

Exam Presentation: Advanced methods in quantitative Finance

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1. Copula Estimation
2. Principal Component Analysis on Implied Volatility
3. Derivation: IBT Downward Nodes



Copula Estimation

- Data: Daily data of biggest stock indices worldwide, 2005 - 2013

USA:	NASDAQ	Switzerland:	SMI
Canada:	S&P/TSX	Russia:	RTS
Great Britain:	FTSE100	India:	SENSEX
France:	CAC40	Japan:	NIKKEI225
Germany:	DAX30	China:	HANGSENG
Singapur:	STRAITS TIMES		

- Procedure:
 - ▶ Estimate Clayton and Gumbel Copula
 - ▶ Investigate tail dependency over time



Animation: Kendall's Tau based on Clayton Copula, quarterly

Figure 1: Changing Kendall's Tau based on Clayton Copula



Animation: Kendall's Tau based on Gumbel Copula, quarterly

Figure 2: Changing Kendall's Tau based on Gumbel Copula



Principal Component Analysis on Implied Volatilities

Data:

- VSMI: Volatility index on swiss stock market
- Monthly data: January 2005 until June 2016
- Time to maturities:
 - ▶ 1-, 2-, 3 months: $\tau_{1M}, \tau_{2M}, \tau_{3M}$
 - ▶ 1-, 2-, 3 quarter: $\tau_{1Q}, \tau_{2Q}, \tau_{3Q}$



Term structure of implied volatilities

Background: While "normal" volatility is a backward measure of uncertainty, implied volatility measures market expectations

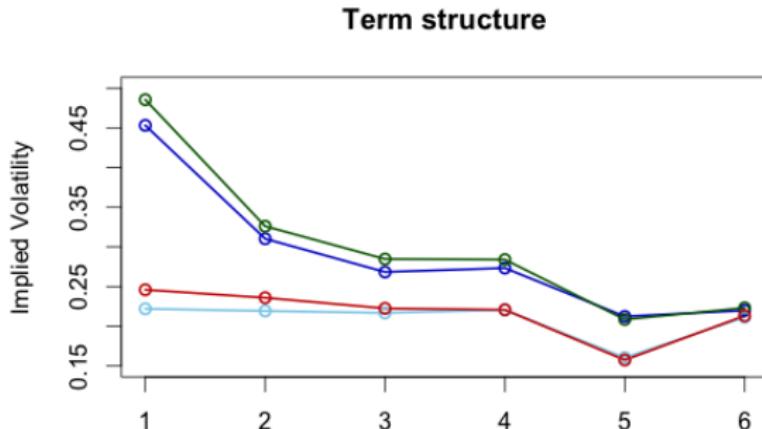


Figure 3: Term Structure for specific ^{Subindex} dates of VSMI: [15.01.2015](#), [16.01.2015](#), [15.09.2008](#), [16.09.2008](#)



Explaining Sample Variance by PCA

PC	Explanatory Power	Cumulative Explanation
1	75.20 %	75.20 %
2	14.92 %	90.12 %
3	6.10 %	96.22 %
4	2.07 %	98.29 %
5	0.96 %	99.26 %
6	0.74 %	100.00 %

Table 1: Explained sample variance using principal components

Conclusion: First two principal components explain 90 % of sample variance



Effect of shocks on first and second Principal Component

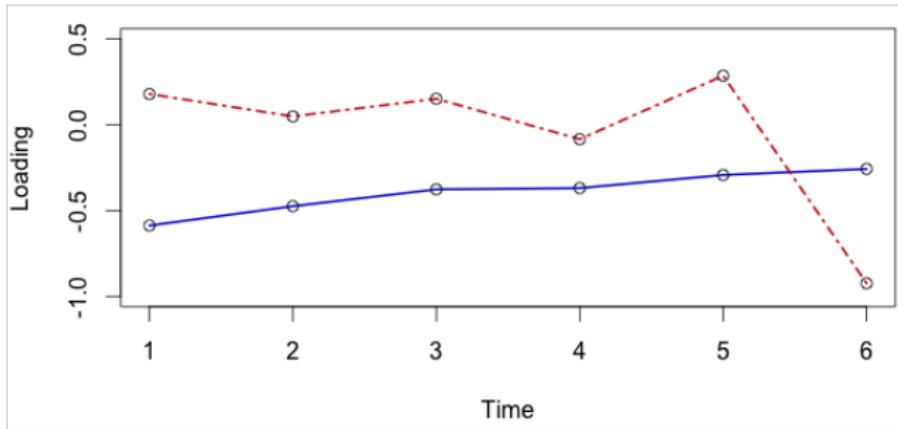


Figure 4: Factor loadings for first and second PC

Conclusion:

