CHAPTER 4: LINEAR PROGRAMMING APPLICATIONS IN MARKETING, FINANCE, AND OPERATIONS MANAGEMENT

MARKETING APPLICATIONS (1 OF 10)

Media Selection

- One application of linear programming in marketing is media selection.
- LP can be used to help marketing managers allocate a fixed budget to various advertising media.
- The objective is to maximize reach, frequency, and quality of exposure.
- Restrictions on the allowable allocation usually arise during consideration of company policy, contract requirements, and media availability.

MARKETING APPLICATIONS (2 OF 10)

Relax-and-Enjoy Lake Development Corporation is developing a lakeside community at a privately owned lake.

The primary market for the lakeside lots and homes includes all middle- and upper-income families within approximately 100 miles of the development.

Relax-and-Enjoy employed the advertising firm of Boone, Phillips, and Jackson (BP&J) to design the promotional campaign.

MARKETING APPLICATIONS (3 OF 10)

BP&J collected data on the number of potential customers reached, the cost per advertisement, the maximum number of times each medium is available, and the exposure quality rating for each of the five media.

The quality rating is measured in terms of an exposure quality unit, a measure of the relative value of one advertisement in each of the media. This measure, based on BP&J's experience in the advertising business, takes into account factors such as audience demographics (age, income, and education of the audience reached), image presented, and quality of the advertisement.

MARKETING APPLICATIONS (4 OF 10)

Advertising media alternatives for the Relax-and-Enjoy Lake Development Corporation:

Advertising Media	Number of Potential Customers Reached	Cost ($) per Advertisement	Maximum Times Available per Month*	Exposure Quality Units
1. Daytime TV (1 min), station WKLA	1000	1500	15	65
2. Evening TV (30 sec), station WKLA	2000	3000	10	90
3. Daily newspaper (full page), *The Morning Journal*	1500	400	25	40
4. Sunday newspaper magazine (1/2 page color), *The Sunday Press*	2500	1000	4	60
5. Radio, 8:00 a.m. or 5:00 p.m. news (30 sec), station KNOP	300	100	30	20

*The maximum number of times the medium is available is either the maximum number of times the advertising medium occurs (e.g., four Sundays per month) or the maximum number of times BP&J recommends that the medium be used.

MARKETING APPLICATIONS (5 OF 10)

The decision to be made is how many times to use each medium. We begin by defining the decision variables:

DTV = number of times daytime TV is used

ETV = number of times evening TV is used

DN = number of times daily newspaper is used

SN = number of times Sunday newspaper is used

R = number of times radio is used

MARKETING APPLICATIONS (6 OF 10)

Each daytime TV (DTV) advertisement is rated at 65 exposure quality units. Evening TV (ETV) is rated at 90 exposure quality units, daily newspaper (DN) rated at 40 exposure quality units, Sunday newspaper (SN) rated at 60 exposure quality units, and radio (R) rated at 20 exposure quality units.

Advertising Media	Number of Potential Customers Reached	Cost ($) per Advertisement	Maximum Times Available per Month*	Exposure Quality Units
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*The maximum number of times the medium is available is either the maximum number of times the advertising medium occurs (e.g., four Sundays per month) or the maximum number of times BP&J recommends that the medium be used.

MARKETING APPLICATIONS (7 OF 10)

With the objective of maximizing the total exposure quality units for the overall media selection plan, the objective function becomes

$$\text{Max} \quad 65DTV + 90ETV + 40DN + 60SN + 20R \quad \text{Exposure quality}$$

We now formulate the constraints for the model from the information given:

$$\left.\begin{array}{lllllll}
DTV & & & & & \leq & 15 \\
 & ETV & & & & \leq & 10 \\
 & & DN & & & \leq & 25 \\
 & & & SN & & \leq & 4 \\
 & & & & R & \leq & 30
\end{array}\right\} \begin{array}{l}\text{Availability} \\ \text{of media}\end{array}$$

$$1500DT + 3000ETV + 400DN + 1000SN + 100R \leq 30,000 \quad \text{Budget}$$

$$\left.\begin{array}{lll}
DTV + \quad ETV & \geq & 10 \\
1500DTV + 3000ETV & \leq & 18,000
\end{array}\right\} \begin{array}{l}\text{Television} \\ \text{restrictions}\end{array}$$

$$1000DTV + 2000ETV + 1500DN + 2500SN + 300R \geq 50,000 \quad \text{Customers reached}$$

$$DTV, ETV, DN, SN, R \geq 0$$

MARKETING APPLICATIONS (8 OF 10)

The optimal solution to this five-variable, nine-constraint linear programming model is shown here.

