CHAPTER 12: SIMULATION

12.1 – What-If Analysis

12.2 – Simulation of Sanotronics Problem

12.3 – Inventory Simulation

12.4 – Waiting Line Simulation

12.5 – Simulation Considerations

SIMULATION (1 OF 5)

This chapter introduces the use of **simulation** to evaluate the impact of uncertainty on a decision.

- Financial applications: investment planning, project selection, and option pricing.
- Marketing applications: new product development and the timing of market entry for a product.
- Operations applications: project management, inventory management, capacity planning, and revenue management (prominent in the airline, hotel, and car rental industries).

SIMULATION (2 OF 5)

A spreadsheet simulation analysis requires a model foundation of logical formulas that correctly express the relationships between **parameters** and decisions to generate outputs of interest.

For example, a simple spreadsheet model may compute a clothing retailer's profit, given values for the number of ski jackets ordered from the manufacturer and the number of ski jackets demanded by customers. A simulation analysis extends this model by replacing the single value used for ski jacket demand with a **probability distribution** of possible values of ski jacket demand. A probability distribution of ski jacket demand represents not only the range of possible values but also the relative likelihood of various levels of demand.

SIMULATION (3 OF 5)

A spreadsheet simulation analysis requires a model foundation of logical formulas that correctly express the relationships between **parameters** and decisions to generate outputs of interest.

For example, a simple spreadsheet model may compute a clothing retailer's profit, given values for the number of ski jackets ordered from the manufacturer and the number of ski jackets demanded by customers. A simulation analysis extends this model by replacing the single value used for ski jacket demand with a **probability distribution** of possible values of ski jacket demand. A probability distribution of ski jacket demand represents not only the range of possible values but also the relative likelihood of various levels of demand.

SIMULATION (4 OF 5)

To evaluate a decision with a simulation model, an analyst identifies parameters that are not known with a high degree of certainty and treats these parameters as random, or uncertain,

variables. The values for the **random variables** or **uncertain variables** are randomly generated from the specified probability distributions.

The simulation model uses the randomly generated values of the random variables and the relationships between parameters and decisions to compute the corresponding values of an output.

SIMULATION (5 OF 5)

After reviewing the simulation results, the analyst is often able to make decision recommendations for the **controllable inputs** that address the *average* output and the *variability* of the output.

When making a decision in the presence of uncertainty, decision makers should not only be interested in the average, or expected, outcome, but they should also be interested in information regarding the variability of possible outcomes.

Specifically, decision makers are interested **risk analysis**, i.e., quantifying the likelihood and magnitude of an undesirable outcome.

WHAT-IF ANALYSIS (1 OF 9)

Sanotronics is a startup company that manufactures medical devices for use in hospital clinics.

Inspired by experiences with family members who have battled cancer, Sanotronics's founders have developed a prototype for a new device that limits health care workers' exposure to chemotherapy treatments while they are preparing, administering, and disposing of these hazardous medications. This new device features an innovative design and has the potential to capture a substantial share of the market.

WHAT-IF ANALYSIS (2 OF 9)

Santronics would like an analysis of the first-year profit potential of the device. Because of Sanotronics's tight cash flow situation, management is particularly concerned about the potential for a loss.

Sanotronics has identified the key parameters in determining first-year profit: selling price per unit (p), first-year administrative and advertising costs (c_a), direct labor cost per unit (c_l), parts cost per unit (c_p), and first-year demand (d).

WHAT-IF ANALYSIS (3 OF 9)

After conducting market research and a financial analysis, Sanotronics estimates with a high level of certainty that the device's selling price will be $249 per unit, and the first-year administrative and advertising costs will total $1,000,000.

Sanotronics is not certain about the values for the cost of direct labor, the cost of parts, and the first-year demand.

At this stage of the planning process, Sanotronics's base estimates of these inputs are $45 per unit for the direct labor cost, $90 per unit for the parts cost, and 15,000 units for the first-year demand.

WHAT-IF ANALYSIS (4 OF 9)

Sanotronics's first-year profit is computed by

$$\text{Profit} = \left(p - c_l - c_p\right) \times d - c_a$$

$$\text{Profit} = \left(249 - c_l - c_p\right) \times d - 1{,}000{,}000$$

Sanotronics's base-case estimates of the direct labor cost per unit, the parts cost per unit, and first-year demand are \$45, \$90, and 15,000 units, respectively. These values constitute the **base-case scenario** for Sanotronics, therefore

$$\text{Profit} = \left(249 - 45 - 90\right)\left(15{,}000\right) - 1{,}000{,}000 = 710{,}000$$

WHAT-IF ANALYSIS (5 OF 9)

While the base-case scenario looks appealing, Sanotronics is aware that the values of direct labor cost per unit, parts cost per unit, and first-year demand are uncertain, so the base-case scenario may not occur.

