CHAPTER 10 - INVENTORY MODELS

10.1 - Economic Order Quantity (EOQ) Model

10.2 - Economic Production Lot Size Model

10.3 - Inventory Model with Planned Shortages

10.4 - Quantity Discounts for the EOQ Model

10.5 – Single-Period Inventory Model with Probabilistic Demand

10.6 – Order-Quantity, Reorder Point Model with Probabilistic Demand

10.7 – Periodic Review Model with Probabilistic Demand

INVENTORY MODELS

- Inventory refers to idle goods or materials held by an organization for use sometime in the future.

- The study of <u>inventory models</u> is concerned with two basic questions:
  - How much should be ordered when the inventory is replenished?
  - When should the inventory be replenished?

ECONOMIC ORDER QUANTITY (EOQ) MODEL

The **economic order quantity (EOQ)** model is applicable when the demand for an item shows a constant, or nearly constant, rate and when the entire quantity ordered arrives in inventory at one point in time.

The **constant demand rate** assumption means that the same number of units is taken from inventory each period of time such as 5 units every day, 25 units every week, 100 units every four-week period, and so on.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (1 OF 21)

R&B Beverage is a distributor of beer, wine, and soft drink products.

- From a main warehouse located in Columbus, Ohio, R&B supplies nearly 1000 retail stores with beverage products.
- The beer inventory, which constitutes about 40% of the company's total inventory, averages approximately 50,000 cases.
- With an average cost per case of approximately $8, R&B estimates the value of its beer inventory to be $400,000.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (2 OF 21)

The warehouse manager decided to conduct a detailed study of the inventory costs associated with Bub Beer, the number-one-selling R&B beer.

The purpose of the study is to establish the how-much-to-order and the when-to-order decisions for Bub Beer that will result in the lowest possible total cost.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL

As the first step in the study, the warehouse manager obtained the following demand data for the past 10 weeks:

Strictly speaking, these weekly demand figures do not show a constant demand rate. However, given the relatively low variability exhibited by the weekly demand, inventory planning with a constant demand rate of 2000 cases per week appears acceptable.

Week	Demand (cases)
1	2000
2	2025
3	1950
4	2000
5	2100
6	2050
7	2000
8	1975
9	1900
10	2000
Total cases	20,000
Average cases per week	2000

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (4 OF 21)

The how-much-to-order decision involves selecting an order quantity that draws a compromise between (1) keeping small inventories and ordering frequently, and (2) keeping large inventories and ordering infrequently.

- The first alternative can result in undesirably high ordering costs, while the second alternative can result in undesirably high inventory holding costs.
- To find an optimal compromise between these conflicting alternatives, let us consider a mathematical model that shows the total cost as the sum of the holding cost and the ordering cost.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (5 OF 21)

Holding costs are the costs associated with maintaining or carrying a given level of inventory; these costs depend on the size of the inventory.

- The first holding cost to consider is the cost of financing the inventory investment.
- When a firm borrows money, it incurs an interest charge. If the firm uses its own money, it experiences an opportunity cost associated with not being able to use the money for other investments.
- In either case, an interest cost exists for the capital tied up in inventory. This **cost of capital** is usually expressed as a percentage of the amount invested. R&B estimates its cost of capital at an annual rate of 18%.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (6 OF 21)

A number of other holding costs, such as insurance, taxes, breakage, pilferage, and warehouse overhead, also depend on the value of the inventory.

- R&B estimates these other costs at an annual rate of approximately 7% of the value of its inventory.
- Thus, the total holding cost for the R&B beer inventory is 18% + 7% = 25% of the value of the inventory.
- The cost of one case of Bub Beer is $8.
- With an annual holding cost rate of 25%, the cost of holding one case of Bub Beer in inventory for 1 year is 0.25($8) = $2.00.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (7 OF 21)

The next step in the inventory analysis is to determine the **ordering cost**.

