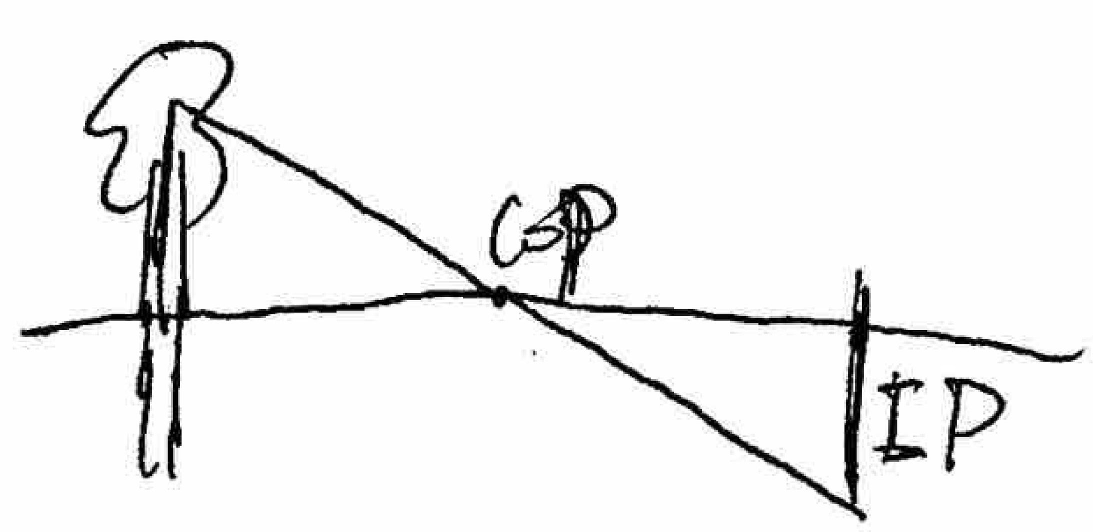
(b)

① image plane behind cop

② image plane in front of cop

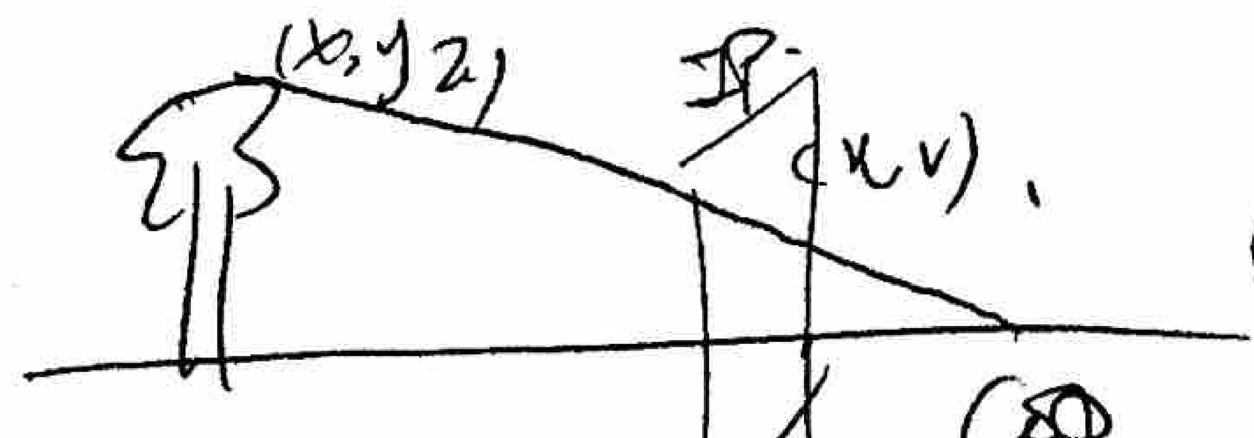
Different?

①



more realistic and better to a physical pinhole camera model

②



② still have

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z}\begin{bmatrix} f & 0 \\ 0 & f \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}.$$

, so it's justified.

(c) $f\uparrow \rightarrow u\uparrow$. focal length gets bigger → progection increa

$Z\uparrow \rightarrow u\downarrow$ distance gets bigger → projection decrease.