To help Sanotronics gauge the impact of the uncertainty, a **what-if analysis** involves considering alternative values for the random variables (direct labor cost, parts cost, and first-year demand) and computing the resulting value for the output (profit).

WHAT-IF ANALYSIS (6 OF 9)

Sanotronics is interested in what happens if the estimates of the direct labor cost per unit, parts cost per unit, and first-year demand do not turn out to be as expected under the basecase scenario.

For instance, suppose that Sanotronics believes that direct labor costs could range from $43 to $47 per unit, the parts cost could range from $80 to $100 per unit, and the first-year demand could range from 0 to 30,000 units. Using these ranges, what-if analysis can be used to evaluate a **worst-case scenario** and a **best-case scenario**.

WHAT-IF ANALYSIS (7 OF 9)

The worst-case value for the direct labor cost is \$47 (the highest value), the worst-case value for the parts cost is \$100 (the highest value), and the worst-case value for demand is 0 units (the lowest value). This leads to the following profit projection:

$$\text{Profit} = (249 - 47 - 100)(0) - 1{,}000{,}000 = -1{,}000{,}000$$

So, the worst-case scenario leads to a projected *loss* of \$1,000,000.

WHAT-IF ANALYSIS (8 OF 9)

The best-case value for the direct labor cost is $43 (the lowest value), the best-case value for the parts cost is $80 (the lowest value), and the best-case value for demand is 30,000 units (the highest value). This leads to the following profit projection:

$$\text{Profit} = (249 - 43 - 80)(30{,}000) - 1{,}000{,}000 = 2{,}780{,}000$$

So the best-case scenario leads to a projected profit of $2,780,000.

WHAT-IF ANALYSIS (9 OF 9)

Simple what-if analyses do not indicate the likelihood of the various profit or loss values.

In particular, we do not know anything about the probability of a loss.

To conduct a more thorough evaluation of risk by obtaining insight on the potential magnitude and probability of undesirable outcomes, we now turn to developing a spreadsheet simulation model.

SIMULATION OF SANTRONICS PROBLEM

The first step in constructing a spreadsheet simulation model is to express the relationship between the inputs and the outputs with appropriate formula logic.

	A	B
1	Sanotronics	
2		
3	Parameters	
4	Selling Price per Unit	249
5	Administrative & Advertising Cost	1000000
6	Direct Labor Cost per Unit	45
7	Parts Cost per Unit	90
8	Demand	15000
9		
10	Model	
11	Profit	=((B4-B6-B7)*B8)-B5
12		

	A	B
1	Sanotronics	
2		
3	Parameters	
4	Selling Price per Unit	$249.00
5	Administrative & Advertising Cost	$1,000,000
6	Direct Labor Cost per Unit	$45.00
7	Parts Cost per Unit	$90.00
8	Demand	15,000
9		
10	Model	
11	Profit	$710,000.00

By changing one or more values for the input parameters, the model can be used to conduct a manual what-if analysis