This cost, which is considered fixed regardless of the order quantity, covers the preparation of the voucher; and the processing of the order, including payment, postage, telephone, transportation, invoice verification, receiving, and so on.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (8 OF 21)

For R&B Beverage, the largest portion of the ordering cost involves the salaries of the purchasers. An analysis of the purchasing process showed that a purchaser spends approximately 45 minutes preparing and processing an order for Bub Beer.

With a wage rate and fringe benefit cost for purchasers of $20 per hour, the labor portion of the ordering cost is $15. Making allowances for paper, postage, telephone, transportation, and receiving costs at $17 per order, the manager estimates that the ordering cost is $32 per order. That is, R&B is paying $32 per order regardless of the quantity requested in the order.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (9 OF 21)

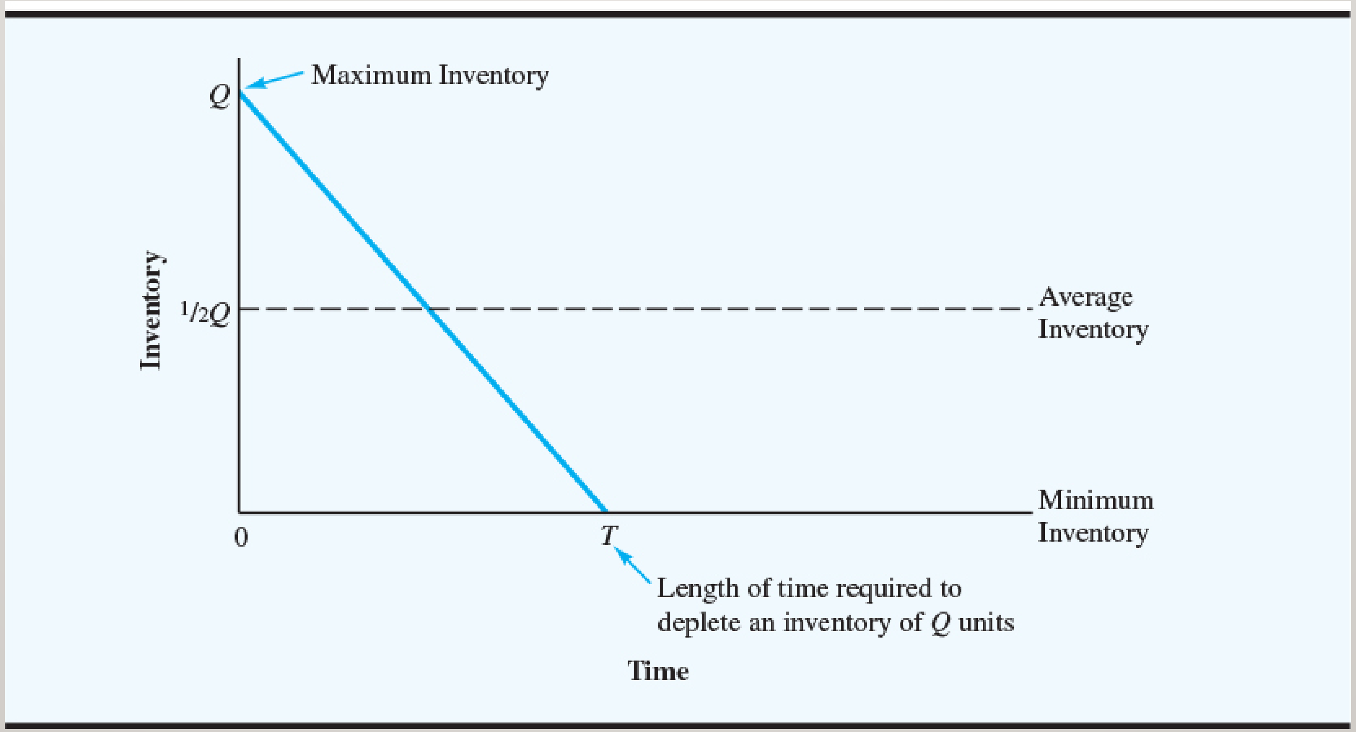
The holding cost, ordering cost, and demand information are the three data items that must be known prior to the use of the EOQ model.

