Exam Presentation: Advanced methods in quantitative Finance

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1. Copula Estimation
2. Principal Component Analysis on Implied Volatility
3. Derivation: IBT Downward Nodes

Copula Estimation

- Data: Daily data of biggest stock indices wordwide, 2005 - 2013

USA:	NASDAQ	Switzerland:	SMI
Canada:	S&P/TSX	Russia:	RTS
Great Britain:	FTSE100	India:	SENSEX
France:	CAC40	Japan:	NIKKEI225
Germany:	DAX30	China:	HANGSENG
Singapur:	STRAITS TIMES		

- Procedure:
  - Estimate Clayton and Gumbel Copula
  - Investigate tail dependency over time

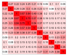
Animation: Kendall's Tau based on Clayton Copula, quarterly

Play Pause

Quarter: 2006-1

	NASDAQ	S&P/TSX	FTSE100	CAC40	DAX	SMI	RTS	SENSEX	NIKKEI225	HANGSENG	STRAITS
NASDAQ	1	0.11	0.04	0.01	0.03	0.08	0.02	0.02	0.07	0.08	0.04
S&P/TSX	0.11	1	0.16	0.13	0.1	0.08	0.09	0.09	0.06	0.1	0.05
FTSE100	0.04	0.16	1	0.49	0.41	0.41	0.18	0.11	0.15	0.11	0.17
CAC40	0.01	0.13	0.49	1	0.64	0.51	0.15	0.13	0.19	0.12	0.18
DAX	0.03	0.1	0.41	0.64	1	0.39	0.12	0.11	0.15	0.1	0.17
SMI	0.08	0.08	0.41	0.51	0.39	1	0.12	0.1	0.17	0.12	0.15
RTS	0.02	0.09	0.18	0.15	0.12	0.12	1	0.08	0.11	0.12	0.1
SENSEX	0.02	0.09	0.11	0.13	0.11	0.1	0.08	1	0.12	0.15	0.11
NIKKEI225	0.07	0.06	0.15	0.19	0.15	0.17	0.11	0.12	1	0.23	0.22
HANGSENG	0.08	0.1	0.11	0.12	0.1	0.12	0.12	0.15	0.23	1	0.22
STRAITS	0.04	0.05	0.17	0.18	0.17	0.15	0.1	0.11	0.22	0.22	1

Figure 1: Changing Kendall's Tau based on Clayton Copula



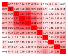
Animation: Kendall's Tau based on Gumbel Copula, quarterly

Play Pause

Quarter: 2006-1

	NASDAQ	S&P/TSX	FTSE100	CAC40	DAX	SMI	RTS	SENSEX	NIKKEI225	HANGSENG	STRAITS
NASDAQ	1	0.05	0.03	0.02	0.02	0.04	0	0	0.03	0	0
S&P/TSX	0.05	1	0.22	0.2	0.2	0.16	0.09	0.14	0.06	0.12	0.1
FTSE100	0.03	0.22	1	0.6	0.52	0.47	0.19	0.12	0.17	0.14	0.15
CAC40	0.02	0.2	0.6	1	0.7	0.56	0.14	0.16	0.19	0.13	0.17
DAX	0.02	0.2	0.52	0.7	1	0.48	0.12	0.16	0.18	0.14	0.17
SMI	0.04	0.16	0.47	0.56	0.48	1	0.12	0.13	0.16	0.14	0.14
RTS	0	0.09	0.19	0.14	0.12	0.12	1	0.13	0.1	0.12	0.11
SENSEX	0	0.14	0.12	0.16	0.16	0.13	0.13	1	0.18	0.22	0.16
NIKKEI225	0.03	0.06	0.17	0.19	0.18	0.16	0.1	0.18	1	0.33	0.28
HANGSENG	0	0.12	0.14	0.13	0.14	0.14	0.12	0.22	0.33	1	0.3
STRAITS	0	0.1	0.15	0.17	0.17	0.14	0.11	0.16	0.28	0.3	1

Figure 2: Changing Kendall's Tau based on Gumbel Copula



Pricincipal Component Analysis on Implied Volatilities

Data:

- VSMI: Volatility index on swiss stock market
- Monthly data: January 2005 until June 2016
- Time to maturities:
  - 1-, 2-, 3 months: τ_{1M}, τ_{2M}, τ_{3M}
  - 1-, 2-, 3 quarter: τ_{1Q}, τ_{2Q}, τ_{3Q}

Term structure of implied volatilites

Background: While "normal" volatility is a backward measure of uncertainty, implied volatility measures market expectations

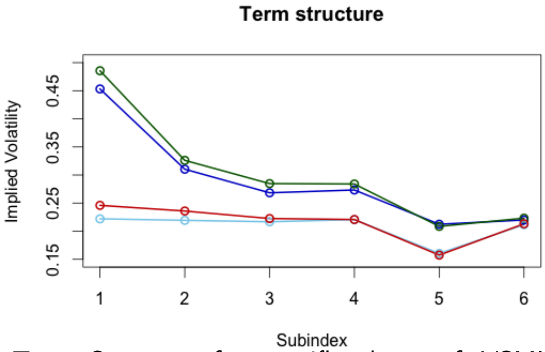


Figure 3: Term Structure for specific dates of VSMI: 15.01.2015, 16.01.2015, 15.09.2008, 16.09.2008