(d) 2D $(1,1) \longrightarrow$ 2DH $(1,1,1)$

$$\downarrow$$

$$(2,2,2)$$

(e) 2DH $(1,1,2) \rightarrow$ 2D $(\frac{1}{2}, \frac{1}{2})$.

(f). Meaning of 2DH point $(1,1,0)$

It is a point at infinity. Direction $(1,1)$.

(g) By adding an extra dimension (coordinate) for matrixs and vectors allows to model n-dimensional affine transformations as $(n+1)\times(n+1)$ matrixes acting on $n+1$-dimensional vectors.

(h) $$\underset{3\times4}{M} = k\,[\underset{3\times3}{I} \mid \underset{3\times1}{0}]$$

(i) $$p = MP$$

$$= k[I|0]P$$

$$\begin{array}{ccc} & \text{2DH} & \text{2D} \\ = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} & = \begin{bmatrix} 18 \\ 46 \\ 10 \end{bmatrix} \rightarrow & \begin{bmatrix} 1.8 \\ 4.6 \end{bmatrix} \end{array}$$

2. Modelling transformations:

(a) (1, 2) translating by (2, 3).

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

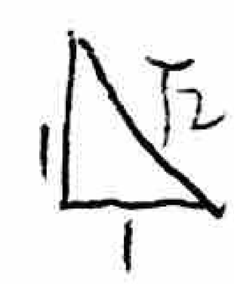
should be (1, 1) instead of (1, 2)
The final answer should be $[3, 4]^T$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

(b). (1, 1) scaling by (2, 2)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

(c) (1, 1) rotating by 45 degree.



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}.$$

(d) (1, 1) rotating by 45° about (2, 2)

$$R_p(\theta) = T(p)\, R(\theta)\, T(-p)$$

$$= T((2,2))\, R(45^\circ)\, T((-2,-2))$$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow{T((-2,-2))} \begin{bmatrix} -1 \\ -1 \end{bmatrix} \xrightarrow{R(45^\circ)} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\sqrt{2} \end{bmatrix}$$

$$\xrightarrow{T((2,2))} \begin{bmatrix} 2 \\ 2-\sqrt{2} \end{bmatrix}.$$

(e) $p'M = TRP$

(f) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 3x \\ 2y \\ 1 \end{bmatrix}$.

Scale P by (3, 2).

(h) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+1 \\ y+2 \\ 1 \end{bmatrix}$.

Translate P by (1, 2).

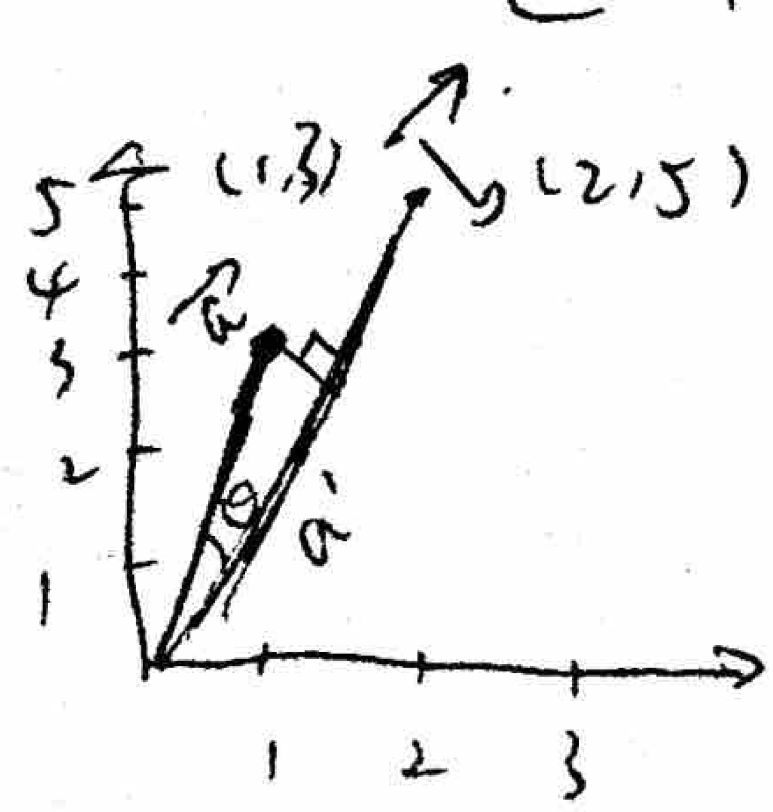
(i) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a scale (3, 2)