After developing these data for the R&B problem, we can look at how they are used to develop a total cost model. We begin by defining *Q* as the order quantity.

Thus, the how-much-to-order decision involves finding the value of *Q* that will minimize the sum of holding and ordering costs.

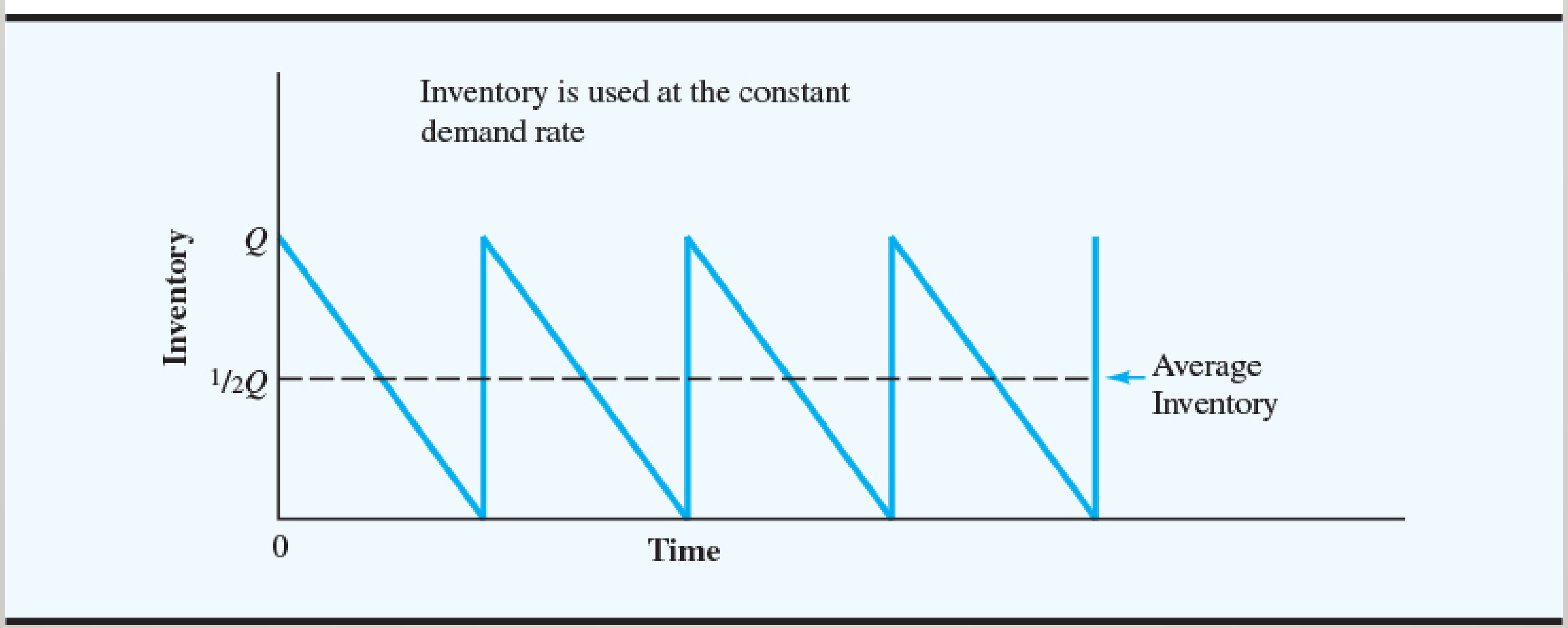
EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (10 OF 21)

The inventory for Bub Beer will have a maximum value of *Q* units when an order of size *Q* is received from the supplier. R&B will then satisfy customer demand from inventory until the inventory is depleted, at which time another shipment of *Q* units will be received.



EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (11 OF 21)

The complete inventory pattern is shown here. If the average inventory during each cycle is 1/2*Q*, the average inventory over any number of cycles is also 1/2*Q*.



EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (12 OF 21)

The holding cost can be calculated using the average inventory. That is, we can calculate the holding cost by multiplying the average inventory by the cost of carrying one unit in inventory for the stated period.