SIMULATION OF SANTRONICS PROBLEM

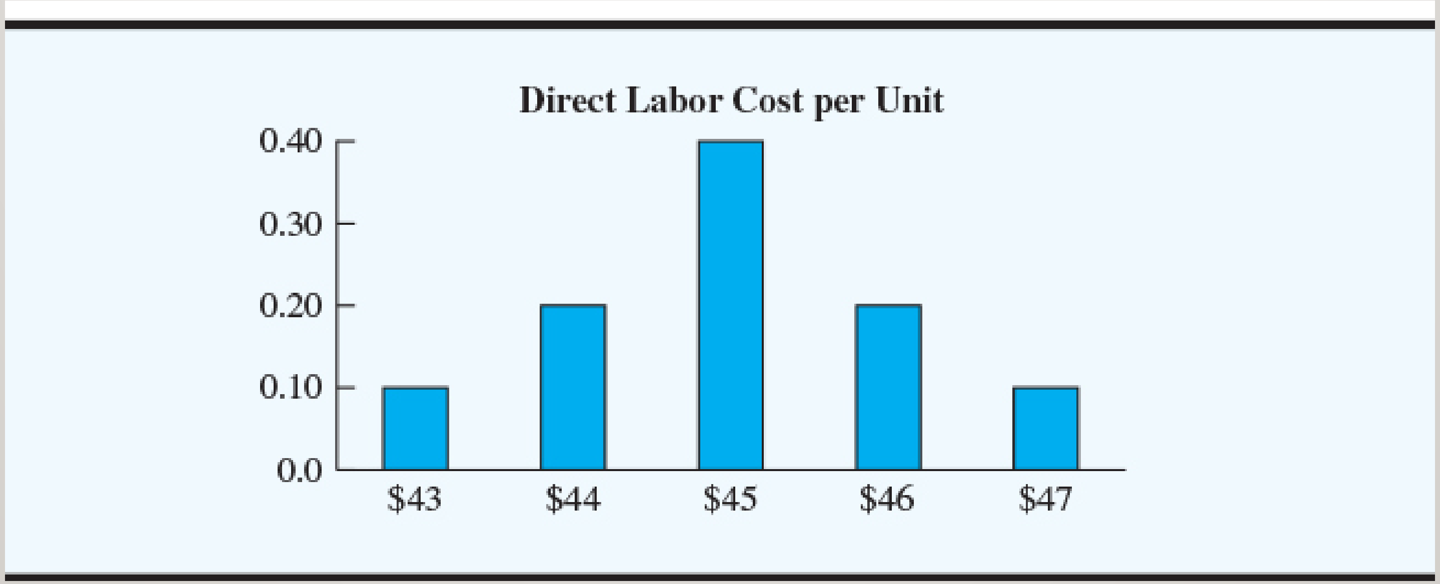
Instead of manually selecting the values for the random variables, a simulation model randomly generates values for the random variables so that the values used reflect what we might observe in practice.

A probability distribution describes the possible values of a random variable and the relative likelihood of the random variable realizing these values.

The analyst can use historical data and knowledge of the random variable (such as the range, mean, mode, standard deviation) to specify the probability distribution for a random variable.

SIMULATION OF SANTRONICS PROBLEM
(3 OF 19)

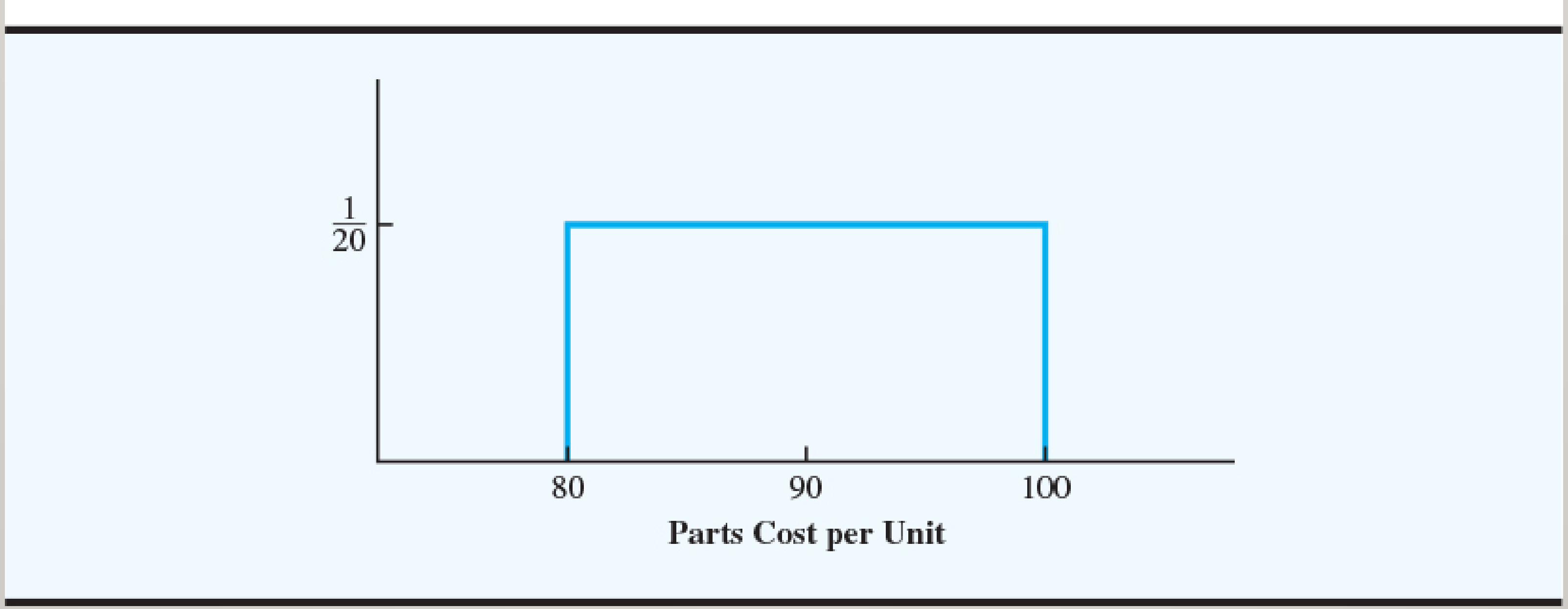
Sanotronics believes that the direct labor cost will range from $43 to $47 per unit and is described by the discrete probability distribution shown here:



Because Sanotronics models the direct labor cost per unit with a **<u>discrete probability distribution</u>**, the direct labor cost per unit can *only* take on a value of $43, $44, $45, $46, or $47.

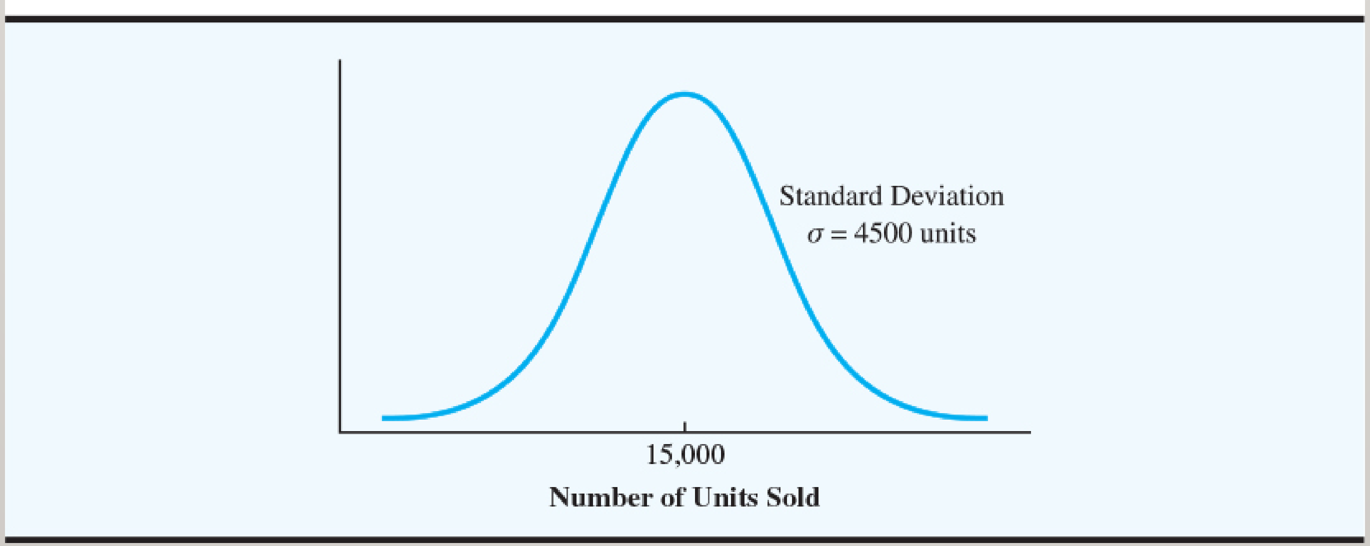
SIMULATION OF SANTRONICS PROBLEM

Sanotronics decides to describe the uncertainty in parts cost with a uniform probability distribution, as shown in Figure 12.3. Costs per unit between \$80 and \$100 are equally likely. A uniform probability distribution is an example of a **continuous probability distribution**; this means that the parts cost can take on *any* value between \$80 and \$100 with equal likelihood.



SIMULATION OF SANTRONICS PROBLEM

Sanotronics believes that first-year demand is described by the normal probability distribution. The mean or expected value of first-year demand is 15,000 units. The standard deviation of 4500 units describes the variability in the first-year demand. The normal probability distribution is a continuous probability distribution in which any value is possible, but values far larger or smaller than the mean are increasingly unlikely.



SIMULATION OF SANTRONICS PROBLEM

To simulate the Sanotronics problem, we must generate values for the three random variables and compute the resulting profit.

A set of values for the random variables is called a trial.

We then generate another trial, compute a second value for profit, and so on.

We continue this process until we are satisfied that sufficient trials have been conducted to describe the probability distribution for profit. Put simply, simulation is the process of generating values of random variables and computing the corresponding output measures.

SIMULATION OF SANTRONICS PROBLEM

In the Sanotronics model, representative values must be generated for the random variables corresponding to the direct labor cost per unit, the parts cost per unit, and the first-year demand.

To illustrate how to generate these values, we need to introduce the concept of computer-generated random numbers.