Optimal Objective Value = 2370.00000

Variable	Value	Reduced Cost
DTV	10.00000	0.00000
ETV	0.00000	-65.00000
DN	25.00000	0.00000
SN	2.00000	0.00000
R	30.00000	0.00000
Constraint	**Slack/Surplus**	**Dual Value**
1	5.00000	0.00000
2	10.00000	0.00000
3	0.00000	16.00000
4	2.00000	0.00000
5	0.00000	14.00000
6	0.00000	0.06000
7	0.00000	-25.00000
8	3000.00000	0.00000
9	11500.00000	0.00000

Media Availability

Budget

Television Restrictions

Audience Coverage

MARKETING APPLICATIONS (9 OF 10)

The optimal solution calls for advertisements to be distributed among daytime TV, daily newspaper, Sunday newspaper, and radio. The maximum number of exposure quality units is 2370, and the total number of customers reached is 61,500.

Media	Frequency	Budget
Daytime TV	10	$15,000
Daily newspaper	25	10,000
Sunday newspaper	2	2,000
Radio	30	3,000
		$30,000

Exposure quality units = 2370

Total customers reached = 61,500

MARKETING APPLICATIONS (10 OF 10)

Marketing Research

- A firm conducts marketing research to learn about consumer characteristics, attitudes, and preferences.
- Marketing research services include designing the study, conducting surveys, analyzing data collected, and providing recommendations for the client.
- In the research design phase, targets or quotas may be established for the number and types of respondents to be surveyed.
- The marketing research firm’s objective is to conduct the survey so as to meet the client’s needs at a minimum cost.

MARKETING RESEARCH

Market Survey, Inc. (MSI) specializes in evaluating consumer reaction to new products, services, and advertising campaigns. A client firm requested MSI's assistance in ascertaining consumer reaction to a recently marketed household product.

During meetings with the client, MSI agreed to conduct door-to-door personal interviews to obtain responses from households with children and households without children. In addition, MSI agreed to conduct both day and evening interviews.

MARKETING RESEARCH (2 OF 8)

The client's contract called for MSI to conduct 1000 interviews under the following quota guidelines:

1. Interview at least 400 households with children.
2. Interview at least 400 households without children.
3. The total number of households interviewed during the evening must be at least as great as the number of households interviewed during the day.
4. At least 40% of the interviews for households with children must be conducted during the evening.
5. At least 60% of the interviews for households without children must be conducted during the evening.

MARKETING RESEARCH (3 OF 8)

Because the interviews for households with children take additional interviewer time and because evening interviewers are paid more than daytime interviewers, the cost varies with the type of interview. Based on previous research studies, estimates of the interview costs are as follows:

	Interview Cost	
Household	Day	Evening
Children	$20	$25
No children	$18	$20

MARKETING RESEARCH (4 OF 8)

In formulating the linear programming model for the MSI problem, we utilize the following decision-variable notation:

DC = the number of daytime interviews of households with children
EC = the number of evening interviews of households with children
DNC = the number of daytime interviews of households without children
ENC = the number of evening interviews of households without children

MARKETING RESEARCH (5 OF 8)

The objective function:

$$\text{Min} \quad 20DC + 25EC + 18DNC + 20ENC$$

The constraint requiring a total of 1000 interviews is:

$$DC + EC + DNC + ENC = 1000$$

The specifications concerning the types of interviews:

- Households with children: $$DC + EC \geq 400$$
- Households without children: $$DC + ENC \geq 400$$
- At least as many evening interviews as day interviews

$$EC + ENC \geq DC + DNC \text{ or } -DC + EC - DNC + ENC \geq 0$$

MARKETING RESEARCH (6 OF 8)

The specifications concerning the types of interviews:

- At least 40% of interviews of households with children during the evening:

$$EC \geq 0.4(DC + EC) \quad \text{or} \quad -0.4DC + 0.6EC \geq 0$$

- At least 60% of interviews of households without children during the evening:

$$ENC \geq 0.6(DC + ENC) \quad \text{or} \quad -0.6DC + 0.4ENC \geq 0$$

- The non-negativity requirements:

$$DC,\ EC,\ DNC,\ ENC \geq 0$$

MARKETING RESEARCH (7 OF 8)