The period selected for the model is up to you; it could be one week, one month, one year, or more. However, because the holding cost for many industries and businesses is expressed as an *annual* percentage, most inventory models are developed on an *annual* cost basis.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (13 OF 21)

Let

I = annual holding cost rate

C = unit cost of the inventory item

C_h = annual cost of holding one unit in inventory

The annual cost of holding one unit in inventory is $C_h = IC$.

The general equation for the annual holding cost for the average inventory of $1/2Q$ units is as follows:

$$\begin{aligned}\begin{pmatrix}\text{Annual}\\ \text{holding cost}\end{pmatrix} &= \begin{pmatrix}\text{Average}\\ \text{inventory}\end{pmatrix}\begin{pmatrix}\text{Annual holding}\\ \text{cost}\\ \text{per unit}\end{pmatrix}\\ &= \frac{1}{2}QC_{\text{h}}\end{aligned}$$

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (14 OF 21)

To complete the total cost model, we must include the annual ordering cost. The goal is to express the annual ordering cost in terms of the order quantity *Q*. How many orders will be placed during the year?

Let *D* denote the annual demand for the product.

For R&B Beverage, *D* = (52 weeks)(2000 cases per week) = 104,000 cases per year.

We know that by ordering *Q* units every time we order, we will have to place *D*/*Q* orders per year. If C_o is the cost of placing one order, the general equation for the annual ordering cost is as follows:

$$\begin{aligned} \text{Annual ordering cost} &= \begin{pmatrix}\text{Number of}\\ \text{orders}\\ \text{per year}\end{pmatrix}\begin{pmatrix}\text{Cost}\\ \text{per}\\ \text{order}\end{pmatrix} \\ &= \left(\frac{D}{Q}\right)C_o \end{aligned}$$

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (15 OF 21)

Thus, the total annual cost, denoted *TC*, can be expressed as follows:

$$\begin{array}{c}\text{Total}\\ \text{annual}\\ \text{cost}\end{array} = \begin{array}{c}\text{Annual}\\ \text{holding}\\ \text{cost}\end{array} + \begin{array}{c}\text{Annual}\\ \text{ordering}\\ \text{cost}\end{array}$$

$$TC = \frac{1}{2}QC_h + \frac{D}{Q}C_o$$

Using the Bub Beer data [$C_h = IC = (0.25)(\$8) = \$2$, $C_o = \$32$, and $D = 104{,}000$], the total annual cost model is

$$TC = \frac{1}{2}Q(\$2) + \frac{104{,}000}{Q}(\$32) = Q + \frac{3{,}328{,}000}{Q}$$

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (16 OF 21)

The next step is to find the order quantity *Q* that will minimize the total annual cost for Bub Beer. Using a trial-and-error approach, we can compute the total annual cost for several possible order quantities.