SIMULATION OF SANTRONICS PROBLEM
(8 OF 19)

Computer-generated random numbers are randomly selected numbers from 0 up to, but not including, 1; this interval is denoted as [0, 1).

All values of the computer-generated random numbers are equally likely and so are uniformly distributed over the interval from 0 to 1.

Computer-generated random numbers can be obtained using built-in functions available in computer simulation packages and spreadsheets. For example, placing the formula =RAND() in a cell of an Excel worksheet will result in a random number between 0 and 1 being placed into that cell.

SIMULATION OF SANTRONICS PROBLEM
(9 OF 19)

The table illustrates the process of partitioning the interval from 0 to 1 into subintervals so that the probability of generating a random number in a subinterval is equal to the probability of the corresponding direct labor cost.

Direct Labor Cost per Unit	Probability	Interval of Random Numbers
$43	0.1	[0.0, 0.1)
$44	0.2	[0.1, 0.3)
$45	0.4	[0.3, 0.7)
$46	0.2	[0.7, 0.9)
$47	0.1	[0.9, 1.0)

SIMULATION OF SANTRONICS PROBLEM (10 OF 19)

Let us now turn to the issue of generating values for the parts cost. The probability distribution for the parts cost per unit is the uniform distribution.

To generate a value for a random variable characterized by a continuous uniform distribution, the following Excel formula is used:

$$\text{Value of uniform random variable} = \text{lower bound} + (\text{upper bound} - \text{lower bound}) \times \text{RAND()}$$

For Sanotronics, parts cost per unit is a uniformly distributed random variable with a lower bound of \$80 and an upper bound of \$100.

$$\text{Parts cost} = 80 + 20 \times \text{RAND()}$$

SIMULATION OF SANTRONICS PROBLEM

$$\text{Parts cost} = 80 + 20 \times \text{RAND()}$$

The first term of the equation is 80, because Sanotronics is assuming that the parts cost will never drop below \$80 per unit.

Because RAND is between 0 and 1, the second term, 20 times RAND(), corresponds to how much more than the lower bound the simulated value of parts cost is.

RAND is equally likely to be any value between 0 and 1, so the simulated value for the parts cost is equally likely to be between the lower bound (80 + 0 = 80) and the upper bound (80 + 20 = 100).

SIMULATION OF SANTRONICS PROBLEM

Lastly, we need a procedure for generating the first-year demand from computer generated random numbers.

To generate a value for a random variable characterized by a normal distribution with a specified mean and standard deviation, the following Excel formula is used:

Value of normal random variable = NORM.INV(RAND(), mean, standard deviation)

For Sanotronics, first-year demand is a normally distributed random variable with a mean of 15,000 and a standard deviation of 4500.

Demand = NORM.INV(RAND(), 15000, 4500)

SIMULATION OF SANTRONICS PROBLEM

Now that we know how to randomly generate values for the random variables (direct labor cost, parts cost, first-year demand) from their respective probability distributions, we modify the spreadsheet by adding this information.

	A	B	C	D	E	F
1	**Sanotronics**					
2						
3	**Parameters**					
4	**Selling Price per Unit**	249				
5	**Administrative & Advertising Cost**	1000000				
6	**Direct Labor Cost per Unit**	=VLOOKUP(RAND(),A15:C19,3,TRUE)				
7	**Parts Cost per Unit**	=F14+(F15-F14)*RAND()				
8	**Demand**	=NORM.INV(RAND(),F18,F19)				
9						
10	**Model**					
11	**Profit**	=((B4-B6-B7)*B8)-B5				
12						
13	**Direct Labor Cost**				**Parts Cost (Uniform Distribution)**	
14	Lower End of Interval	Upper End of Interval	Cost per Unit	Probability	Lower Bound	80
15	0	=D15+A15	43	0.1	Upper Bound	100
16	=B15	=D16+A16	44	0.2		
17	=B16	=D17+A17	45	0.4	**Demand (Normal Distribution)**	
18	=B17	=D18+A18	46	0.2	Mean	15000
19	=B18	1	47	0.1	Standard Deviation	4500
20						