so $S^{-1} = S\left(\frac{1}{Sx}, \frac{1}{Sy}\right) = \left(\frac{1}{3}, \frac{1}{2}\right)$.

(j) $\begin{bmatrix} x \\ y \end{bmatrix} [1, 3] = x + 3y = 0$

$\therefore \begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

(k)



a' is the $\vec{a}$'s projection onto $\vec{b}$

$a' = \vec{a} \cos\theta \, \vec{b}$

$= \vec{a} \cdot \frac{\vec{b}}{|\vec{b}|}$

$= \frac{1\times 2 + 3\times 5}{\sqrt{4+25}} = \frac{17}{\sqrt{29}}$

3.

(a) We need the general projection matrix to align the camera coordinates with the world coordinates.

(b) $p^c = M_{c\leftarrow w}\, p^w$

$$M_{c\leftarrow w} = R^{-1} T^{-1}$$

$$= \left[\begin{array}{c|c} R^T & 0 \\ \hline 0 & 1 \end{array}\right] \left[\begin{array}{c|c} I & -T \\ \hline 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} R^T & -R^T T \\ \hline 0 & 1 \end{array}\right] = \left[\begin{array}{c|c} R^* & T^* \\ \hline 0 & 1 \end{array}\right]$$

where. $R^* = R^T$, $T^* = -R^T T$

(c) $R[\hat{x}, \hat{y}, \hat{z}]$

(d) $M = \begin{bmatrix} R^* & T^* \\ 0 & 1 \end{bmatrix}$ R^* and T^* are rotation + translation of world. with respect to camera.

$R^* = R^T$, $T^* = -R^T T$

(e) $M_{c\leftarrow i} = \left[\begin{array}{cc|c} 1/k_u & & 0 \\ & 1/k_v & \\ \hline & 0 & 1 \end{array}\right] \left[\begin{array}{c|c} I & -\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} \\ \hline 0 & 1 \end{array}\right]$

(f) $K^*[R^* | T^*]$, where T^*, R^* extrinsic parameters

K^* intrinsic parameters

(g) when image coordinates are not orthogonal, the skew parameter makes camera model more accurate.

(h) When taken account radial lens distortion, the image will get distorted, the more distance away from center, the more distortion. The complication is the lens distortion ~~matrix~~ matrix is not linear.

(i) The weak-perspective camera is a ~~simplication~~ simplification (approximation) for camera model which assumes all lines are parallel and no vanishing point. (Depth is small compare to distance)

Affine camera is a ~~base~~ branch of arbitany ~~coef~~ coefficient with 0001.

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Some affine transmations.

worse than weak-perspective cam

4. (a) surface radiance is the power of light per surface area reflected from surface.

image irradiance is the power of light per surface area received at each pixel.

(b) $E(p) = L(p) \frac{\pi}{4} \left(\frac{d}{f}\right) (\cos\alpha)^4$

(c) Albedo of the surface is a measure for reflectance or optical brightness of a surface. range from 0 to 1.

(d) Green, Red and Blue mixed with specific percentage can produce a broad array of colors. Because RGB are three primary colors.

(e) The line connects $(0,0,0)$ to $(1,1,1)$ changes
Black / White

from black to white.

(f) We can use color quantization to represent real-world colors, ie. 8 bit per chanel $1 \sim 255$. (R, G, B)

(g) Defining Y as luminace has the useful result that for any given Y value, the XZ plane will contain all possible chromaticities at that luminace. —— Wikipedia, CIE 1931 color space.

(h) LAB color model is designed to approximate human vision. It can be used to make accurate color balance corrections.