The 4-variable, 6-constraint LP problem formulation is:

$$
\begin{array}{llll}
\text{Min} & 20DC + 25EC + 18DNC + 20ENC & & \\
\text{s.t.} & & & \\
& DC + EC + DNC + ENC & = 1000 & \text{Total interviews} \\
& DC + EC & \geq 400 & \text{Households with children} \\
& DNC + ENC & \geq 400 & \text{Households without children} \\
& -DC + EC - DNC + ENC & \geq 0 & \text{Evening interviews} \\
& -0.4DC + 0.6EC & \geq 0 & \text{Evening interviews in households with children} \\
& -0.6DNC + 0.4ENC & \geq 0 & \text{Evening interviews in households without children} \\
& DC, EC, DNC, ENC \geq 0 & &
\end{array}
$$

MARKETING RESEARCH (8 OF 8)

Optimal Solution: Minimum total cost = $20,320

	Number of Interviews		
Household	Day	Evening	Totals
Children	240	160	400
No children	240	360	600
Totals	480	520	1000

FINANCIAL APPLICATIONS (1 OF 8)

- LP can be used in financial decision-making that involves capital budgeting, make-or-buy, asset allocation, portfolio selection, financial planning, and more.
- Portfolio selection problems involve choosing specific investments – for example, stocks and bonds – from a variety of investment alternatives.
- This type of problem is faced by managers of banks, mutual funds, and insurance companies.
- The objective function usually is maximization of expected return or minimization of risk.

FINANCIAL APPLICATIONS (2 OF 8)

- Portfolio selection problems involve situations in which a financial manager must select specific investments—for example, stocks and bonds—from a variety of investment alternatives.
- The objective function for portfolio selection problems usually is maximization of expected return or minimization of risk.
- The constraints usually take the form of restrictions on the type of permissible investments, state laws, company policy, maximum permissible risk, and so on.

FINANCIAL APPLICATIONS (3 OF 8)

Consider Welte Mutual Funds, Inc., located in New York City. Welte just obtained $100,000 and is looking for investment opportunities for these funds. Based on Welte's current investments, the firm's top financial analyst recommends that all new investments be made in the oil industry, steel industry, or in government bonds. Specifically, the analyst identified five investment opportunities and projected their annual rates of return.

Investment	Projected Rate of Return (%)
Atlantic Oil	7.3
Pacific Oil	10.3
Midwest Steel	6.4
Huber Steel	7.5
Government bonds	4.5

FINANCIAL APPLICATIONS (4 OF 8)

Management of Welte imposed the following guidelines:

1. Neither industry (oil or steel) should receive more than \$50,000.
2. Government bonds should be at least 25% of the steel industry investments.
3. The investment in Pacific Oil, the high-return but high-risk investment, cannot be more than 60% of the total oil industry investment.

What portfolio recommendations—investments and amounts—should be made for the available \$100,000?

FINANCIAL APPLICATIONS (5 OF 8)

Let:

A = dollars invested in Atlantic Oil

P = dollars invested in Pacific Oil

M = dollars invested in Midwest Steel

H = dollars invested in Huber Steel

G = dollars invested in government bonds

The objective function for maximizing the total return for the portfolio is

$$\text{Max} \quad 0.073A + 0.103P + 0.064M + 0.075H + 0.045G$$

FINANCIAL APPLICATIONS (6 OF 8)

The complete linear programming model for the Welte Mutual Funds investment problem:

Max	$0.073A + 0.103P + 0.064M + 0.075H + 0.045G$	
s.t.		
	$A + P + M + H + G = 100{,}000$	Available funds
	$A + P \leq 50{,}000$	Oil industry maximum
	$M + H \leq 50{,}000$	Steel industry maximum
	$G \geq 0.25\,(M + H)$	Government bonds minimum
	$P \leq 0.60\,(A + P)$	Pacific Oil restriction
	$A, P, M, H, G \geq 0$	

FINANCIAL APPLICATIONS (7 OF 8)

Here is the computer solution to the linear programming model for the Welte Mutual Funds investment problem:

Optimal Objective Value =8000.00000

Variable	Value	Reduced Cost
A	20000.00000	0.00000
P	30000.00000	0.00000
M	0.00000	-0.01100
H	40000.00000	0.00000
G	10000.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	0.06900
2	0.00000	0.02200
3	10000.00000	0.00000
4	0.00000	-0.02400
5	0.00000	0.03000

