

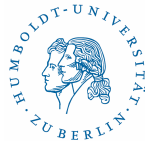
# Selected Topics in Mathematical Statistics, Quiz 15a

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## Quiz 15a

### Properties of $M_X$ :

- ▣  $M_X(0) = 1$
- ▣  $M_{aX}(t) = M_X(at), \forall a \in \mathbb{R}$
- ▣  $EX^n = \frac{d^n}{dt^n} M_X(0)$
- ▣ Let  $X_1, \dots, X_n$  iid,  $S_n = \sum_{i=1}^n X_i$  then  $M_{S_n}(t) = \prod_{i=1}^n M_{X_i}(t)$

Quiz 15a: Verify the properties of  $M_X$



## Quiz 15a

The Moment Generating Function is defined as:

$$M_X(t) = E[e^{t^T X}] = \int \dots \int \exp(t^T X) dF_X, \quad t \in \mathbb{R}^k$$

The idea behind is, that

$$e^{tX} = 1 + tX + \frac{t^2 X^2}{2!} + \frac{t^3 X^3}{3!} + \dots,$$

Thus

$$M_X(t) = E[e^{tX}] = 1 + tm_1 + \frac{t^2 m_2}{2!} + \frac{t^3 m_3}{3!} + \dots,$$

where  $m_i$  is the  $i$ th moment.



## Quiz 15a

First and second property:

□  $M_X(0) = 1$ :

$$M_X(0) = \mathbb{E}[e^{0X}] = \mathbb{E}[1] = 1$$

□  $M_{aX}(t) = M_X(at), \forall a \in \mathbb{R}$

$$M_{aX} = \mathbb{E}[e^{atX}] = M_X(at)$$

for any  $a \in \mathbb{R}$



## Quiz 15a

Third property:

$$\square \quad \mathbf{E}X^n = \frac{d^n}{dt^n} M_X(0):$$

$$\begin{aligned} \frac{d^n}{dt^n} M_X(t) &= \frac{d^n}{dt^n} E[\exp(tX)] = \\ &= E \left[ \frac{d^n}{dt^n} \exp(tX) \right] = E[X^n \exp(tX)] \end{aligned}$$

calculate for  $t = 0$  and we get:

$$E[X^n \exp(0X)] = E[X^n]$$



## Quiz 15a

Fourth property:

▣ **Let  $X_1, \dots, X_n$  iid,  $S_n = \sum_{i=1}^n X_i$  then  $M_{S_n}(t) = \prod_{i=1}^n M_{X_i}(t)$**

$$\begin{aligned} M_{S_n}(t) &= E[\exp(tS_n)] = E\left[\exp\left(t \sum_{i=1}^n X_i\right)\right] = E\left[\exp\left(\sum_{i=1}^n tX_i\right)\right] \\ &= E\left[\prod_{i=1}^n \exp(tX_i)\right] = \prod_{i=1}^n E[\exp(tX_i)] = \prod_{i=1}^n M_{X_i}(t) \end{aligned}$$

Last two steps were possible because of the mutual independence of variables  $X_i$ .

