Selected Topics in Mathematical Statistics, Quiz 15a

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Quiz 15a

Properties of M_X:

- $M_X(0) = 1$
- $M_{aX}(t) = M_X(at), \; \forall a \in \mathbb{R}$
- $EX^n = \frac{d^n}{dt^n} M_X(0)$
- Let $X_1, \ldots, X_n$ iid, $S_n = \sum_{i=1}^n X_i$ then $M_{S_n}(t) = \prod_{i=1}^n M_{X_i}(t)$

Quiz 15a: Verify the properties of M_X

Quiz 15a

The Moment Generating Function is defined as:

$$M_X(t) = E[e^{t^T X}] = \int \ldots \int exp(t^T X) dF_X, \ \ t \in \mathbb{R}^k$$

The idea behind is, that

$$e^{tX} = 1 + tX + \frac{t^2 X^2}{2!} + \frac{t^3 X^3}{3!} + \cdots ,$$

Thus

$$M_X(t) = E[e^{tX}] = 1 + tm_1 + \frac{t^2 m_2}{2!} + \frac{t^3 m_3}{3!} + \cdots ,$$

where m_i is the ith moment.

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First and second property:

- $\mathbf{M_X(0) = 1}$:

$$M_X(0) = \mathbb{E}[e^{0X}] = \mathbb{E}[1] = 1$$

- $\mathbf{M_{aX}(t) = M_X(at), \ \forall a \in \mathbb{R}}$

$$M_{aX} = \mathbb{E}[e^{atX}] = M_X(at)$$

  for any $a \in \mathbb{R}$

Quiz 15a

Third property:

- $\mathbf{EX^n = \frac{d^n}{dt^n} M_X(0)}$:

$$\frac{d^n}{dt^n} M_X(t) = \frac{d^n}{dt^n} E[exp(tX)] =$$
$$= E\left[\frac{d^n}{dt^n} exp(tX)\right] = E[X^n exp(tX)]$$

calculate for $t = 0$ and we get:

$$E[X^n exp(0X)] = E[X^n]$$

Quiz 15a

Fourth property:

- **Let** $X_1, ..., X_n$ **iid,** $S_n = \sum_{i=1}^{n} X_i$ **then** $M_{S_n}(t) = \prod_{i=1}^{n} M_{X_i}(t)$

$$M_{S_n}(t) = E[exp(tS_n)] = E\left[exp(t\sum_{i=1}^{n} X_i)\right] = E\left[exp(\sum_{i=1}^{n} tX_i)\right]$$
$$= E\left[\prod_{i=1}^{n} exp(tX_i)\right] = \prod_{i=1}^{n} E[exp(tX_i)] = \prod_{i=1}^{n} M_{X_i}(t)$$

Last two steps were possible because of the mutual independence of variables X_i.