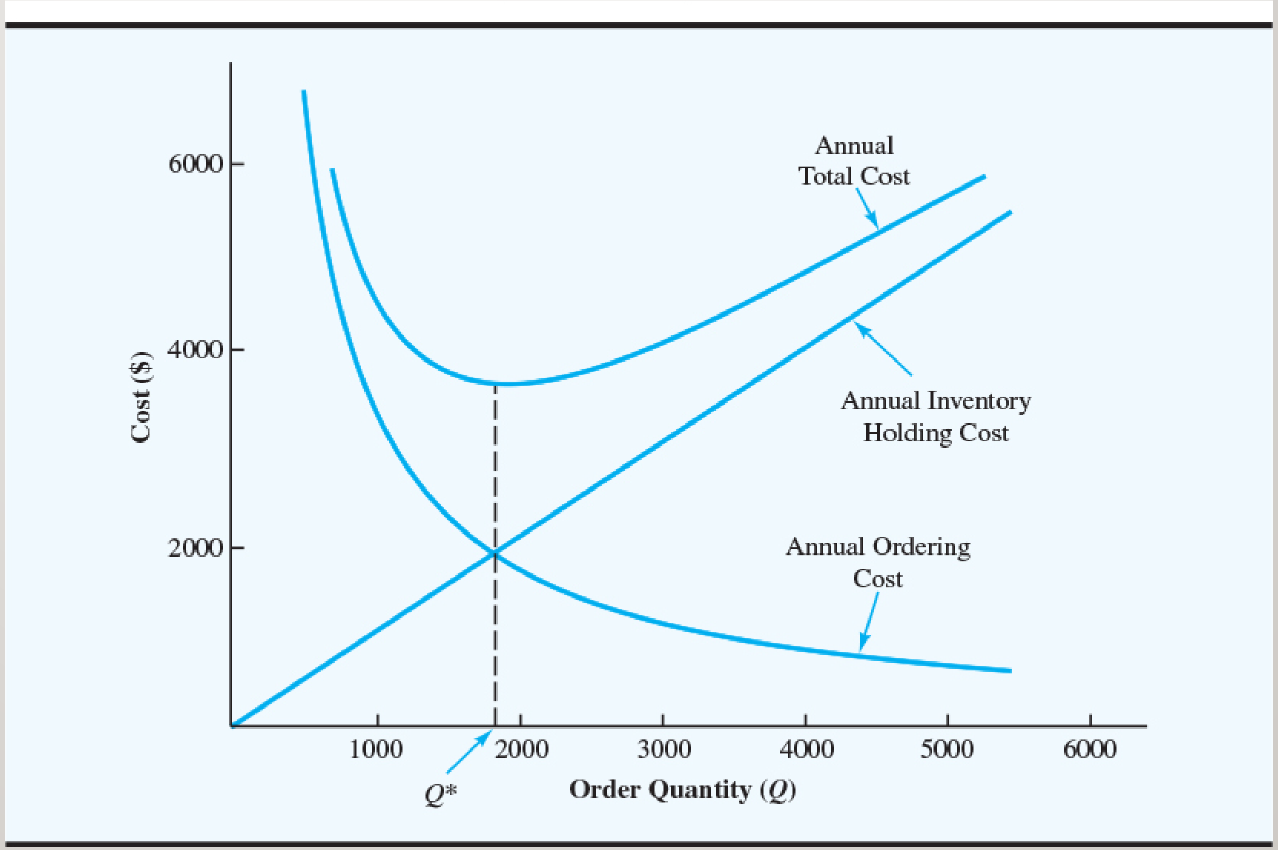
		Annual Cost	
Order Quantity	**Holding**	**Ordering**	**Total**
5000	$5000	$666	$5666
4000	$4000	$832	$4832
3000	$3000	$1109	$4109
2000	$2000	$1664	$3664
1000	$1000	$3328	$4328

The disadvantage of this approach, however, is that it does not provide the exact minimum cost order quantity.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (17 OF 21)

A graphical/analytical approach can also be taken. The minimum total cost order quantity is denoted by an order size of *Q**.

The minimum total annual cost order quantity for Bub beer is 1824 cases and the minimum cost inventory policy for Bub beer has a total annual cost of $3649.



EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (18 OF 21)

Now that we know how much to order, we want to address the question of *when* to order. To answer this question, we need to introduce the concept of inventory position.

The **inventory position** is defined as the amount of inventory on hand plus the amount of inventory on order.

The when-to-order decision is expressed in terms of a **reorder point**—the inventory position at which a new order should be placed.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (19 OF 21)

- The manufacturer of Bub Beer guarantees a two-day delivery on any order placed by R&B Beverage.
- Hence, assuming R&B Beverage operates 250 days per year, the annual demand of 104,000 cases implies a daily demand of 104,000 / 250 = 416 cases.
- Thus, we expect (2 days)(416 cases per day) = 832 cases of Bub to be sold during the two days it takes a new order to reach the R&B warehouse.

In inventory terminology, the two-day delivery period is referred to as the **lead time** for a new order, and the 832-case demand anticipated during this period is referred to as the **lead-time demand**.

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL

R&B should order a new shipment of Bub Beer from the manufacturer when the inventory reaches 832 cases.

