Econ 8307
Assignment 3 (Spring 2019)

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Question 1

1. $M^* = (1-\delta)M^* + \epsilon$ gives
   $M^* = \frac{\epsilon}{\delta}$

2. Let $\mu_t(z_i)$ represent the number of firms of type z_i at time t. Then:
   $\mu_{t+1}(z_i) = (1-\delta)\sum_{j=1}^{N} \mu_t(z_j) f(z_i|z_j) + \epsilon\psi(z_i)$
   Or in terms of matrix algebra:
   $\mu_{t+1} = (1-\delta)\mu_t T + \epsilon\Psi$
   Where μ_t is the 1xN measure over firm types, T is the NxN transition matrix (i.e. $t_{ij} = f(z_j|z_i)$), and Ψ is the 1xN probability distribution for new firms (i.e. $\Psi_i = \psi(z_i)$.)

3. $\mu^* = (1-\delta)\mu^* T + \epsilon\Psi$ gives:
   $\mu^* = \epsilon\Psi(I-(1-\delta)T)^{-1}$

4. Computing steady state measure over firm types.

```
N = 10;
e = 1;
Δ = .1;
beta = 1;
gamma = .1;

psi = zeros(1,N);
T = zeros(N);
I = eye(N);
A= zeros(1,N);

```

```
relprob=@(new, old) max(0, beta - gamma*(new-old)^2);

for i = 1:N
    psi(i) = 1/N;
    for j= 1:N
        A(i) = A(i) + relprob(j, i);
    end
end

for i=1:N
    for j = 1:N
        T(i,j) = relprob(j, i) / A(i);
    end
end

mustar = e*psi*inv(I-(1 - Δ)*T);

mustar'
```

Prints transposed steady state measure over firm types.

```
ans =

    0.7503
    0.9625
    1.0950
    1.1011
    1.0912
    1.0912
    1.1011
    1.0950
    0.9625
    0.7503
```

Question 2

1. Firm's value function:

   $$V(z_{it}) = \max_{n_{it}} \left[z_{it} n_{it}^{\alpha} - w_t n_{it} + \beta(1-\lambda)E(V(z_{i,t+1})) \right]$$

   Or equivalently:

   $$V(z_{it}) = \max_{n_{it}} \left[z_{it} n_{it}^{\alpha} - w_t n_{it} + \beta(1-\lambda) \sum_{j=1}^{N} V(z_j) f(z_j | z_{it}) \right]$$

Transition function of measure of types:

$$\mu_{t+1}(z_i) = \sum_{j=1}^{N} [(1-\lambda) f(z_i|z_j)\mu_t(z_j)] + M_t \psi(z_i)$$

2. Suppressing firm index i, firm's value function:

$$V(z_t, n_{t-1}) = \max_{n_t} [z_t n_t^{\alpha} - w_t n_t - \tau w_t \max(o, n_{t-1} - n_t) + \beta(1-\lambda)E(V(z_{t+1}, n_t)) - \lambda \tau w_t n_t]$$

Transition function of measure of types:

$$\mu_{t+1}(z', n') = \sum_{z}\sum_{n} [(1-\lambda)\mathbf{1}(n^*(z,n) = n') f(z'|z)\mu_t(z,n)] + M_t \psi(z)\mathbf{1}(n' = 0)$$

Where $n^*(z,n)$ is the argmax from the value function with arguments z and n. $\mathbf{1}(a) = 1$ if a is true and 0 otherwise.

3. Firm's value function:

$$V(z_t) = \max_{n_t, X} [z_t n_t^{\alpha} - w_t n_t + \beta(1-X)E(V(z_{t+1})) - (1-X)k]$$

$$X \in \{0, 1\}$$

Transition function of measure of types:

$$\mu_{t+1}(z_i) = \sum_{j=1}^{N} [(1 - X^*(z_j)) f(z_i|z_j)\mu_t(z_j)] + M_t \psi(z_i)$$

Where $X^*(z)$ is the argmax from the value function with argument z.

Question 3

1. Finding $n^*(z)$ where $z \in \{1, 2, ..., N\}$:

```
N = 5;
p = .8;
w = 1;
alpha = .7;
beta = .95;

Z = zeros(1,N);
nstar = zeros(N,1);
for i =1:N
    Z(i) = i;
    nstar(i) = (w/(Z(i)*alpha))^(1/(alpha-1));
end

nstar
```

Prints n^*:

```
nstar =

    0.3046
    3.0697
   11.8594
   30.9405
   65.0969
```

The aggregate labor input is: $\sum_z \mu_t(z)n^*(z)$ Or in matrix notation: $\mu_t n^*$ where μ_t is the 1xN measure over types and n^* is the the Nx1 vector of optimal employment over types. Since the distribution of firms is uniform over firm types (see part 2) the aggregate labor input is $Number\ of\ firms * \sum_z .2 * n^*(z) = Number\ of\ firms * 22.2542$.

