Econ 8307
Assignment 3 (Spring 2019)

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Question 1

1. $M^* = (1-\delta)M^* + \epsilon$ gives

   $M^* = \frac{\epsilon}{\delta}$

2. Let $\mu_t(z_i)$ represent the number of firms of type z_i at time t. Then:

   $\mu_{t+1}(z_i) = (1-\delta)\sum_{j=1}^{N}\mu_t(z_j)f(z_i|z_j) + \epsilon\psi(z_i)$

   Or in terms of matrix algebra:

   $\mu_{t+1} = (1-\delta)\mu_t T + \epsilon\Psi$

   Where μ_t is the 1xN measure over firm types, T is the NxN transition matrix (i.e. $t_{ij} = f(z_j|z_i)$), and Ψ is the 1xN probability distribution for new firms (i.e. $\Psi_i = \psi(z_i)$.)

3. $\mu^* = (1-\delta)\mu^*T + \epsilon\Psi$ gives:

   $\mu^* = \epsilon\Psi(I - (1-\delta)T)^{-1}$

4. Computing steady state measure over firm types.

```
1   N = 10;
2   e = 1;
3   Δ = .1;
4   beta = 1;
5   gamma = .1;
6
7   psi = zeros(1,N);
8   T = zeros(N);
9   I = eye(N);
10  A= zeros(1,N);
11
```

```
12   relprob=@(new, old) max(0, beta − gamma*(new−old)^2);

13

14   for i = 1:N
15        psi(i) = 1/N;
16        for j= 1:N
17             A(i) = A(i) + relprob(j, i);
18        end
19   end

20

21   for i=1:N
22        for j = 1:N
23             T(i,j) = relprob(j, i) / A(i);
24        end
25   end

26

27   mustar = e*psi*inv(I−(1 − Δ)*T);

28

29   mustar'
```

Prints transposed steady state measure over firm types.

```
1    ans =

2

3        0.7503
4        0.9625
5        1.0950
6        1.1011
7        1.0912
8        1.0912
9        1.1011
10       1.0950
11       0.9625
12       0.7503
```

Question 2

1. Firm's value function:
$$V(z_{it}) = \max_{n_{it}} \left[z_{it} n_{it}^\alpha - w_t n_{it} + \beta(1-\lambda) E(V(z_{i,t+1})) \right]$$
Or equivalently:
$$V(z_{it}) = \max_{n_{it}} \left[z_{it} n_{it}^\alpha - w_t n_{it} + \beta(1-\lambda) \sum_{j=1}^{N} V(z_j) f(z_j | z_{it}) \right]$$

Transition function of measure of types:

$$\mu_{t+1}(z_i) = \sum_{j=1}^{N} [(1-\lambda)f(z_i|z_j)\mu_t(z_j)] + M_t\psi(z_i)$$

2. Suppressing firm index i, firm's value function:

$$V(z_t, n_{t-1}) = \max_{n_t} [z_t n_t^\alpha - w_t n_t - \tau w_t \max(o, n_{t-1} - n_t) + \beta(1-\lambda)E(V(z_{t+1}, n_t)) - \lambda\tau w_t n_t]$$

Transition function of measure of types:

$$\mu_{t+1}(z', n') = \sum_z \sum_n [(1-\lambda)\mathbf{1}(n^*(z,n) = n')f(z'|z)\mu_t(z,n)] + M_t\psi(z)\mathbf{1}(n' = 0)$$

Where $n^*(z, n)$ is the argmax from the value function with arguments z and n. $\mathbf{1}(a) = 1$ if a is true and 0 otherwise.

3. Firm's value function:

$$V(z_t) = \max_{n_t, X} [z_t n_t^\alpha - w_t n_t + \beta(1-X)E(V(z_{t+1})) - (1-X)k]$$
$$X \in \{0, 1\}$$

Transition function of measure of types:

$$\mu_{t+1}(z_i) = \sum_{j=1}^{N} [(1 - X^*(z_j))f(z_i|z_j)\mu_t(z_j)] + M_t\psi(z_i)$$

Where $X^*(z)$ is the argmax from the value function with argument z.

Question 3

1. Finding $n^*(z)$ where $z \in \{1, 2, ..., N\}$:

```
1  N = 5;
2  p = .8;
3  w = 1;
4  alpha = .7;
5  beta = .95;
6
7  Z = zeros(1,N);
8  nstar = zeros(N,1);
9  for i =1:N
10     Z(i) = i;
11     nstar(i) = (w/(Z(i)*alpha))^(1/(alpha-1));
12  end
13
14  nstar
```

Prints n^*:

```
1  nstar =
2
3       0.3046
4       3.0697
5      11.8594
6      30.9405
7      65.0969
```

The aggregate labor input is: $\sum_z \mu_t(z) n^*(z)$ Or in matrix notation: $\mu_t n^*$ where μ_t is the 1xN measure over types and n^* is the the Nx1 vector of optimal employment over types. Since the distribution of firms is uniform over firm types (see part 2) the aggregate labor input is $Number\ of\ firms * \sum_z .2 * n^*(z) = Number\ of\ firms * 22.2542$.