For inventory systems using the constant demand rate assumption and a fixed lead time, the reorder point is the same as the lead-time demand. For these systems, the general expression for the reorder point is as follows:

$$r = dm$$

Where

r = reorder point

d = demand per day

m = lead time for a new order in days

EXAMPLE: ECONOMIC ORDER QUANTITY (EOQ) MODEL (21 OF 21)

The question of how frequently the order will be placed can now be answered. The period between orders is referred to as the **cycle time**.

Recall that *D*/*Q* = the number of orders that will be placed in a year. Thus, *D*/*Q** = 104,000 / 1824 = 57 is the number of orders R&B Beverage will place for Bub Beer each year.

If R&B places 57 orders over 250 working days, it will order approximately every 250 / 57 = 4.39 working days.

Thus, the cycle time is 4.39 working days.

ECONOMIC PRODUCTION LOT SIZE MODEL

The inventory model presented in this section is similar to the EOQ model in that we are attempting to determine *how much* we should order and *when* the order should be placed.

We again assume a constant demand rate. However, instead of assuming that the order arrives in a shipment of size Q^*, as in the EOQ model, we assume that units are supplied to inventory at a constant rate over several days or several weeks.

ECONOMIC PRODUCTION LOT SIZE MODEL

The **constant supply rate** assumption implies that the same number of units is supplied to inventory each period of time (e.g., 10 units every day or 50 units every week).

This model is designed for production situations for which, once an order is placed, production begins and a constant number of units is added to inventory each day until the production run has been completed.

ECONOMIC PRODUCTION LOT SIZE MODEL

The **<u>lot size</u>** is the number of units in an order.

- If we have a production system that produces 50 units per day and we decide to schedule 10 days of production, we have a 50(10) = 500-unit production lot size.
- If we let Q = the production lot size, the approach to the inventory decisions is similar to the EOQ model; that is, we build a holding and ordering cost model that expresses the total cost as a function of the production lot size.
- We want to find the production lot size that minimizes the total cost.

ECONOMIC PRODUCTION LOT SIZE MODEL

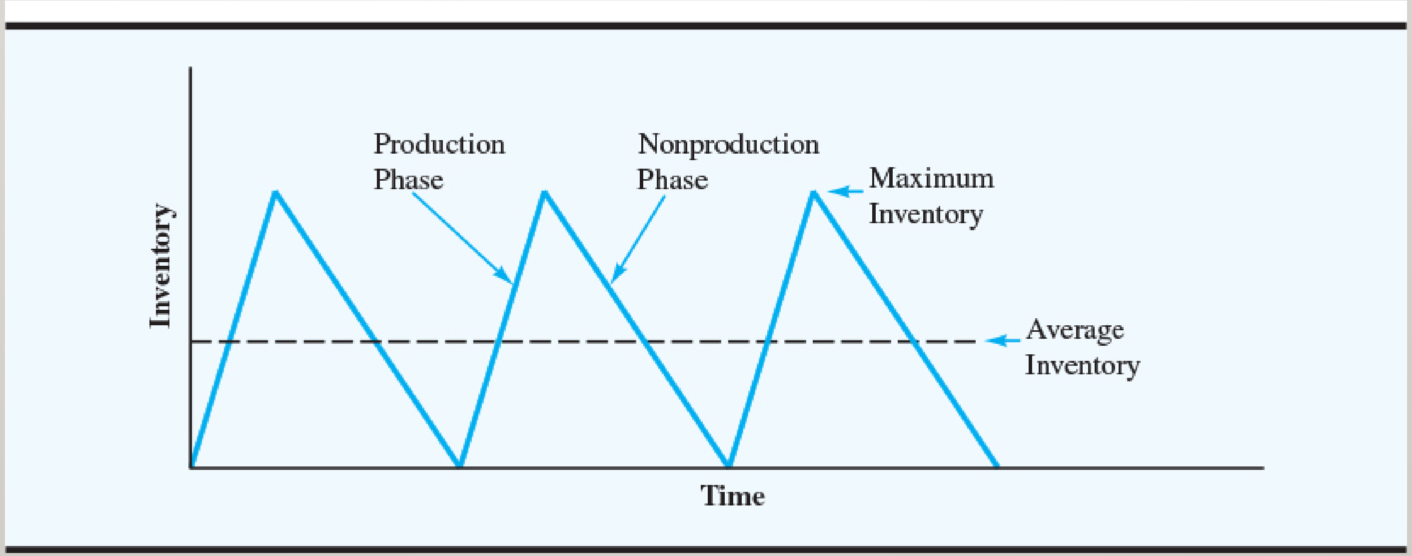
The model only applies to situations where the production rate is greater than the demand rate; the production system must be able to satisfy demand.