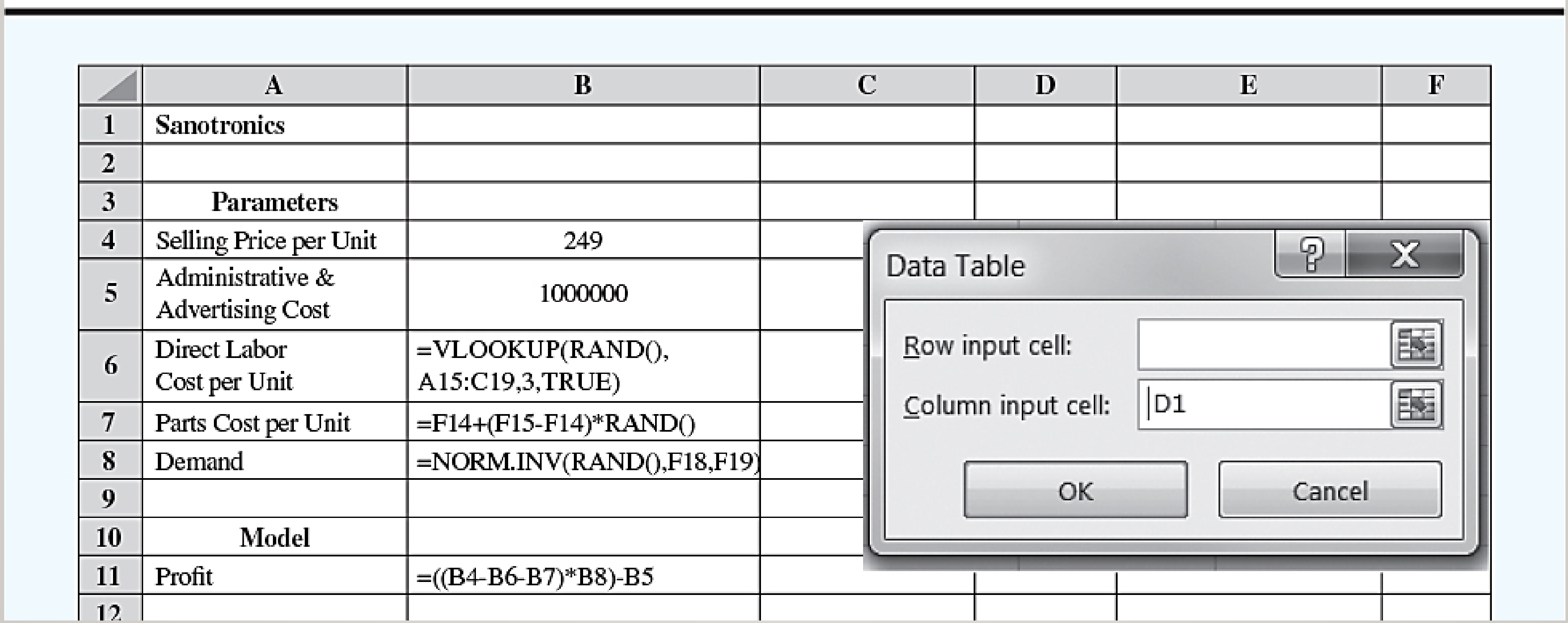
SIMULATION OF SANTRONICS PROBLEM
(14 OF 19)

Each trial in the simulation involves randomly generating values for the random variables (direct labor cost, parts cost, and first-year demand) and computing profit.

Next, we set up a spreadsheet for the execution of 1000 simulation trials using cells A22:A1021 to contain the simulation results.

The spreadsheet is shown on the next two slides.

SIMULATION OF SANTRONICS PROBLEM
(16 OF 19)



	A	B	C	D	E	F
1	Sanotronics					
2						
3	Parameters					
4	Selling Price per Unit	249				
5	Administrative & Advertising Cost	1000000				
6	Direct Labor Cost per Unit	=VLOOKUP(RAND(), A15:C19,3,TRUE)				
7	Parts Cost per Unit	=F14+(F15-F14)*RAND()				
8	Demand	=NORM.INV(RAND(),F18,F19)				
9						
10	Model					
11	Profit	=((B4-B6-B7)*B8)-B5				
12						

SIMULATION OF SANTRONICS PROBLEM
(15 OF 19)

12						
13	Direct Labor Cost				Parts Cost (Uniform Distribution)	
14	Lower End of Interval	Upper End of Interval	Cost per Unit	Probability	Lower Bound	80
15	0	=D15+A15	43	0.1	Upper Bound	100
16	-B15	=D16+A16	44	0.2		
17	-B16	=D17+A17	45	0.4	Demand (Normal Distribution)	
18	-B17	=D18+A18	46	0.2	Mean	15000
19	-B18	1	47	0.1	Standard Deviation	4500
20						
21	Simulation Trial	Direct Labor Cost per Unit	Parts Cost per Unit	Demand	Profit	
22	1	=B6	=B7	=B8	=B11	
23	2					
24	3					
1019	998					
1020	999					
1021	1000					

