Pareto Type 2 with fix s

2022-10-13

Simulate Data

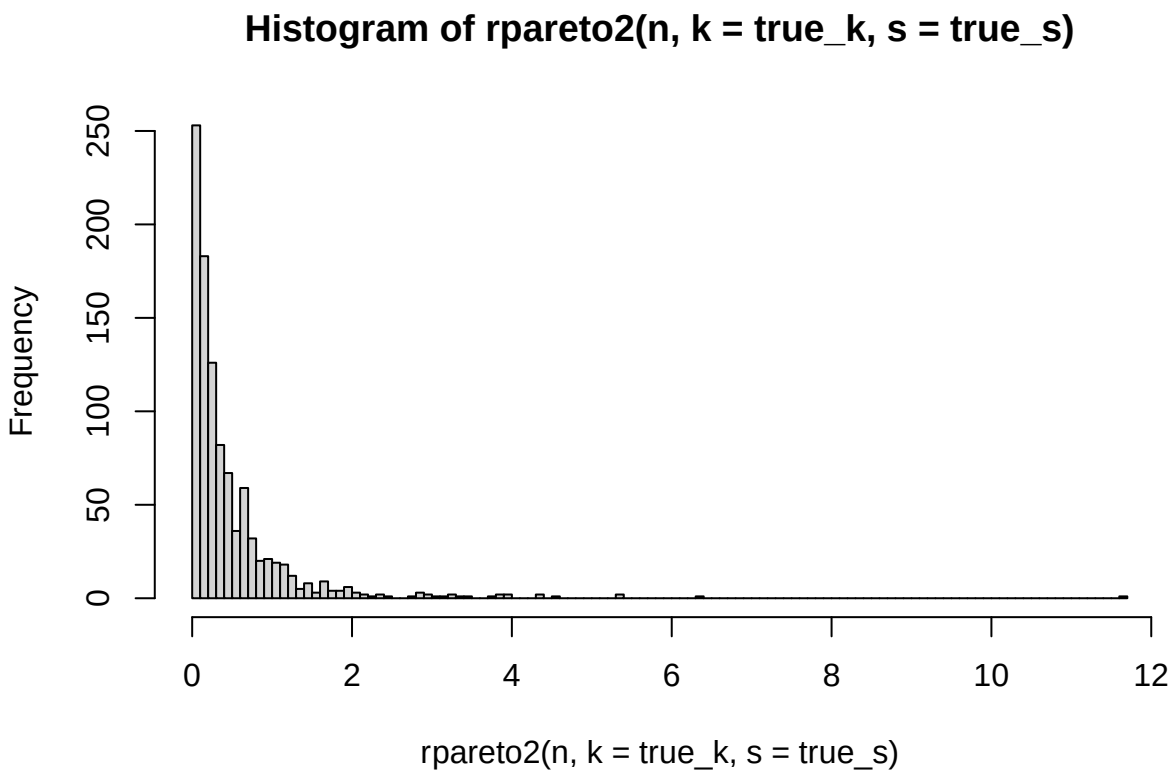
Hazard rate for Lomax(pareto 2) distribution is $S(x) = \frac{k}{s+x}$, where s is a vector of scale parameters and k is a vector of shape parameters.

```
knitr::opts_chunk$set(echo = TRUE)
#install.packages("truncdist")
library(survival)
library(truncdist)
```

```
## Loading required package: stats4
```

```
## Loading required package: evd
```

```
set.seed(3)
#install.packages("agop")
# type-II Pareto is the true distribution (the one that generate data)
library("agop")

n <- 1000
A <- rbinom(n, 1, .5)
X <- model.matrix(~ A)
true_beta <- matrix(c(-.1, .2), ncol=1)
true_mu <- X %*% true_beta
true_k = 3
true_s <- exp(-1*true_mu*true_k)
hist(rpareto2(n, k=true_k, s=true_s), breaks=100)
```




```
# simulate censoring and survival times
survt = rpareto2(n, k=true_k, s = true_s)
cent = rpareto2(n, k=true_k, s = true_s)

## observed data:
#censoring indicator
delta <- cent < survt
survt[delta==1] <- cent[delta==1] # censor survival time.


# survt_all will combine observed and imputed survival times.
survt_all <- survt

# count number of missing/censored survival times
n_miss <- sum(delta)
row_miss <- c(1:n)[delta] # index for which rows are censored
```

