Econ 8307
Assignment 7 (Spring 2019)

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Question 1

1. The number of firms in steady state satisfies

$$
\begin{aligned}
E &= (1-\delta)M \\
M &= \frac{E}{(1-\delta)} = 50
\end{aligned}
\tag{1}
$$

   The measure of firms in steady state satisfies

$$
\begin{aligned}
\begin{pmatrix} \mu_l \\ \mu_h \end{pmatrix} &= (1-\delta)\begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix}^{'} \begin{pmatrix} \mu_l \\ \mu_h \end{pmatrix} + E\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
\mu &= (1-\delta)T^{'}\mu + E\psi \\
\mu &= (I_2 - (1-\delta)T^{'})^{-1}E\psi = \begin{pmatrix} 18.7898089171975 \\ 31.2101910828026 \end{pmatrix}
\end{aligned}
\tag{2}
$$

2. From FOCs of firms' profit maximization problem, we have

$$
w = \theta z_l n_l^{\theta-1} = \theta z_h n_h^{\theta-1} \tag{3}
$$

   The household problem is formulated as

$$
\max_{\{n_{li}\},\{n_{hi}\}} \ln c - \alpha\left(\int_0^{\mu_l} n_{li}dx + \int_0^{\mu_h} n_{hi}dx\right)
$$

s.t. $c = y = \int_0^{\mu_l} z_{li} n_{li}^{\theta} dx + \int_0^{\mu_h} z_{hi} n_{hi}^{\theta} dx$ FOC gives

$$\frac{w}{y} = \alpha \tag{4}$$

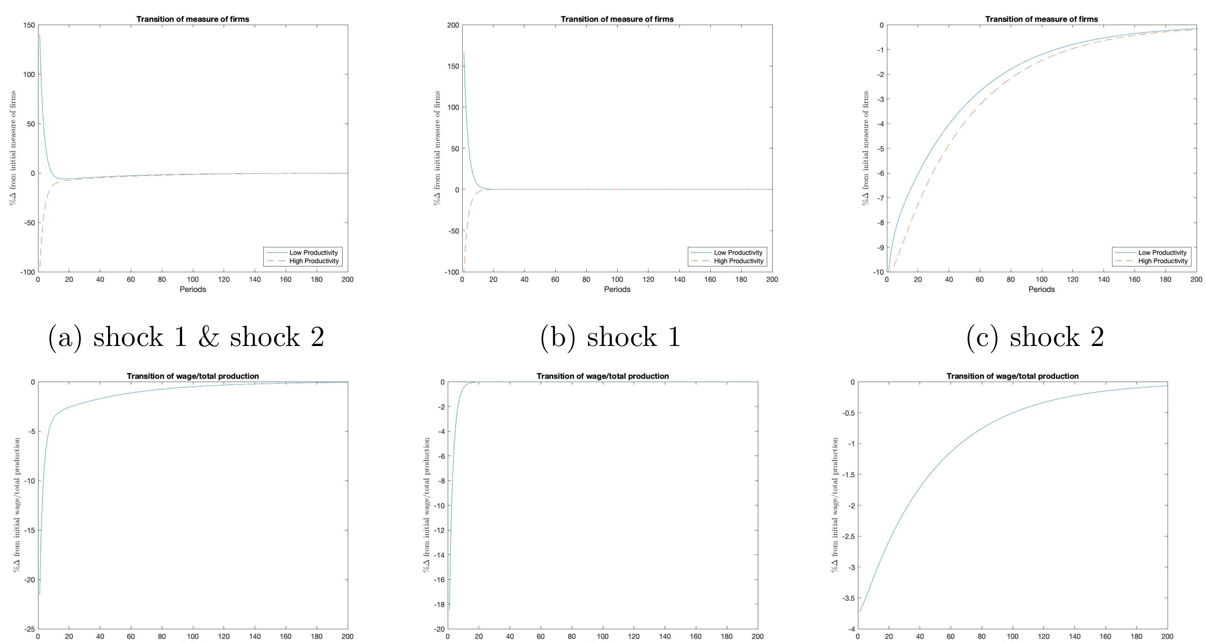
Combine equation (3) and equation (4), we have

$$\begin{aligned} w &= \alpha(\mu_l z_l n_l^{\theta} + \mu_h z_h n_h^{\theta}) \\ w &= \mu_l z_l (\frac{w}{\theta z_l})^{\frac{\theta}{\theta-1}} + \mu_h z_h (\frac{w}{\theta z_h})^{\frac{\theta}{\theta-1}} \end{aligned} \tag{5}$$

w can be then solved using Matlab and we have $w = 2.82724832831722$. It can also be solved given the measure of firms. From (5), we can solve for w

$$\begin{aligned} w^{\frac{1}{1-\theta}} &= \theta^{\frac{\theta}{1-\theta}} (\mu_l z_l^{\frac{1}{1-\theta}} + \mu_h z_h^{\frac{1}{1-\theta}}) \\ w &= \theta^{\theta} (\mu_l z_l^{\frac{1}{1-\theta}} + \mu_h z_h^{\frac{1}{1-\theta}})^{1-\theta} \end{aligned} \tag{6}$$

Question 2&3



(a) shock 1 & shock 2 (b) shock 1 (c) shock 2

(d) shock 1 & shock 2 (e) shock 1 (f) shock 2

Figure 1: Transition dynamics with different shocks

Shock 1 provides amplification while shock 2 provides propagation. We have shown in (4) that $w = y$ because $\alpha = 1$. Thus, the lower graphs show the dynamics for both wage and total production. We can also easily show from (3) and (6) that $\mu_l n_l + \mu_h n_h = \theta$. So there is no dynamics of total employment to be shown.

$$\begin{aligned} \mu_l n_l + \mu_h n_h &= \mu_l (\frac{w}{\theta z_l})^{\frac{1}{\theta-1}} + \mu_h (\frac{w}{\theta z_h})^{\frac{1}{\theta-1}} \\ &= (\frac{w}{\theta})^{\frac{1}{\theta-1}} (\mu_l z_l^{\frac{1}{1-\theta}} + \mu_h z_h^{\frac{1}{1-\theta}}) \\ &= (\theta^{\theta-1})^{\frac{1}{\theta-1}} (\mu_l z_l^{\frac{1}{1-\theta}} + \mu_h z_h^{\frac{1}{1-\theta}})^{-1} (\mu_l z_l^{\frac{1}{1-\theta}} + \mu_h z_h^{\frac{1}{1-\theta}}) \\ &= \theta \end{aligned} \tag{7}$$