Econ 8307

Assignment 7 (Spring 2019)

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Question 1

1. The number of firms in steady state satisfies

$$E = (1 - \delta)M$$
$$M = \frac{E}{(1 - \delta)} = 50 \tag{1}$$

The measure of firms in steady state satisfies

$$\begin{pmatrix} \mu_l \\ \mu_h \end{pmatrix} = (1 - \delta) \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix}' \begin{pmatrix} \mu_l \\ \mu_h \end{pmatrix} + E \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mu = (1 - \delta)T'\mu + E\psi \tag{2}$$

$$\mu = (I_2 - (1 - \delta)T')^{-1}E\psi = \begin{pmatrix} 18.7898089171975 \\ 31.2101910828026 \end{pmatrix}$$

2. From FOCs of firms' profit maximization problem, we have

$$w = \theta z_l n_l^{\theta-1} = \theta z_h n_h^{\theta-1} \tag{3}$$

The household problem is formulated as

$$\max_{\{n_{li}\},\{n_{hi}\}} \ln c - \alpha \left(\int_0^{\mu_l} n_{li}dx + \int_0^{\mu_h} n_{hi}dx \right)$$

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s.t. $c = y = \int_0^{\mu_l} z_{li} n_{li}^\theta dx + \int_0^{\mu_h} z_{hi} n_{hi}^\theta dx$ FOC gives

$$\frac{w}{y} = \alpha \tag{4}$$

Combine equation (3) and equation (4), we have

$$w = \alpha(\mu_l z_l n_l^\theta + \mu_h z_h n_h^\theta)$$
$$w = \mu_l z_l \left(\frac{w}{\theta z_l}\right)^{\frac{\theta}{\theta-1}} + \mu_h z_h \left(\frac{w}{\theta z_h}\right)^{\frac{\theta}{\theta-1}} \tag{5}$$

w can be then solved using Matlab and we have $w = 2.82724832831722$

Question 23

(a) shock 1 & shock 2

(b) shock 1

(c) shock 2

(d) shock 1 & shock 2
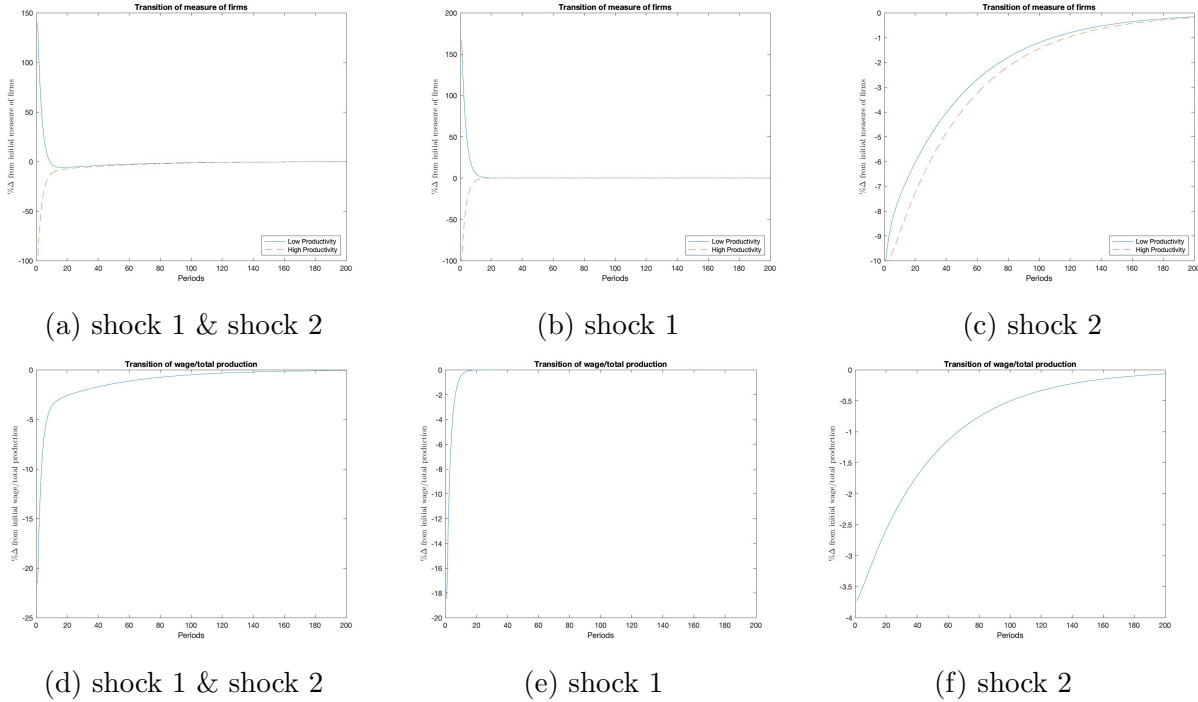
(e) shock 1

(f) shock 2

Figure 1: Transition dynamics with different shocks

Shock 1 provides amplification while shock 2 provides propagation. We have shown in (4) that $w = y$ because $\alpha = 1$. Thus, the lower graphs show the dynamics for both wage and total production. We can also show from (3) and (5) that $\mu_l n_l + \mu_h n_h = \theta$. So there is no dynamics of total employment to be shown. To show this, first rearrange (5) to get

$$\left(\frac{w}{\theta}\right)^{\frac{1}{\theta-1}} = \theta\left(\mu_l z_l^{\frac{1}{1-\theta}} + \mu_h z_h^{\frac{1}{1-\theta}}\right)^{-1} \tag{6}$$

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Then we can compute total employment

$$
\begin{aligned}
\mu_l n_l + \mu_h n_h &= \mu_l \left(\frac{w}{\theta z_l}\right)^{\frac{1}{\theta-1}} + \mu_h \left(\frac{w}{\theta z_h}\right)^{\frac{1}{\theta-1}} \\
&= \left(\frac{w}{\theta}\right)^{\frac{1}{\theta-1}} \left(\mu_l z_l^{\frac{1}{1-\theta}} + \mu_h z_h^{\frac{1}{1-\theta}}\right) \\
&= \theta
\end{aligned}
\tag{7}
$$