Econ 8307
Assignment 1 (Spring 2019)

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Question 1

(a) 1. returns a pseudorandom scalar integer N between 1 and 10

```
N = randi(10);
```

2. Generating a vector of N real numbers, where each value is drawn from the uniform distribution over [-1;1]. Expected value of mean is 0.

```
X = rand(N,1)*2  1;
```

3. 3.1 Computing the mean of the N real numbers using loops

```
sum1 = 0;
for i=1:N
    sum1 = sum1 + X(i);
end
result1 = sum1/N;
```

3.2 Computing the mean of the N real numbers using Matlab built-in function

```
result2 = mean(X);
```

4. Printing both results

```
 result1
 result2

result1 =

    0.0111


result2 =

    0.0111
```

(b) 1. Generating three random positive integers N, M and L

```
N = randi(10);
M = randi(10);
L = randi(10);
```

2. Generating a NxM matrix of real numbers as before, and also a MxL matrix

```
matrix1 = rand(N,M)*2  1;
matrix2 = rand(M,L)*2  1;
```

3. 3.1 Multiplying matrix 1 and matrix 2 using loops

```
resultmatrix1 = zeros(N,L);
for i =1:N
    for j = 1:L
val = 0;
for h = 1:M
    val = val + matrix1(i,h) * matrix2(h,j);
end
resultmatrix1(i,j) = val;
    end
end
```

3.2 Multiplying matrix 1 and matrix 2 using Matlab function

```
resultmatrix2 = matrix1 * matrix2;
```

4. Printing both results

```
 resultmatrix1
 resultmatrix2

resultmatrix1 =

    1.3531    0.7949    1.2546    0.7810    1.6366    0.7368
    1.0276    1.9193    0.8117    0.9999    1.9131    0.5385
    0.0565    1.8597    0.2167    0.9230    0.0832    0.3726
    0.4876    0.0426    0.2310    0.1071    0.1139    0.7056
    0.2503    0.1662    1.0685    0.1601    0.3765    0.2254
    0.9476    0.8543    0.8032    0.0655    0.5442    0.7607
    0.9518    0.5732    0.6697    0.2639    0.2710    0.5262
    0.6902    0.0980    1.2289    0.5333    1.1795    0.8939
    0.3438    0.7926    0.3158    0.4998    0.1655    0.8317


resultmatrix2 =

    1.3531    0.7949    1.2546    0.7810    1.6366    0.7368
    1.0276    1.9193    0.8117    0.9999    1.9131    0.5385
    0.0565    1.8597    0.2167    0.9230    0.0832    0.3726
    0.4876    0.0426    0.2310    0.1071    0.1139    0.7056
    0.2503    0.1662    1.0685    0.1601    0.3765    0.2254
    0.9476    0.8543    0.8032    0.0655    0.5442    0.7607
    0.9518    0.5732    0.6697    0.2639    0.2710    0.5262
    0.6902    0.0980    1.2289    0.5333    1.1795    0.8939
    0.3438    0.7926    0.3158    0.4998    0.1655    0.8317
```