Functions

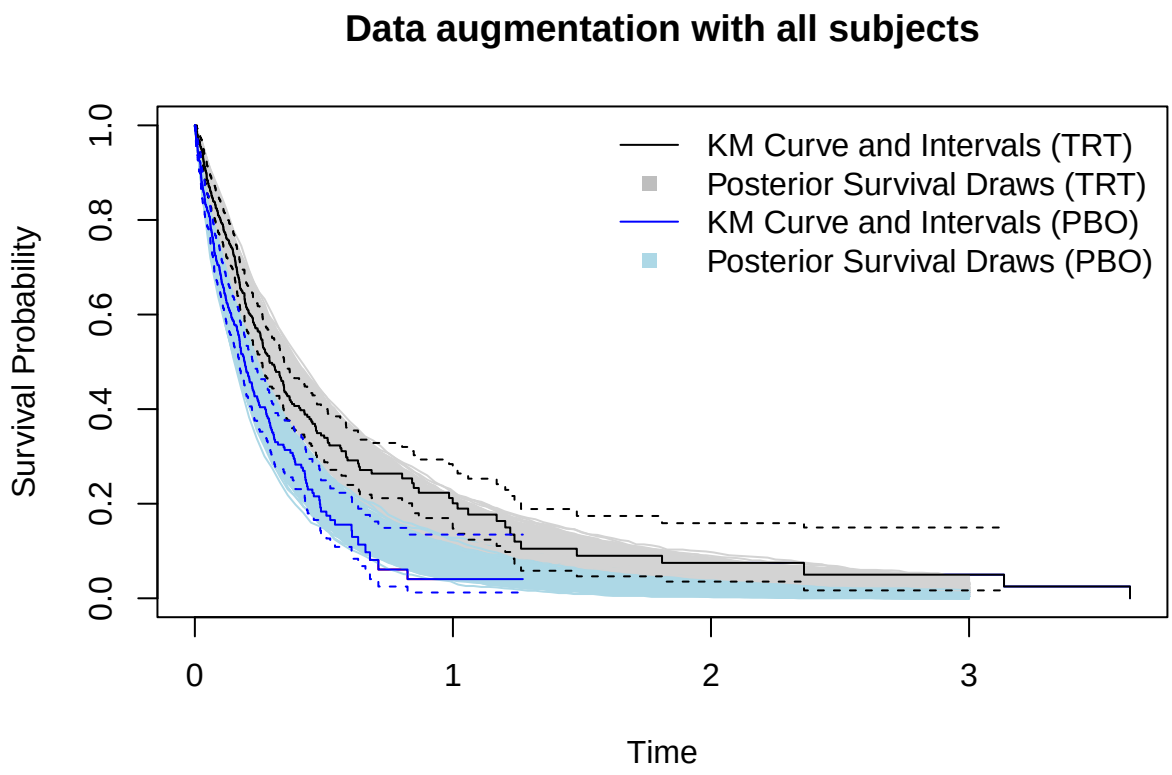
```
log_post_beta <- function(beta, log_k, X, survt){
  k <- exp(log_k)
  mu <-  X %*% beta
  s <- exp(-1*mu*k)

  lik <- sum(log(dpareto2(survt, k = k, s = s)))
  pr <- dexp(x = k, rate = 1, log = T)
  return(lik + pr)
```

```
}
log_post_k <- function(log_k, beta, X, survt){
  k <- exp(log_k)

  mu <-  X %*% beta
  s <- exp(-1*mu*k)

  lik <- sum(log(dpareto2(survt, k = k, s = s)))
  pr <- dexp(x = k, rate = 1, log = T)
  return(lik + pr)
}
metrop_hastings<-function(x_0, iter=1, log_post_density,
                          proposal_dist = function(x, prop_sigma){
                            MASS::mvrnorm(1, mu = x, Sigma = prop_sigma )
                          },
                          lower=-Inf, upper=Inf, prop_sigma,
                          ... ){
  for(i in 1:iter){
    # draw from proposal distribution
    x_star <- proposal_dist(x_0, prop_sigma)

    # calculate ratio of conditional posterior densities
    r_num <- do.call(log_post_density, c(list(x_star), list(...)) )
    r_denom <- do.call(log_post_density, c(list(x_0), list(...)) )
    r <- exp(r_num - r_denom)
    rmin<-min(r,1)
    if(is.na(rmin)) browser()
    # accept / reject proposal
    if(rbinom(1,1,rmin)==1){
      x_0<-x_star
    }
  }

  res<-list(x_0 = x_0, accept_prob = rmin )
  return(res)
}
rpareto2_trunc <- function(n, a, k, s) {

  # range is a vector of two values

  F.a <- ppareto2(a, k=k, s = s)
  F.b <- 1

  u <- runif(n, min = F.a, max = F.b)

  qpareto2(u, k=k, s=s)

}
```

