# Selected topics in Mathematical Statistics, Quiz 16

Nina Lesnik

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E. – Center for Applied Statistics
and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de
http://case.hu-berlin.de

# Motivation

- i.i.d random variables - with CLT we can approximate $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} \sim N(\mu, \frac{\sigma^2}{n})$
- we would like to increase our approximation
- therefore we need some additional information
- **Edgeworth approximation** - use information about higher order moments to increase the accuracy
- especially important if we have small sample

# Edgeworth Expansion

Edgeworth Expansion can be applied in case 3rd and 4th moment are known.

$$G_n(x) \sim \phi(x) - \frac{\beta_1(x^2-1)}{6n^(1/2)}\varphi(x) - \{\frac{\beta_2(x^3-3x)}{24n} + \frac{\beta_1^2(x^5-10x^3+15x)}{72n}\}\varphi(x)$$

Where $\varphi$ and $\phi$ are pdf and cdf of standard normal distribution. $\beta_1$ is standardized 3rd moment and $\beta_2$ is the excess kurtosis, $\frac{\mu_4}{\sigma^4} - 3$

# Quiz 16

**Example** A sample of 5 values from the standard exponential distribution gives us:

$$\mu = 1, \sigma^2 = 1, \beta_1 = 2, \beta_2 = 6$$

Prove the result

standard exponential distribution: $f(x) = e^{-x}$

**Calculate $\mu$:**

$$\mu = E[x] = \int_0^\infty xe^{-x}dx = -xe^{-x}\Big|_0^\infty + \int_0^\infty e^{-x}dx = 1$$

**Calculate $\sigma^2$:**

$$\sigma^2 = E[x^2] - E[x]^2$$

$$E[x^2] = \int_0^\infty x^2e^{-x}dx = -x^2e^{-x}\Big|_0^\infty + \int_0^\infty 2xe^{-x}dx =$$
$$0 - 0 + 2\int_0^\infty xe^{-x}dx = 2$$

$$\sigma^2 = 2 - 1 = 1$$

**Calculate $\beta_1$:**

$$\beta_1 = \frac{\mu_3}{\sigma^3} = \frac{E[(X-\mu)^3]}{\sqrt{E[(X-\mu)^2]}^3}$$

$E[(X-\mu)^3] = \int_0^\infty (x-1)^3 e^{-x} dx = \int_0^\infty x^3 e^{-x} dx - \int_0^\infty 3x^2 e^{-x} dx + \int_0^\infty 3x e^{-x} dx - \int_0^\infty e^{-x} dx = \int_0^\infty x^3 e^{-x} dx - 6 + 3 - 1 = -x^3 e^{-x}\Big|_0^\infty + \int_0^\infty 3x^2 e^{-x} dx - 24 = 6 - 4 = 2$

$E[(X-\mu)^2] = \int_0^\infty (x-1)^2 e^{-x} dx = \int_0^\infty x^2 e^{-x} dx - \int_0^\infty 2x e^{-x} dx + \int_0^\infty e^{-x} dx = 2 - 2 + 1 = 1$

$$\beta_1 = \frac{2}{\sqrt{1}^3} = 2$$

**Calculate $\beta_2$:**

$$\beta_2 = \frac{\mu_4}{\sigma^4} - 3 = \frac{E[(X-\mu)^4]}{\sqrt{E[(X-\mu)^2]}^4} - 3$$

$E[(X-\mu)^4] = \int_0^\infty (x-1)^4 e^{-x} dx = \int_0^\infty x^4 e^{-x} dx - \int_0^\infty 4x^3 e^{-x} dx + \int_0^\infty 6x^2 e^{-x} dx - \int_0^\infty 4xe^{-x} dx + \int_0^\infty e^{-x} dx = -x^4 e^{-x}\Big|_0^\infty + \int_0^\infty 4x^3 e^{-x} dx - 24 + 12 - 4 + 1 = 24 - 24 + 9 = 9$

$$\beta_2 = \frac{9}{\sqrt{1}^4} - 3 = 6$$