Question 2

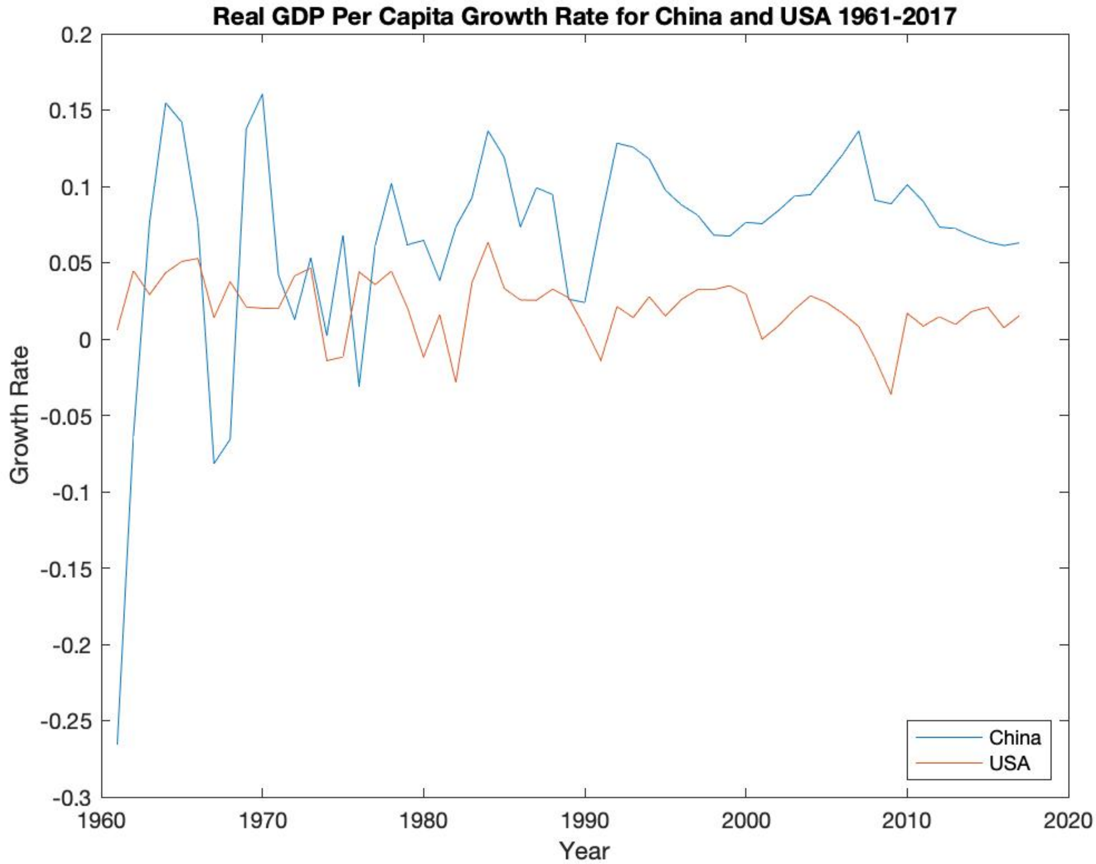
(a) Use World Bank Indicator data for GDP per capita in constant 2010 US$ for China and USA for years 1960-2017 available here `https://data.worldbank.org/indicator/NY.GDP.PCAP.KD?locations=CN-US`

(b) Load GDP per capita data into Matlab as vectors.

```
China = [191.791179910216;..;7329.08929913216];
USA = [17036.8851695882;..;53128.5396999252];
```

(c) Computing GDP per capita growth rate and plotting over time

```
sz = size(China,1)   1;
Growth = zeros(sz ,2);
Year = zeros(sz,1);
for i = 1:sz
    Growth(i,1) = China(i+1,1) / China(i,1)  1;
    Growth(i,2) = USA(i+1,1) / USA(i,1)  1;
    Year(i,1) = 1960 + i;
end
figure
plot(Year, Growth)
title('Real GDP Per Capita Growth Rate for China and USA 19612017')
xlabel('Year')
ylabel('Growth Rate')
legend({'China','USA'},'Location','southeast')
```



Question 3

(a) Agent's budget constraint is:

$$\begin{gathered} f(k_t) + (1-\delta) * k_t >= c_t + k_{t+1} \\ k_{t+1} >= 0 \end{gathered} \tag{1}$$

(b) From t=1 to T-1 the first order condition that charachterizes the solution is:

$$u'(c_t) = \beta * u'(c_{t+1}) * [f'(k_{t+1}) + 1 - \delta] \tag{2}$$

The optimality condtion at period T is:

$$c_T = f(k_T) + (1 - delta) * k_T \tag{3}$$

At the end of period T there should be zero capital left over

$$k_{T+1} = 0 \tag{4}$$

(c) Finding the solution numerically

```
%parameters
beta = .97;
Δ = .1;
theta = .3;
T = 4;
epsilon=1e 15;
initialcapital = 1;
guessinitialcons = initialcapital; %could be any guess

if guessinitialcons < 0
guessinitialcons = 0;
end %fail safe

c = zeros(T,1);
k = zeros (T+1,1);
k(1) = initialcapital;

guess = guessinitialcons;
last =  2*epsilon; % to allow for guess of 0
step = k(1);

while abs(guess   last) > epsilon || abs(k(T+1)) > epsilon
c(1) = guess;
ind = 0;
for i= 1: T 1
   if k(i)^theta + (1   Δ)*k(i)   c(i) ≥ 0
   k(i+1) = k(i)^theta + (1   Δ)*k(i)   c(i);
   c(i+1) = beta * c(i) * (theta * k(i+1)^(theta 1) + 1   Δ);
   else
   ind = 1;
   end
end
k(T+1) = k(T)^theta + (1   Δ)*k(T)   c(T);
if k(T+1) < 0 || ind == 1
step = step/2;
```

```
guess = last + step;
else
last = guess;
guess = guess + step;
end
end

%Print optimal consumption and capital series
c
k

c =

    0.8629
    0.9980
    1.1731
    1.4797


k =

    1.0000
    1.0371
    0.9464
    0.6622
   0.0000
```

We have set the tolerance level equal to epsilon (initally 10^{-15}). This means our answer, while approximate, will have leftover capital of less than epsilon in absolute value. Each period's consumption is therefore at least that "close" to the actual optimal vlaue.