Econ 8307
Assignment 2 (Spring 2019)

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Question 1

```
beta = .97;
Δ = .1;
theta = .3;
steady=@(x) [ x(1) + beta*x(1)*(theta * x(2)^(theta 1) + 1   Δ);
    x(2)^theta   Δ*x(2)   x(1)];

initial_guess=[1;1];


steady_state=fsolve(steady,initial_guess)
```

Prints the steady state solution. Thew first entry is steady state consumption and the second entry is steady state capital.

```
steady_state =

    1.0998
    3.2690
```

Question 2

1. Define E as a row matrix with two variables. Each variable is an indicator for employed

or unemplyed i.e. $E = [1, 0]$ if employed and $E = [0, 1]$ if unemployed. This is the state variable. Value function becomes Value(E) = EV where V is a 1x2 column vector: $V = [value\ if\ employed; value\ if\ unemployed]$.
$Value(E) = EV = EU + \beta * ETV$
Where T is a transition matrix: $T = [pi_e, 1 - pi_e; 1 - pi_u, pi_u]$, and U is the column matrix $U = [u(w_h); u(w_l)]$

2.

```
w_h = 1;
w_l = .9;
beta = .96;
pi_e = .95;
pi_u = .9;
utility=@(c) log(c);

T = [pi_e, 1 pi_e; 1 pi_u, pi_u];
U = [utility(1); utility(.9)];
I = eye(2);

V=inv(I beta*T)*U;
V
```

Prints value function vector.

```
    V =

    0.6871
    1.2597
```

3.

```
steady2=@(x) (1 pi_e)*(1 x) + pi_u*x   x;
initial_guess2=.5;
unemployment_rate=fsolve(steady2,initial_guess2)
```

Prints steady state unemployment rate.

```
    unemployment_rate =

    0.3333
```

Question 3

$n^*(z) = (\frac{w}{z\alpha})^{\frac{1}{\alpha-1}}$

Let $S_{it} = [s_{it1}; s_{it2}; ...; s_{itN}]$ be a Nx1 column vector where $s_{itx} = 1$ if $z_{it} = z_x$ and 0 otherwise. Therefore, $S_{it}Z = z_{it}$. S˙it is the state variable.
The value function is: $Value(S) = SV = S\pi + \beta(1-\lambda)STV$ Where T is the NxN transition matrix i.e. $T_{jk} = f(z_{i,t+1} = k|z_{it} = j)$ and π is the optimal profit row vector $[\pi_1, \pi_2, ..., \pi_N]$ where $\pi_x = z_x n^*(z_x)^\alpha - wn^*(z_x)$

Question 4

1.

```
N = 5;
p = .8;
w = 1;
alpha = .7;
beta = .95;
lambda = .1;

Z = zeros(1,N);
T = zeros(N);
nstar = zeros(N,1);
pi = zeros(N,1);
I = eye(N);
T=T+(1 p)/(N 1)+eye(N)*(p (1 p)/(N 1));
for i =1:N
    Z(i) = i/N;
    nstar(i) = (w/(Z(i)*alpha))^(1/(alpha 1));
    pi(i)=Z(i)*nstar(i)^alpha  w*nstar(i);
end

V = inv(I beta*(1 lambda)*T)*pi
```

Prints value vector V^*

```
   V =

```

```
    0.1851
    0.2005
    0.2496
    0.3563
    0.5472
```

2.

```
Vguess = zeros(5,1);
for i= 1:2000
    Vguess = pi + beta*(1 lambda)*T*Vguess;
end

Vguess
```