Explaining Sample Variance by PCA

PC	Explanatory Power	Cumulative Explanation
1	75.20 %	75.20 %
2	14.92 %	90.12 %
3	6.10 %	96.22 %
4	2.07 %	98.29 %
5	0.96 %	99.26 %
6	0.74 %	100.00 %

Table 1: Explained sample variance using principal components

Conclusion: First two principal components explain 90 % of sample variance

Effect of shocks on first and second Principal Component

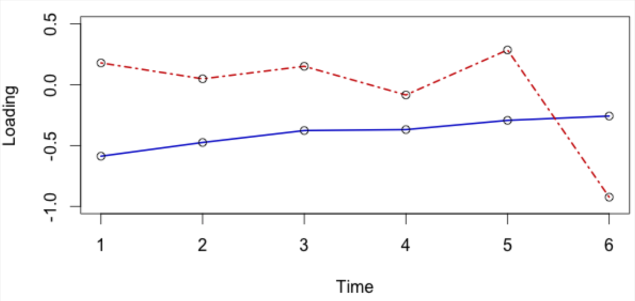


Figure 4: Factor loadings for first and second PC

Conclusion:

- first PC: similar effect for all time to maturities
- second PC: strong negative effect for τ_{3Q}

Implied Binomal Tree - Downward Nodes

Risk neutral condition:

$$F_n^i = p_{i+1}^n S_{n+1}^{i+1} + (1 - p_{i+1}^n) S_{n+1}^i$$

yields transition probabilities:

$$p_{i+1}^n = P(S_{n+1} = S_{n+1}^{i+1} | S_n = S_n^i$$

Arrow-Debreu prices (discounted risk neutral probabilities):

$$\begin{aligned}
\lambda_{n+1}^0 &= e^{-r\Delta t} \left\{ (1 - p_i^n) \lambda_0^0 \right\} \\
\lambda_{n+1}^{i+1} &= e^{-r\Delta t} \left\{ \lambda_n^i p_{i+1}^n + \lambda_n^{i+1} (1 - p_{i+2}^n) \right\} \\
\lambda_{n+1}^{n+1} &= e^{-r\Delta t} \left\{ \lambda_n^n p_{i+1}^n \right\}
\end{aligned}$$

Put option price

$$P(K, n\Delta) = \sum_{i=0}^{n} \lambda_{n+1}^{i+1} max\left(S_{n+1}^{i+1} - K, 0\right)$$

We center the tree by setting $K = S = S_n^i$:

$$\begin{aligned}
P(S, n\Delta t)\; e^{r\Delta t} &= \lambda_n^0 (1 - p_1^n)\, max(S - S_{n+1}^0) \\
&\quad + \sum_{j=0}^{n-1} \left\{ \lambda_n^j p_{j+1}^n + \lambda_n^{j+1} (1 - p_{j+2}^n) \right\}\; max(S - S_{n+1}^{i+1}) \\
&\quad + \lambda_n^n p_{i+1}^n\, max(S - S_{n+1}^{n+1}) \\
&= \lambda_n^0 (1 - p_1^n)(S - S_{n+1}^0) \\
&\quad + \sum_{j=0}^{i-1} \left\{ \lambda_n^j p_{j+1}^n + \lambda_n^{j+1} (1 - p_{j+2}^n) \right\} (S - S_{n+1}^{i+1})
\end{aligned}$$

$$
\begin{aligned}
P(S, n\Delta t)\, e^{r\Delta t} &= \lambda_n^0 (1 - p_1^n)\,(S - S_{n+1}^0) \\
&\quad + \lambda_n^0 p_1^n\,(S - S_{n+1}^1) + \lambda_n^1 (1 - p_2^n)\,(S - S_{n+1}^1) \\
&\quad + \sum_{j=1}^{i-1} \lambda_n^j p_{j+1}^n (S - S_{n+1}^{j+1}) + \sum_{j=1}^{i-1} \lambda_n^{j+1} (1 - p_{j+2}^n)(S - S_{n+1}^{j+1}) \\
&= \sum_{j=0}^{i} \lambda_n^j (1 - p_{j+1}^n)(S - S_{n+1}^j) + \sum_{j=0}^{i-1} \lambda_n^j p_{j+1}^n (S - S_{n+1}^{j+1}) \\
&= \lambda_n^i (1 - p_{i+1}^n)(S - S_{n+1}^i) \\
&\quad + \underbrace{\sum_{j=0}^{i-1} \lambda_n^j \left[p_{j+1}^n (S - S_{n+1}^{j+1}) + (1 - p_{j+1}^n)(S - S_{n+1}^j) \right]}_{= \sum_{j=0}^{i-1} \lambda_n^j (S - F_n^j) := \rho_l}
\end{aligned}
$$

Using option price formula:

$$P(s, n\Delta t) = e^{-r\Delta t}\{\lambda_n^i(1 - p_{i+1}^n)(S - S_{n+1}^i) + \underbrace{\sum_{j=0}^{i-1} \lambda_n^j(S - F_n^j)\}}_{:=\rho_l}$$

for transition probabilities, we can calculate downward nodes from risk free condition:

$$S_{n+1}^i = \frac{S^{i+1}n + 1\left[P(S, n\Delta t)e^{r\Delta t} - \rho_l\right] - \lambda_n^i S(F_n^i - S_{n+1}^{i+1})}{P(S, n\Delta t)e^{r\Delta t} - \rho_l + \lambda_n^i(F_n^i - S_{n+1}^{i+1})}$$