SIMULATION OF SANTRONICS PROBLEM

The analysis of the output observed over the set of simulation trials is a critical part of the simulation process. For the collection of simulation trials, it is helpful to compute descriptive statistics such as sample average, sample standard deviation, minimum, maximum, and sample proportion. To compute these statistics for the Sanotronics example, we use the following Excel functions:

Cell H22	= AVERAGE(E22:E1021)
Cell H23	= STDEV.S(E22:E1021)
Cell H24	= MIN(E22:E1021)
Cell H25	= MAX(E22:E1021)
Cell H26	= COUNTIF(E22:E1021,“,0”) / COUNT(E22:E1021)

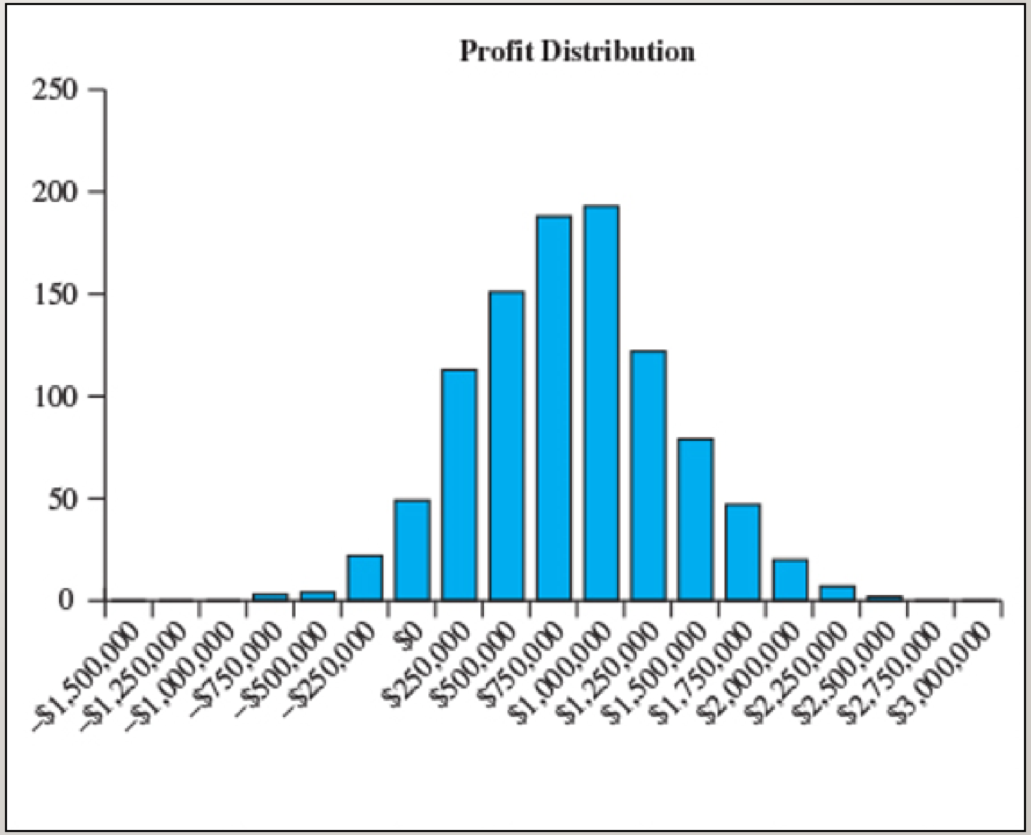
SIMULATION OF SANTRONICS PROBLEM

We observe a mean profit of \$717,663, standard deviation of \$521,536, extremes ranging between -\$996,547 and \$2,253,674, and a 0.078 estimated probability of a loss.

Profit Summary Statistics	
Mean	\$717,663
Standard Deviation	\$521,536
Minimum Profit	-\$996,547
Maximum Profit	\$2,253,674
P(Profit<\$0)	0.078

SIMULATION OF SANTRONICS PROBLEM

The distribution of profit values is fairly symmetric, with a large number of values in the range of $250,000 to $1,250,000. The probability of a large loss or a large gain is small. Only 7 trials out of 1000 resulted in a loss of more than $500,000, and only 9 trials resulted in a profit greater than $2,000,000. The bin with the largest number of values has profit ranging between $750,000 and $1,000,000.



INVENTORY SIMULATION (1 OF 8)

In this section, we describe how simulation can be used to establish an inventory policy for a product that has an uncertain demand.