2. We will assume that that the distribution for new firms ψ is a uniform distribution.

   (a) Without n_{t-1} as a state variable. $\mu^* = \mu^*(z)$

```
T = zeros(N);
T=T+(1-p)/(N-1)+eye(N)*(p-(1-p)/(N-1));
I=eye(N);
psi= zeros(1,N);
for i =1:N
    psi(i) = 1/N;
end

mustar = psi*inv(I-(1-lambda)*T);
mustar1 = mustar/sum(mustar);
mustar1'
```

   Prints steady state distribution of types.

```
ans =

    0.2000
    0.2000
    0.2000
    0.2000
    0.2000
```

(b) With n_{t-1} as a state variable. $\mu^* = \mu^*(z_t, n_{t-1})$

```
N = 5;
p = .8;
alpha = .7;
beta = .95;
lambda = .1;
E = 1;
wguess = 1;
tau = .0;

Z = zeros(1,N);

for i =1:N
    Z(i) = i;
end

T = zeros(N);
T=T+(1-p)/(N-1)+eye(N)*(p-(1-p)/(N-1));
I=eye(N);
psi= zeros(1,N);
for i =1:N
    psi(i) = 1/N;
end

%Find equilibrium wage
step = 1;
gridsize = 100;
gridmax = 100;

while step > .00001
% Find Value function given wguess

nvalues = zeros(gridsize+1,1);
for i = 1:gridsize+1
    nvalues(i)= gridmax*(i-1)/gridsize;
end

nstar = zeros(N, gridsize+1);
Vold = zeros(N, gridsize+1);
temp = zeros(gridsize+1,1);
loss = 1;

```

```
while loss > .00001
Vnew = zeros(N, gridsize+1);
for i = 1:N
    for j = 1:gridsize+1
        temp = zeros(gridsize+1,1);
        for k = 1:gridsize+1
            temp(k) = Z(i)*nvalues(k)^alpha - wguess*nvalues(k) -...
                 tau*wguess*max(0,nvalues(j)-nvalues(k)) + ...
               beta*(1 lambda)*T(i,:)*Vold(:,k) - ...
               beta*lambda*tau*wguess*nvalues(k);
        end
        [M, I]= max(temp);
        nstar(i,j) = nvalues(I);
        Vnew(i,j) = Z(i)*nstar(i,j)^alpha - wguess*nstar(i,j) - ...
           tau*wguess*max(0,nvalues(j)-nstar(i,j)) + ...
           beta*(1-lambda)*T(i,:)*Vold(:,I) - ...
           beta*lambda*tau*wguess*nstar(i,j);
    end
end
loss = sum(abs(Vnew-Vold), 'all');
Vold = Vnew;
end
V = Vnew;
gridmax = round(max(nstar,[],'all')*1.1,2,'significant');
Ecalc = beta*psi*V(:,1);
if Ecalc-E >0
    wguess = wguess + step;
else
    wguess = wguess - step;
    step = step/2;
end
end

%iterate to find steady state measure
muold = ones(N, gridsize+1);
loss2=1;

while loss2 > .00001
munew = zeros(N, gridsize+1);
for i = 1:N
    for j = 1:gridsize+1
        for ii = 1:N
            for jj = 1:gridsize+1
```

```
                    if nstar(ii,jj) == nvalues(j)
                        munew(i,j) = munew(i,j) + ...
                            (1-lambda)*T(i,ii)*muold(ii,jj);
                    end
                end
            end
            if nvalues(j) == 0
                munew(i,j) = munew(i,j) + psi(i);
            end
        end
    end
    loss2 = sum(abs(munew-muold), 'all');
    muold = munew;
end

mm = munew/sum(munew,'all');
mu = sum(mm')
```

Prints steady state distribution of types.

```
mu =

    0.2000    0.2000    0.2000    0.2000    0.2000
```

Computing proportion of jobs destroyed.

```
jobs = 0;
dest = 0;
for i= 1:N
    for j = 1:gridsize+1
        jobs = jobs + mm(i,j)*nstar(i,j);
        dest = dest + lambda*mm(i,j)*nstar(i,j);
        dest = dest + mm(i,j) * max(0,(nvalues(j) - nstar(i,j)));
        end
end
propdest = dest/jobs
```

```
propdest =

```

```
0.2267
```

3. Uses same code as 2b with tau parameter changed. List only results for equilibrium wage and job destruction for each value of tau.

```
tau =  0, wage = 5.8801, job destruction = .2267
tau = .5, wage = 5.3305, job destruction = .2031
tau =  1, wage = 4.9192, job destruction = .1813
```

The cost of firing lowers wages. Job destruction varies a similar amount as wages.

4. Finding steady state employment.

```
omega = 1;
c = wguess/omega;
y = 0;
emp = 0;
for i = 1:N
    for j = 1:gridsize+1
        y = y + Z(i)*nstar(i,j)^alpha * munew(i,j);
        emp = emp + nstar(i,j)*munew(i,j);
    end
end
Mstar = c/(y-E);
emp = emp*Mstar
```

Prints steady state employment. Showing results for each value of tau.

```
tau =  0, employment = .8717
tau = .5, employment = .7966
tau =  1, employment = .7469
```

5. Since $\omega = 1$ consumption = wages which is 5.88 in the base (tau = 0) case. Steady state one period utility is $log(C) - \omega n = .8999$ in the base case. To maintain this utility, consumption would need to be increased by $exp\,(.8999 + .7966 - log(5.3305)) - 1 = 2.33\%$ if tau = .5, and by $exp\,(.8999 + .7469 - log(4.9192)) - 1 = 5.51\%$ if tau = 1.