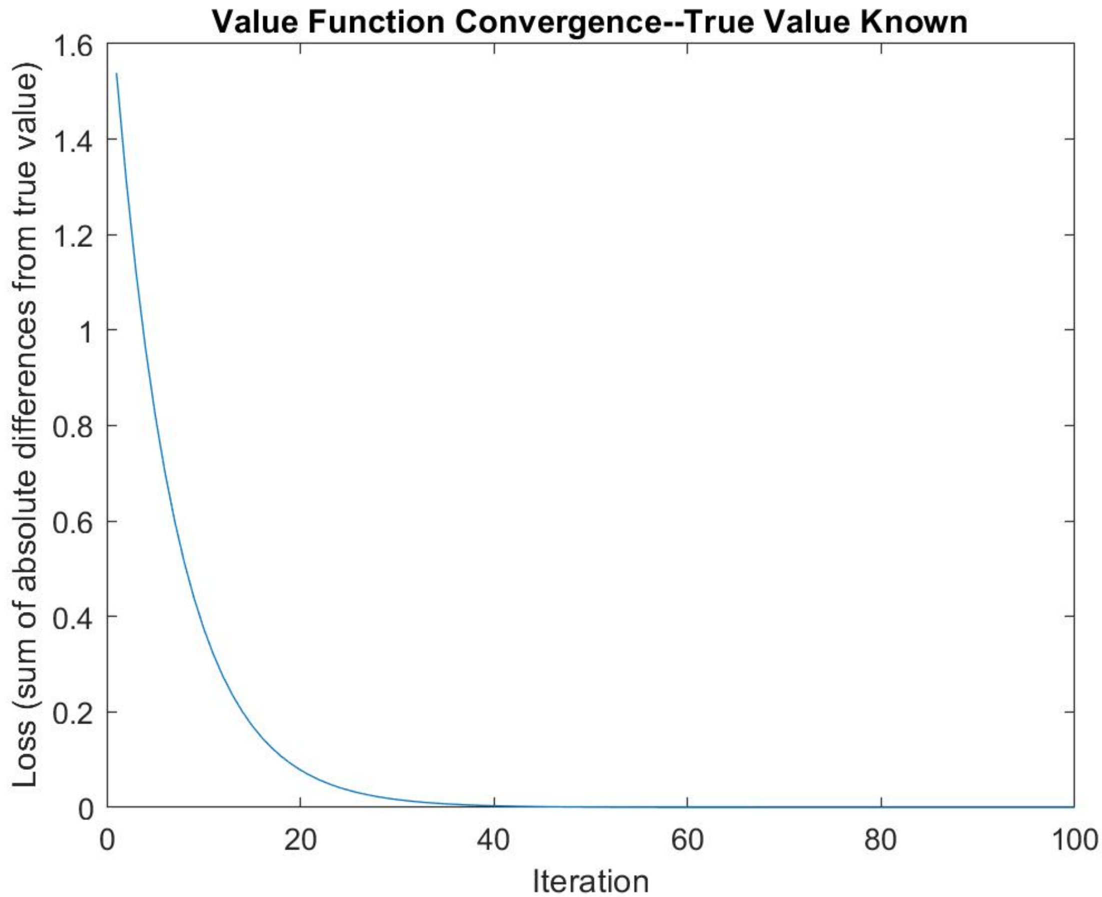
Prints 2000th iteration.

```
    Vguess =

    0.1851
    0.2005
    0.2496
    0.3563
    0.5472
```

3.

```
iterations = 100;
loss = zeros(iterations,1);
Vguess = zeros(5,1);
for j =1:5
    loss(1) = loss(1) + abs(V(j) Vguess(j));
end
for i= 1:iterations 1
    Vguess = pi + beta*(1 lambda)*T*Vguess;
    for j =1:5
        loss(i+1) = loss(i+1) + abs(V(j) Vguess(j));
    end
end

plot(loss)
title('Value Function Convergence  True Value Known')
xlabel('Iteration')
```

```
ylabel('Loss (sum of absolute differences from true value)')
```