FINANCIAL APPLICATIONS (8 OF 8)

The optimal solution to the linear programming model for the Welte Mutual Funds investment problem is:

Investment	Amount	Expected Annual Return
Atlantic Oil	$20,000	$1460
Pacific Oil	30,000	3090
Huber Steel	40,000	3,000
Government bonds	10,000	450
Totals	$100,000	$8000

Exposure annual return of $8000

Overall rate of return = 8%

FINANCIAL PLANNING (1 OF 7)

Hewlitt Corporation established an early retirement program as part of its corporate restructuring. At the close of the voluntary sign-up period, 68 employees had elected early retirement. As a result of these early retirements, the company incurs the following obligations over the next eight years:

Year	1	2	3	4	5	6	7	8
$ Required	430	210	222	231	240	195	225	255

The cash requirements (in thousands of dollars) are due at the beginning of each year.

FINANCIAL PLANNING (2 OF 7)

The corporate treasurer must determine how much money must be set aside today to meet the eight yearly financial obligations as they come due. The financing plan for the retirement program includes investments in government bonds as well as savings. The investments in government bonds are limited to three choices:

Bond	Price	Rate (%)	Years to Maturity
1	$1150	8.875	5
2	1000	5.500	6
3	1350	11.750	7

FINANCIAL PLANNING (3 OF 7)

The government bonds have a par value of $1000, which means that even with different prices each bond pays $1000 at maturity. The rates shown are based on the par value. For purposes of planning, the treasurer assumed that any funds not invested in bonds will be placed in savings and earn interest at an annual rate of 4%.

FINANCIAL PLANNING (4 OF 7)

Define the Decision Variables

F = total dollars required to meet the retirement plan's eight-year obligation

$B1$ = units of bond 1 purchased at the beginning of year 1

$B2$ = units of bond 2 purchased at the beginning of year 1

$B3$ = units of bond 3 purchased at the beginning of year 1

S = amount placed in savings at the beginning of year i for $i = 1, \ldots, 8$

FINANCIAL PLANNING (5 OF 7)

Define the Objective Function

The objective function is to minimize the total dollars needed to meet the retirement plan's eight-year obligation:

$$\text{Min } F$$

Define the Constraints

A key feature of this type of financial planning problem is that a constraint must be formulated for each year of the planning horizon. It's form is:

(Funds available at the beginning of the year)

- (Funds invested in bonds and placed in savings)

= (Cash obligation for the current year)

FINANCIAL PLANNING (6 OF 7)

Define the Constraints

A constraint must be formulated for each year of the planning horizon in the following form:

$$\text{Year 1: } F - 1.15B_1 - 1B_2 - 1.35B_3 - S_1 = 430$$

$$\text{Year 2: } 0.08875B_1 + 0.055B_2 + 0.1175B_3 - 1.04S_1 - S_2 = 210$$

$$\text{Year 3: } 0.08875B_1 + 0.055B_2 + 0.1175B_3 - 1.04S_2 - S_3 = 222$$

$$\text{Year 4: } 0.08875B_1 + 0.055B_2 + 0.1175B_3 - 1.04S_3 - S_4 = 231$$

$$\text{Year 5: } 0.08875B_1 + 0.055B_2 + 0.1175B_3 - 1.04S_4 - S_5 = 240$$

$$\text{Year 6: } 1.08875B_1 + 0.055B_2 + 0.1175B_3 - 1.04S_5 - S_6 = 195$$

$$\text{Year 7: } 1.055B_2 + 0.1175B_3 - 1.04S_6 - S_7 = 225$$

$$\text{Year 8: } 1.1175B_3 - 1.04S_7 - S_8 = 255$$

FINANCIAL PLANNING (7 OF 7)

Optimal solution to the 12-variable, 8-constraint LP problem:

Minimum total obligation = $1,728,794

Bond	Units Purchased	Investment Amount
1	$B1$ = 144.988	$1150(144.988) = $166,736
2	$B2$ = 187.856	$1000(187.856) = $187,856
3	$B3$ = 228.188	$1350(228.188) = $308,054

OPERATIONS MANAGEMENT APPLICATIONS (1 OF 8)

- LP can be used in operations management to aid in decision-making about product mix, production scheduling, staffing, inventory control, capacity planning, and other issues.
- An important application of LP is multi-period planning such as <u>production scheduling</u>.
- Usually the objective is to establish an efficient, low-cost production schedule for one or more products over several time periods.
- Typical constraints include limitations on production capacity, labor capacity, storage space, and more.