For instance, if the constant demand rate is 400 units per day, the production rate must be at least 400 units per day to satisfy demand.

During the production run, demand reduces the inventory while production adds to inventory.

ECONOMIC PRODUCTION LOT SIZE MODEL

Because we assume that the production rate exceeds the demand rate, each day during a production run we produce more units than are demanded. Thus, the excess production causes a gradual inventory buildup during the production period. When the production run is completed, the continuing demand causes the inventory to gradually decline until a new production run is started.



ECONOMIC PRODUCTION LOT SIZE MODEL

As in the EOQ model, we are now dealing with two costs, the holding cost and the ordering cost.

- The holding cost is identical to the definition in the EOQ model, but the interpretation of the ordering cost is slightly different.
- In fact, in a production situation the ordering cost is more correctly referred to as the production **setup cost**.
- This cost, which includes labor, material, and lost production costs incurred while preparing the production system for operation, is a fixed cost that occurs for every production run regardless of the production lot size.

ECONOMIC PRODUCTION LOT SIZE MODEL

Let us build the production lot size model by writing the holding cost in terms of the production lot size *Q*. We will develop an expression for average inventory and then establish the holding costs associated with the average inventory. We use

a one-year time period and an annual cost for the model.

In the EOQ model the average inventory is one-half the maximum inventory, or 1/2*Q*. For a production lot size model, a constant inventory buildup rate occurs during the production run, and a constant inventory depletion rate occurs during the nonproduction period; thus, the average inventory will be one-half the maximum inventory. However, in this inventory system the production lot size *Q* does not go into inventory at one point in time, and thus the inventory never reaches a level of *Q* units.

ECONOMIC PRODUCTION LOT SIZE MODEL

To show how we can compute the maximum inventory, let

d = daily demand rate

p = daily production rate

t = number of days for a production run

Maximum inventory = $(p - d)t$

The length of the production run is $t = Q / p$ days

$$\text{Maximum inventory} = (p-d)t = (p-d)\left(\frac{Q}{P}\right) = \left(1-\frac{d}{p}\right)Q$$

ECONOMIC PRODUCTION LOT SIZE MODEL

The average inventory, which is one-half the maximum inventory, is given by

$$\text{Average inventory} = \frac{1}{2}\left(1 - \frac{d}{p}\right)Q$$

With an annual per-unit holding cost of C_h, the general equation for annual holding cost is as follows:

$$\begin{array}{c}\text{Annual}\\ \text{holding cost}\end{array} = \left(\begin{array}{c}\text{Average}\\ \text{inventory}\end{array}\right)\left(\begin{array}{c}\text{Annual}\\ \text{cost}\\ \text{per unit}\end{array}\right)$$

$$= \frac{1}{2}\left(1 - \frac{d}{p}\right)QC_h$$

ECONOMIC PRODUCTION LOT SIZE MODEL (10 OF 13)

If *D* is the annual demand for the product and *Co* is the setup cost for a production run, then the annual setup cost, which takes the place of the annual ordering cost in the EOQ model, is as follows:

$$\text{Annual setup cost} = \begin{pmatrix}\text{Number of production}\\ \text{runs per year}\end{pmatrix}\begin{pmatrix}\text{Setup cost}\\ \text{per run}\end{pmatrix} = \frac{D}{Q}C_o$$