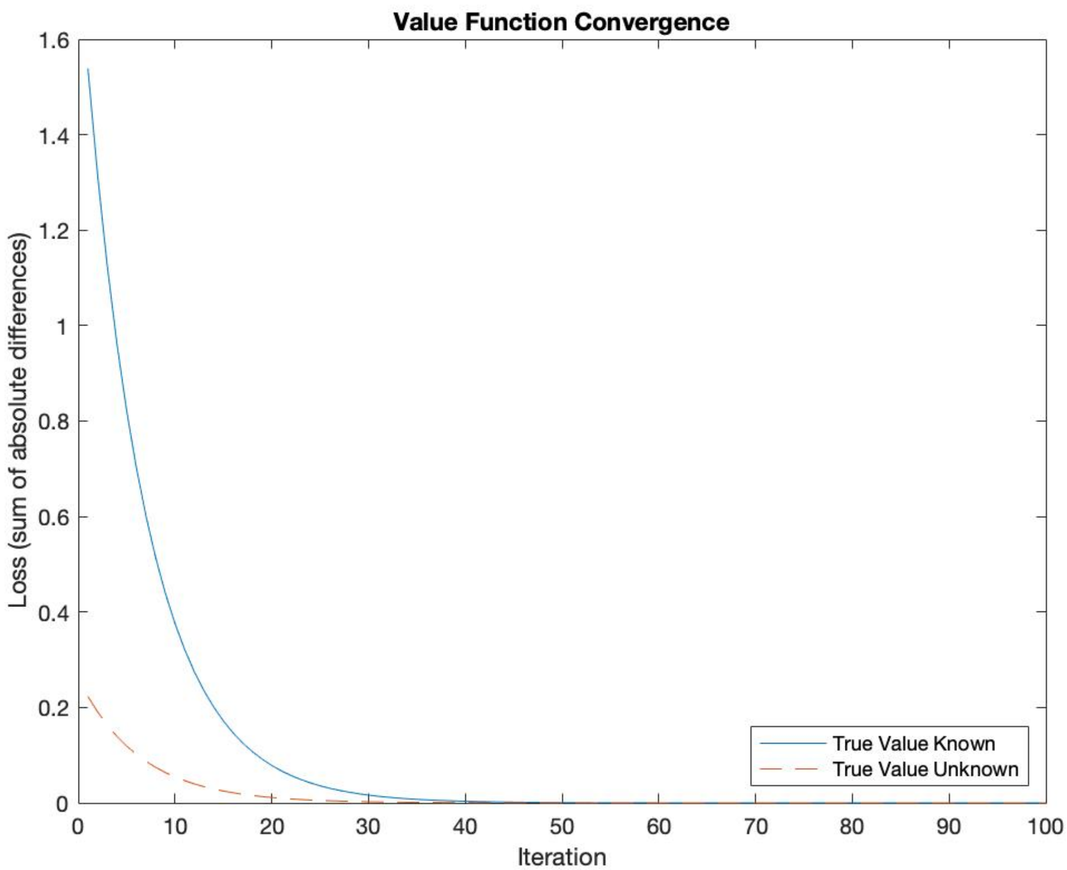


Around 67 iterations gives a fairly precise estimate of the value function. Note 67 is the first iteration where loss function rounds to .0000.

4.

```
loss2 = zeros(iterations,2);
Vold = zeros(5,1);
Vnew = zeros(5,1);
for i= 1:iterations
    loss2(i,1) = loss(i);
    Vold=Vnew;
    Vnew = pi + beta*(1 lambda)*T*Vold;
    for j =1:5
       loss2(i,2) = loss2(i,2) + abs(Vnew(j) Vold(j));
    end
end

plot(loss2)
```

```
title('Value Function Convergence')
xlabel('Iteration')
ylabel('Loss (sum of absolute differences)')
legend({'True Value Known','True Value ...
    Unknown'},'Location','southeast')
```



Now around 55 iterations appears to give a fairly precise estimate of the value function. Note 55 is the first iteration where the new loss function rounds to .0000.