In our example, we consider the Butler Internet Company, which distributes a wireless router. Each router costs Butler \$75 and sells for \$125. Thus Butler realizes a gross profit of \$125 - \$75 = \$50 for each router sold. Monthly demand for the router is described by a normal probability distribution with a mean of 100 units and a standard deviation of 20 units.

INVENTORY SIMULATION (2 OF 8)

Butler receives monthly deliveries from its supplier and replenishes its inventory to a level of *Q* at the beginning of each month. This beginning inventory level is referred to as the replenishment level.

- If monthly demand is less than the replenishment level, an inventory holding cost of $15 is charged for each unit that is not sold.
- If monthly demand is greater than the replenishment level, a stock-out occurs and a shortage cost is incurred.

INVENTORY SIMULATION (3 OF 8)

Because Butler assigns a loss-of-goodwill cost of $30 for each customer turned away, a shortage cost of $30 is charged for each unit of demand that cannot be satisfied.

Management would like to use a simulation model to determine the average monthly net profit resulting from using particular replenishment levels.

Management would also like information on the percentage of total demand that will be satisfied. This percentage is referred to as the *service level*.

INVENTORY SIMULATION (4 OF 8)

The controllable input to the Butler simulation model is the replenishment level, *Q*. The monthly demand, *D*, is a random variable.

The two output measures are the average monthly net profit and the service level. Computation of the service level requires that we keep track of the number of routers sold each month and the total demand for routers for each month.

The service level will be computed at the end of the simulation run as the ratio of total units sold to total demand.

INVENTORY SIMULATION (5 OF 8)

When demand is less than or equal to the replenishment level ($D \le Q$), D units are sold, and an inventory holding cost of \$15 is incurred for each of the $Q - D$ units that remain in inventory. Net profit for this case is computed as follows:

Case 1: $D \le Q$

Gross profit = \$50D

Holding cost = \$15($Q - D$)

Net profit = Gross profit − Holding cost = \$50D − \$15($Q - D$)

INVENTORY SIMULATION (6 OF 8)

When demand is greater than the replenishment level ($D > Q$), Q routers are sold, and a shortage cost of \$30 is imposed for each of the $D – Q$ units of demand not satisfied. Net profit for this case is computed as follows:

Case 2: $D > Q$

Gross profit = $\$50Q$

Shortage cost = $\$30(D – Q)$

Net profit = Gross profit – Shortage cost = $\$50Q – \$30(D – Q)$

INVENTORY SIMULATION (7 OF 8)

Each trial in the simulation represents one month of operation. The simulation is run for 1000 months using a given replenishment level, *Q*. Then, the average profit and service level output measures are computed. Here are the results:

Month	Demand	Sales	Gross Profit ($)	Holding Cost ($)	Shortage Cost ($)	Net Profit ($)
1	79	79	3,950	315	0	3,635
2	111	100	5,000	0	330	4,670
3	93	93	4,650	105	0	4,545
4	100	100	5,000	0	0	5,000
5	118	100	5,000	0	540	4,460
Totals	501	472	23,600	420	870	22,310
Average	100	94	$4,720	$84	$174	$4,462

INVENTORY SIMULATION (8 OF 8)

By varying the values of controllable inputs, simulation models can be used to identify good operating policies and decisions. For Butler, the simulation model can be used to test the impact of different replenishment levels on the monthly net profit. The table summarizes the results of varying the replenishment levels of 110, 120, 130, and 140 units.

Replenishment Level	Average Net Profit ($)	Standard Deviation Profit ($)	Service Level (%)
100	4276	661	92.4
110	4498	853	96.2
120	4573	1078	98.1
130	4462	1201	99.4
140	4327	1247	99.9

SIMULATION CONSIDERATIONS (1 OF 2)

Verification is the process of determining that the computer procedure performing the simulation calculations is logically correct. Verification is largely a debugging task to ensure that there are no errors in the computer procedure that implements the simulation.

In some cases, an analyst may compare computer results for a limited number of events with independent hand calculations. In other cases, tests may be performed to verify that the random variables are being generated correctly and that the output from the simulation model appears to be reasonable. The verification step is not complete until the user develops a high degree of confidence that the computer procedure is error free.

SIMULATION CONSIDERATIONS (2 OF 2)

Validation is the process of ensuring that the simulation model provides an accurate representation of a real system. Validation requires an agreement among analysts and managers that the logic and the assumptions used in the design of the simulation model accurately reflect how the real system operates.

The first phase of the validation process is done prior to, or in conjunction with, the development of the computer procedure for the simulation process. Validation continues after the computer program has been developed, with the analyst reviewing the simulation output to see whether the simulation results closely approximate the performance of the real system.