Thus, the total annual cost (*TC*) model is

$$\text{TC} = \frac{1}{2}\left(1-\frac{d}{p}\right)QC_h + \frac{D}{Q}C_o$$

ECONOMIC PRODUCTION LOT SIZE MODEL

Given estimates of the holding cost (*C*h), setup cost (*C*o), annual demand rate (*D*), and annual production rate (*P*), we could use a trial-and-error approach to compute the total annual cost for various production lot sizes (*Q*). However, trial and error is not necessary; we can use the minimum cost formula for *Q** that has been developed using differential calculus.

$$Q^* = \sqrt{\frac{2DC_o}{\left(1 - D/P\right)C_h}}$$

ECONOMIC PRODUCTION LOT SIZE MODEL (12 OF 13)

What is the recommended production lot size?

- Beauty Bar Soap is produced on a production line that has an annual capacity of 60,000 cases.
- The annual demand is estimated at 26,000 cases, with the demand rate essentially constant throughout the year.
- The cleaning, preparation, and setup of the production line cost approximately $135.
- The manufacturing cost per case is $4.50, and the annual holding cost is figured at a 24% rate.
- Thus, $C_h = IC = 0.24(\$4.50) = \$1.08$.

ECONOMIC PRODUCTION LOT SIZE MODEL

Other relevant data include a five-day lead time to schedule and set up a production run and 250 working days per year.

Thus, the lead-time demand of (26,000 / 250)(5) = 520 cases is the reorder point.

The cycle time is the time between production runs. The cycle time is $T = 250Q^* / D = [(250)(3387)] / 26{,}000 = 33$ working days.

Thus, we should plan a production run of 3387 units every 33 working days.

INVENTORY MODEL WITH PLANNED SHORTAGES (1 OF 2)

A **shortage** or **stock-out** occurs when demand exceeds the amount of inventory on hand. In many situations, shortages are undesirable and should be avoided if at all possible. However, in other cases it may be desirable—from an economic point of view—to plan for and allow shortages.

In practice, these types of situations are most commonly found where the value of the inventory per unit is high and hence the holding cost is high. An example of this type of situation is a new car dealer's inventory. Often a specific car that a customer wants is not in stock. However, if the customer is willing to wait a few weeks, the dealer is usually able to order the car.

INVENTORY MODEL WITH PLANNED SHORTAGES (2 OF 2)

The model developed in this section takes into account a type of shortage known as a **backorder**.

In a backorder situation, we assume that when a customer places an order and discovers that the supplier is out of stock, the customer waits until the new shipment arrives, and then the order is filled.

Frequently, the waiting period in backorder situations is relatively short. Thus, by promising the customer top priority and immediate delivery when the goods become available, companies may be able to convince the customer to wait until the order arrives. In these cases, the backorder assumption is valid.

QUANTITY DISCOUNTS FOR THE EOQ MODEL

Quantity discounts occur in numerous situations for which suppliers provide an incentive for large order quantities by offering a lower purchase cost when items are ordered in larger quantities.

In this section we show how the EOQ model can be used when quantity discounts are available.

ORDER-QUANTITY, REORDER POINT MODEL WITH PROBABILISTIC DEMAND

In the previous section we considered a single-period inventory model with probabilistic demand. In this section we extend our discussion to a multiperiod order-quantity, reorder point inventory model with probabilistic demand.

In the multiperiod model, the inventory system operates continuously with many repeating periods or cycles; inventory can be carried from one period to the next. Whenever the inventory position reaches the reorder point, an order for Q units is placed. Because demand is probabilistic, the time the reorder point will be reached, the time between orders, and the time the order of Q units will arrive in inventory cannot be determined in advance.

PERIODIC REVIEW MODEL WITH PROBABILISTIC DEMAND

The order-quantity, reorder point inventory models previously discussed require a **continuous review inventory system**.

In a continuous review inventory system, the inventory position is monitored continuously so that an order can be placed whenever the reorder point is reached. Computerized inventory systems can easily provide the continuous review required by the order-quantity, reorder point models.

An alternative to the continuous review system is the **periodic review inventory system**. With a periodic review system, the inventory is checked and reordering is done only at specified points in time.