OPERATIONS MANAGEMENT APPLICATIONS (2 OF 8)

The Janders Company markets various business and engineering products. Currently, Janders is preparing to introduce two new calculators: one for the business market called the Financial Manager and one for the engineering market called the Technician.

Each calculator has three components: a base, an electronic cartridge, and a faceplate or top. The same base is used for both calculators, but the cartridges and tops are different. All components can be manufactured by the company or purchased from outside suppliers.

OPERATIONS MANAGEMENT APPLICATIONS (3 OF 8)

Here are the manufacturing costs and purchase prices for Janders calculator components:

	Cost per Unit	
Component	**Manufacture (regular time)**	**Purchase**
Base	$0.50	$0.60
Financial cartridge	$3.75	$4.00
Technician cartridge	$3.30	$3.90
Financial top	$0.60	$0.65
Technician top	$0.75	$0.78

OPERATIONS MANAGEMENT APPLICATIONS (4 OF 8)

Company forecasters indicate that 3000 Financial Manager calculators and 2000 Technician calculators will be needed. However, manufacturing capacity is limited. The company has 200 hours of regular manufacturing time and 50 hours of overtime that can be scheduled for the calculators. Overtime involves a premium at the additional cost of $9 per hour.

Component	Manufacturing Time
Base	1.0
Financial cartridge	3.0
Technician cartridge	2.5
Financial top	1.0
Technician top	1.5

OPERATIONS MANAGEMENT APPLICATIONS (5 OF 8)

The problem for Janders is to determine how many units of each component to manufacture and how many units of each component to purchase. We define the decision variables as follows:

BM = number of bases manufactured

BP = number of bases purchased

FCM = number of Financial cartridges manufactured

FCP = number of Financial cartridges purchased

TCM = number of Technician cartridges manufactured

TCP = number of Technician cartridges purchased

FTM = number of Financial tops manufactured

FTP = number of Financial tops purchased

TTM = number of Technician tops manufactured

TTP = number of Technician tops purchased

One additional decision variable is needed to determine the hours of overtime that must be scheduled:

OT = number of hours of overtime to be scheduled

OPERATIONS MANAGEMENT APPLICATIONS (6 OF 8)

The complete formulation of the Janders make-or-buy problem with all decision variables greater than or equal to zero is

$$
\begin{aligned}
\text{Min}\quad & 0.5BM + 0.6BP + 3.75FCM + 4FCP + 3.3TCM + 3.9TCP \\
& + 0.6FTM + 0.65FTP + 0.75TTM + 0.78TTP + 9OT
\end{aligned}
$$

s.t.

$BM + BP =$	5000	Bases
$FCM + FCP =$	3000	Financial cartridges
$TCM + TCP =$	2000	Technician cartridges
$FTM + FTP =$	3000	Financial tops
$TTM + TTP =$	2000	Technician tops
$OT \leq$	50	Overtime hours
$BM + 3FCM + 2.5TCM + FTM + 1.5TTM \leq$	$12{,}000 + 60OT$	Manufacturing capacity

OPERATIONS MANAGEMENT APPLICATIONS (7 OF 8)

The optimal solution of the Janders make-or-buy problem is

Optimal Objective Value = 24443.33333

Variable	Value	Reduced Cost
BM	5000.00000	0.00000
BP	0.00000	0.01667
FCM	666.66667	0.00000
FCP	2333.33333	0.00000
TCM	2000.00000	0.00000
TCP	0.00000	0.39167
FTM	0.00000	0.03333
FTP	3000.00000	0.00000
TTM	0.00000	0.09500
TTP	2000.00000	0.00000
OT	0.00000	4.00000

(CONTINUED)

OPERATIONS MANAGEMENT APPLICATIONS (8 OF 8)

The optimal solution of the Janders make-or-buy problem is

Constraint	Slack/Surplus	Dual Value
1	0.00000	0.58333
2	0.00000	4.00000
3	0.00000	3.50833
4	0.00000	0.65000
5	0.00000	0.78000
6	50.00000	0.00000
7	0.00000	-0.08333