2. We will assume that that the distribution for new firms ψ is a uniform distribution.

   (a) Without n_{t-1} as a state variable. $\mu^* = \mu^*(z)$

```
1   T = zeros(N);
2   T=T+(1-p)/(N-1)+eye(N)*(p-(1-p)/(N-1));
3   I=eye(N);
4   psi= zeros(1,N);
5   for i =1:N
6       psi(i) = 1/N;
7   end
8
9   mustar = psi*inv(I-(1-lambda)*T);
10  mustar1 = mustar/sum(mustar);
11  mustar1'
```

   Prints steady state distribution of types.

```
1  ans =
2
3       0.2000
4       0.2000
5       0.2000
6       0.2000
7       0.2000
```

4

(b) With n_{t-1} as a state variable. $\mu^* = \mu^*(z_t, n_{t-1})$

```matlab
1  N = 5;
2  p = .8;
3  alpha = .7;
4  beta = .95;
5  lambda = .1;
6  E = 1;
7  wguess = 1;
8  tau = .0;
9
10 Z = zeros(1,N);
11
12 for i =1:N
13     Z(i) = i;
14 end
15
16 T = zeros(N);
17 T=T+(1-p)/(N-1)+eye(N)*(p-(1-p)/(N-1));
18 I=eye(N);
19 psi= zeros(1,N);
20 for i =1:N
21     psi(i) = 1/N;
22 end
23
24 %Find equilibrium wage
25 step = 1;
26 gridsize = 100;
27 gridmax = 100;
28
29 while step > .00001
30 % Find Value function given wguess
31
32 nvalues = zeros(gridsize+1,1);
33 for i = 1:gridsize+1
34     nvalues(i)= gridmax*(i-1)/gridsize;
35 end
36
37 nstar = zeros(N, gridsize+1);
38 Vold = zeros(N, gridsize+1);
39 temp = zeros(gridsize+1,1);
40 loss = 1;
41
```

```matlab
while loss > .00001
Vnew = zeros(N, gridsize+1);
for i = 1:N
    for j = 1:gridsize+1
        temp = zeros(gridsize+1,1);
        for k = 1:gridsize+1
            temp(k) = Z(i)*nvalues(k)^alpha - wguess*nvalues(k) -...
                tau*wguess*max(0,nvalues(j)-nvalues(k)) + ...
                beta*(1 lambda)*T(i,:)*Vold(:,k) - ...
                beta*lambda*tau*wguess*nvalues(k);
        end
        [M, I]= max(temp);
        nstar(i,j) = nvalues(I);
        Vnew(i,j) = Z(i)*nstar(i,j)^alpha - wguess*nstar(i,j) - ...
            tau*wguess*max(0,nvalues(j)-nstar(i,j)) + ...
            beta*(1-lambda)*T(i,:)*Vold(:,I) - ...
            beta*lambda*tau*wguess*nstar(i,j);
    end
end
loss = sum(abs(Vnew-Vold), 'all');
Vold = Vnew;
end
V = Vnew;
gridmax = round(max(nstar,[],'all')*1.1,2,'significant');
Ecalc = beta*psi*V(:,1);
if Ecalc-E >0
    wguess = wguess + step;
else
    wguess = wguess - step;
    step = step/2;
end
end

%iterate to find steady state measure
muold = ones(N, gridsize+1);
loss2=1;

while loss2 > .00001
munew = zeros(N, gridsize+1);
for i = 1:N
    for j = 1:gridsize+1
        for ii = 1:N
            for jj = 1:gridsize+1
```

```matlab
79                    if nstar(ii,jj) == nvalues(j)
80                        munew(i,j) = munew(i,j) + ...
                              (1-lambda)*T(i,ii)*muold(ii,jj);
81                    end
82                end
83            end
84            if nvalues(j) == 0
85                munew(i,j) = munew(i,j) + psi(i);
86            end
87        end
88  end
89  loss2 = sum(abs(munew-muold), 'all');
90  muold = munew;
91  end
92
93  mm = munew/sum(munew,'all');
94  mu = sum(mm')
```

Prints steady state distribution of types.

```matlab
1  mu =
2
3     0.2000    0.2000    0.2000    0.2000    0.2000
```

Computing proportion of jobs destroyed.

```matlab
1  jobs = 0;
2  dest = 0;
3  for i= 1:N
4      for j = 1:gridsize+1
5          jobs = jobs + mm(i,j)*nstar(i,j);
6          dest = dest + lambda*mm(i,j)*nstar(i,j);
7          dest = dest + mm(i,j) * max(0,(nvalues(j)   nstar(i,j)));
8      end
9  end
10  propdest = dest/jobs
```

```matlab
1  propdest =
2
```

```
3        0.2267
```

3. Uses same code as 2b with tau parameter changed. List only results for equilibrium wage and job destruction for each value of tau.

```
1  tau =   0, wage = 5.8801, job destruction = .2267
2  tau = .5, wage = 5.3305, job destruction = .2031
3  tau =   1, wage = 4.9192, job destruction = .1813
```

The cost of firing lowers wages. Job destruction varies a similar amount as wages.

4. Finding steady state employment.

```
1  omega = 1;
2  c = wguess/omega;
3  y = 0;
4  emp = 0;
5  for i = 1:N
6      for j = 1:gridsize+1
7          y = y + Z(i)*nstar(i,j)^alpha * munew(i,j);
8          emp = emp + nstar(i,j)*munew(i,j);
9      end
10  end
11  Mstar = c/(y−E);
12  emp = emp*Mstar
```

Prints steady state employment. Showing results for each value of tau.

```
1  tau =   0, employment = .8717
2  tau = .5, employment = .7966
3  tau =   1, employment = .7469
```

5. Since $\omega = 1$ consumption $=$ wages which is 5.88 in the base (tau $= 0$) case. Steady state one period utility is $log(C) - \omega n = .8999$ in the base case. To maintain this utility, consumption would need to be increased by $exp\left(.8999 + .7966 - log(5.3305)\right) - 1 = 2.33\%$ if tau $= .5$, and by $exp\left(.8999 + .7469 - log(4.9192)\right) - 1 = 5.51\%$ if tau $= 1$.