- first PC: similar effect for all time to maturities
- second PC: strong negative effect for τ_{3Q}



Implied Binomial Tree - Downward Nodes

Risk neutral condition

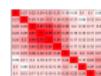
$$F_n^i = p_{i+1}^n S_{n+1}^{i+1} + (1 - p_{i+1}^n) S_{n+1}^i \quad (1)$$

Arrow-Debreu prices

$$\begin{aligned}\lambda_{n+1}^0 &= e^{-r\Delta t} \{(1 - p_i^n) \lambda_0^0\} \\ \lambda_{n+1}^{i+1} &= e^{-r\Delta t} \{\lambda_n^i p_{i+1}^n + \lambda_n^{i+1} (1 - p_{i+2}^n)\} \\ \lambda_{n+1}^{n+1} &= e^{-r\Delta t} \{\lambda_n^n p_{i+1}^n\}\end{aligned}$$

Put option price

$$P(K, n\Delta) = \sum_{i=0}^n \lambda_{n+1}^{i+1} \max(S_{n+1}^{i+1} - K, 0) \quad (2)$$



$$\begin{aligned} P(S, n\Delta t) e^{-r\Delta t} &= \lambda_n^0(1 - p_1^n) \max(S - S_{n+1}^0) \\ &\quad + \sum_{j=0}^{n-1} \{ \lambda_n^j p_{j+1}^n + \lambda_n^{j+1}(1 - p_{j+2}^n) \} \max(S - S_{n+1}^{j+1}) \\ &\quad + \lambda_n^n p_{n+1}^n \max(S - S_{n+1}^{n+1}) \\ &= \lambda_n^0(1 - p_1^n)(S - S_{n+1}^0) \\ &\quad + \sum_{j=0}^{i-1} \{ \lambda_n^j p_{j+1}^n + \lambda_n^{j+1}(1 - p_{j+2}^n) \} (S - S_{n+1}^{j+1}) \end{aligned}$$



$$\begin{aligned}
 P(S, n\Delta t) e^{-r\Delta t} &= \lambda_n^0(1 - p_1^n)(S - S_{n+1}^0) \\
 &\quad + \lambda_n^0 p_1^n (S - S_{n+1}^1) + \lambda_n^1(1 - p_2^n)(S - S_{n+1}^1) \\
 &\quad + \sum_{j=1}^{i-1} \lambda_n^j p_{j+1}^n (S - S_{n+1}^{j+1}) + \sum_{j=1}^{i-1} \lambda_n^{j+1}(1 - p_{j+2}^n)(S - S_{n+1}^{j+1}) \\
 &= \sum_{j=0}^i \lambda_n^j(1 - p_{j+1}^n)(S - S_{n+1}^j) + \sum_{j=0}^{i-1} \lambda_n^j p_{j+1}^n (S - S_{n+1}^{j+1}) \\
 &= \lambda_n^i(1 - p_{i+1}^n)(S - S_{n+1}^i) \\
 &\quad + \underbrace{\sum_{j=0}^{i-1} \lambda_n^j \left[p_{j+1}^n (S - S_{n+1}^{j+1}) + (1 - p_{j+1}^n)(S - S_{n+1}^j) \right]}_{=\sum_{j=0}^{i-1} \lambda_n^j (S - F_n^j) := \rho_I}
 \end{aligned}$$



Using option price formula:

$$P(s, n\Delta t) = e^{-r\Delta t} \{ \lambda_n^i (1 - p_{i+1}^n) (S - S_{n+1}^i) + \underbrace{\sum_{j=0}^{i-1} \lambda_n^j (S - F_n^j)}_{:=\rho_I} \}$$

and using this formula for transition probabilities, we can calculate downward nodes from risk free condition:

$$S_{n+1}^i = \frac{S^i n + 1 [P(S, n\Delta t) e^{r\Delta t} - \rho_I] - \lambda_n^i S (F_n^i - S_{n+1}^{i+1})}{P(S, n\Delta t) e^{r\Delta t} - \rho_I + \lambda_n^i (F_n^i - S_{n+1}^{i+1})}$$