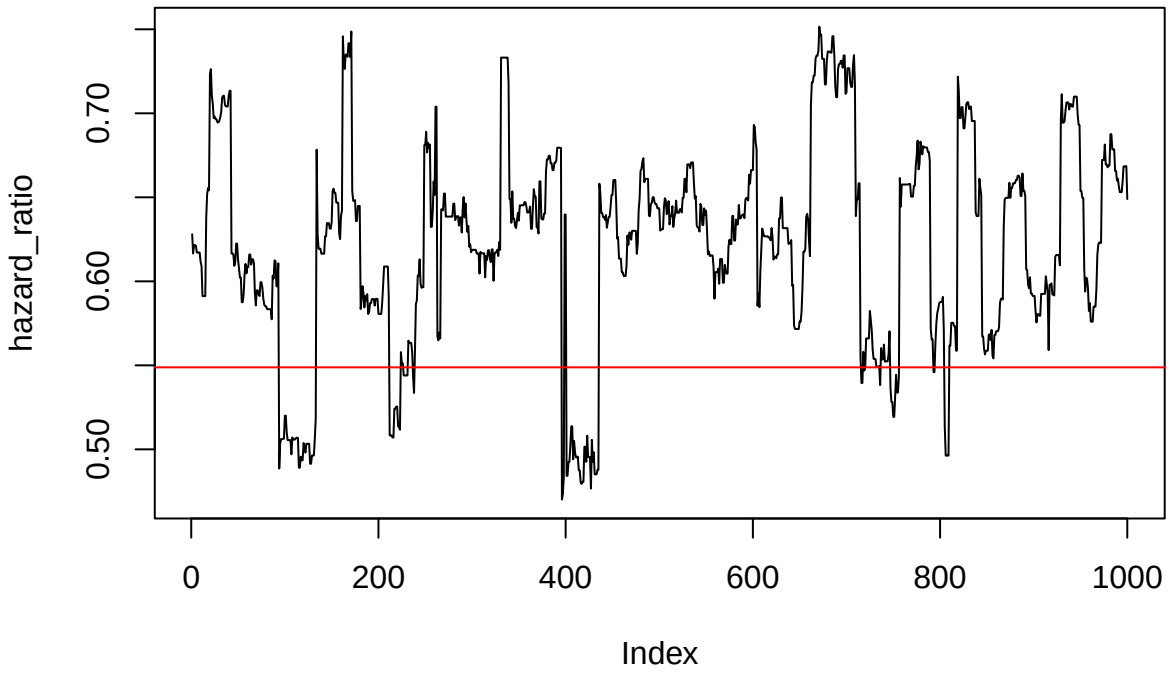
Run Augmented Sampler Accounting for Censoring

```
iter <- 10000 # number of gibbs iterations
burnin <- 9000 # burn-in iterations

# shells for storing parameters
hazard_ratio <- numeric(iter - burnin)

# initial values
beta_shell <- matrix(c(0,0), ncol=1)
l_k_shell <- c(0)

prop_covar <- diag(c(.01,.01))
# plot stratified Kaplan-Meier
par(mfrow=c(1,1))
plot(survfit(Surv(survt, 1-delta) ~ A),conf.int = F, col=c('blue','red'),
     xlab=c('Time'),ylab='Survival Probability',
     main = 'Data augmentation with all subjects')
for(i in 2:iter){
  ## sample from posterior of parameters,
  ## conditional on observed and missing survival times

  # metrop_hastings() is a custom function for generating a draw
  # from conditional posterior of beta: log_post_beta
  beta_shell <- metrop_hastings(x_0 = beta_shell,
                                iter = 1,
                                log_post_density = log_post_beta,
                                prop_sigma = prop_covar,
                                X=X, survt=survt_all,
                                log_k=l_k_shell )$x_0

  # sample from conditional posterior of k: log_post_k
  l_k_shell <- metrop_hastings(x_0 = l_k_shell,
                               iter = 1,
                               log_post_density = log_post_k,
                               prop_sigma = matrix(.002),
                               X=X, survt=survt_all,
                               beta=beta_shell)$x_0

  ## sample from conditional posterior of missing survival times
  mu_curr <-  X %*% beta_shell
  k_curr <- exp(l_k_shell)

  for(m in row_miss){
    s_curr <- exp(-1*mu_curr[m]*k_curr)

    survt_all[m] <- rpareto2_trunc(1, survt[m], k_curr, s_curr)

  }

  if(i>burnin){
    # plot 500 posterior survival curve draws for treated and placebo
    mu_trt <-  sum(beta_shell)
```

```
    mu_pbo <-  beta_shell[1]

    post_draw <- rpareto2(n, k = k_curr, s = exp(-1*mu_trt*k_curr)  )
    post_ecdf <- ecdf(post_draw)
    curve(1-post_ecdf(x), add=T, from=0, to=3, col='lightblue')

    post_draw <- rpareto2(n, k = k_curr, s = exp(-1*mu_pbo*k_curr)  )
    post_ecdf <- ecdf(post_draw)
    curve(1-post_ecdf(x), add=T, from=0, to=3, col='lightgray')

    # store hazard ratio
    hazard_ratio[i-burnin] <- exp(-beta_shell[2]*k_curr)
  }

}
# overlay KM curve and plot legend
lines(survfit(Surv(survt, 1-delta) ~ A),conf.int = T, col=c('black','blue'))
legend('topright',
       legend = c('KM Curve and Intervals (TRT)',
                  'Posterior Survival Draws (TRT)',
                  'KM Curve and Intervals (PBO)',
                  'Posterior Survival Draws (PBO)'),
       col=c('black','gray','blue','lightblue'),
       lty=c(1,0,1,0), pch=c(NA,15,NA,15), bty='n')
```




```
plot(hazard_ratio, type='l')
abline(h=exp(-true_beta[2]*true_k), col='red')
```




```
hist(hazard_ratio)
abline(v=exp(-true_beta[2]*true_k), col='red')
```