Econ 8307
Assignment 5 (Spring 2019)

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Question 1

1. Bellman Equations:
   $\frac{1}{c_t} = \rho E_t \frac{1}{c_{t+1}} \left(\alpha e^{z_{t+1}} u_{t+1}^{\alpha} k_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + 1 - \frac{u_{t+1}^{\phi}}{\phi} \right)$
   $\theta(1-N_t)^{-\gamma} = \frac{1}{c_t}(1-\alpha)e^{z_t} u_t^{\alpha} k_t^{\alpha} N_t^{-\alpha}$
   $\alpha e^{z_t} u_t^{\alpha-\phi} k_t^{\alpha-1} N_t^{1-\alpha} = 1$

2. Deterministic steady state equations:
   $\frac{1}{c^*} = \rho E_t \frac{1}{c^*} \left(\alpha e^{z^*} u^{*\alpha} k^{*\alpha-1} N^{*1-\alpha} + 1 - \frac{u^{*\phi}}{\phi} \right)$
   $\theta(1-N^*)^{-\gamma} = \frac{1}{c^*}(1-\alpha)e^{z^*} u^{*\alpha} k^{*\alpha} N^{*-\alpha}$
   $\alpha e^{z^*} u^{*\alpha-\phi} k^{*\alpha-1} N^{*1-\alpha} = 1$
   $c^* + k^* = e^{z^*} u^{*\alpha} k^{*\alpha} N^{*1-\alpha} + (1 - \frac{u^{*\phi}}{\phi})k^*$
   $z^* = \mu z^*$

3. Steady state values if $\phi = 2$

```
STEADY STATE RESULTS:

c        0.36592
k        7.95205
n        0.264572
z        0
u        0.142134
```

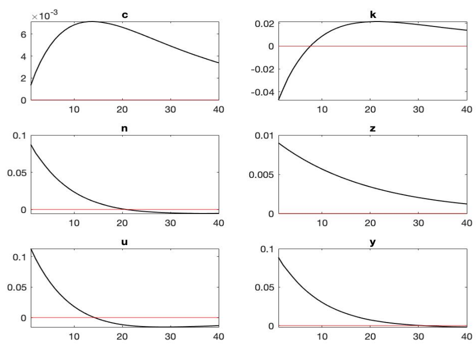
4. Steady state values if $\phi = 1.2$

```
STEADY STATE RESULTS:

c         0.158824
k         1.34774
n         0.309927
z         0
u         0.0967008
```

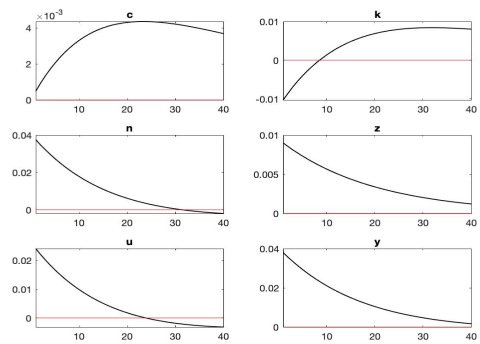
Steady state values if $\phi = 4$

```
STEADY STATE RESULTS:

c         0.751585
k         22.0768
n         0.238406
z         0
u         0.340663
```

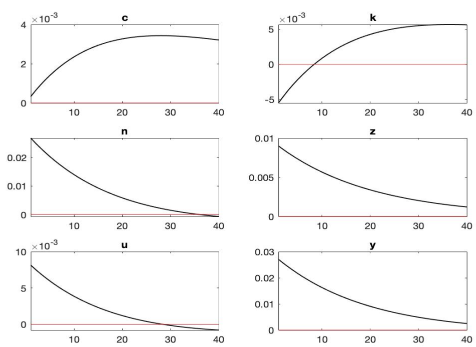
5. The impulse responses are plotted in percentage terms. Higher values of ϕ dampen the effects of a productivity shock on capital utilization. This in turn dampens the the effect of the shock on consumption, output, capital and hours worked relative to the size of the economy. With high ϕ, the percentage change in consumption, output, capital, hours worked, and capital utilization are smaller. Higher values of ϕ also increase the persistence of the changes in consumption, capital, labor, capital utilization, and output.



(a) $\phi = 1.2$



(b) $\phi = 2$



(c) $\phi = 4$

Figure 1: Dynare simulation

6. .mod code:

```
var c, k, n, z, u, y, w;
varexo e f;
parameters beta, alpha, theta, gam, mu, phi;

alpha = 0.36;
mu = 0.95;
beta = 0.99;
```

```
gam =0;
theta = 2.95;
phi = 2;

model;
beta*exp(c)^(-1)*(1-exp(u)^phi/(phi*exp(w))+alpha*exp(z)*exp(u)^alpha
*exp(k)^(alpha-1)*exp(n)^(1-alpha))=exp(c(-1))^(-1);
theta*(1-exp(n))^(-gam)=exp(c)^(-1)*(1-alpha)*exp(z)*exp(u)^alpha
*k^alpha*exp(n)^(-alpha);
alpha*exp(z)*exp(u)^(alpha-phi)*exp(k)^(alpha-1)*exp(n)^(1-alpha)=1;
exp(k(+1))=exp(z)*exp(u)^alpha*exp(k)^alpha*exp(n)^(1-alpha)
+(1-exp(u)^phi/(phi*exp(w)))*exp(k)-exp(c);
z = mu*z(-1)+ e;
exp(y)  = exp(z)*exp(u)^alpha*exp(k)^alpha*exp(n)^(1-alpha);
w = mu*w(-1)+ f;
end;

initval;
c=log(0.8036);
k=log(11.0836);
n=log(0.2918);
u=log(.5);
z=0;
w=0;
e=0;
f=0;
y = log(1);
end;

shocks;
var e = 0.009^2;
var f = 0.009^2;
corr e, f = .3;
end;

steady;
stoch_simul(periods=10100);
```

Log of steady state values:

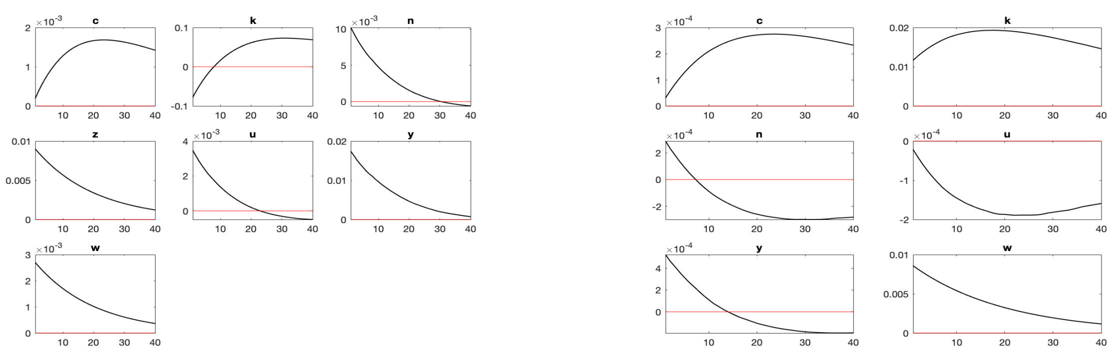
```
STEADY STATE RESULTS:

```

```
c         -1.00536
k         2.07337
n         -1.32964
z         0
u         -1.95096
y         -0.806905
w         0
```

Steady state values:

```
STEADY STATE RESULTS:

c         0.36592
k         7.95205
n         0.264572
z         0
u         0.142134
y         0.446243
w         0
```

The impulse responses are also in percentage term.



(a) Productivity shock (b) w shock

Figure 2: Dynare simulation