ecgr5105_Assignment1

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1 ECGR 5105 Assignment 1: Linear Regression with Gradient Descent

Assignment: 1

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GitHub Repository: https://github.com/xuy50/ecgr5105/tree/main/Assignment_1

In this Notebook, we solve two problems using the dataset D3.csv.

The first three columns (x1, x2, x3) are the explanatory variables,and the fourth column (y) is the dependent variable.

Problem 1:

- Develop a code that runs linear regression with a gradient descent algorithm for each explanatory variable (x1, x2, x3) in isolation. In each iteration, only one explanatory variable is used to predict y.
- Compare three learning rates (e.g., 0.1, 0.05, 0.01).
- Initialize parameters (theta) to zero.
- Report the linear model for each variable.
- Plot the final regression model (regression line) and the loss over iterations for each variable.
- Identify which variable yields the lowest final loss.
- Discuss the impact of different learning rates on final loss and training iterations.

Problem 2:

- Run linear regression with a gradient descent algorithm using all three explanatory variables together.
- Compare three learning rates (e.g., 0.1, 0.05, 0.01).
- Initialize parameters (theta) to zero.
- Report the final linear model you found the best.
- Plot the loss over iterations.
- Describe the impact of different learning rates on final loss and training iterations.
- Predict y for new inputs: (1, 1, 1), (2, 0, 4), (3, 2, 1).

Common Code: Import Libraries and Load Dataset

[ ]:
```python
# Import necessary libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
```

[ ]:
```python
# Load the dataset from D3.csv file
data = pd.read_csv('/content/drive/MyDrive/ecge5105/hw/1/D3.csv')

# Display the first few rows of the dataset to check its structure
print("Dataset preview:")
print(data.head())
```

```
Dataset preview:
         X1        X2        X3         Y
0  0.000000  3.440000  0.440000  4.387545
1  0.040404  0.134949  0.888485  2.679650
2  0.080808  0.829899  1.336970  2.968490
3  0.121212  1.524848  1.785455  3.254065
4  0.161616  2.219798  2.233939  3.536375
```

1.1 Problem 1: Single-Feature Linear Regression

For Problem 1, we perform linear regression on each feature (x1, x2, x3) separately using gradient descent. I compare three learning rates (0.1, 0.05, 0.01) and observe the final loss and the regression line.

Define Gradient Descent Function for a Single Feature

[ ]:
```python
def gradient_descent_single(x, y, learning_rate=0.1, num_iterations=1000):
    """
    Performs gradient descent for single-feature linear regression.

    Parameters:
      x -- numpy array of a single input feature (shape: [m,])
      y -- numpy array of target values (shape: [m,])
      learning_rate -- learning rate for gradient descent
      num_iterations -- number of iterations

    Returns:
      theta -- learned parameters (array of length 2: [theta0, theta1])
      losses -- list of loss values over iterations
    """
    m = len(y)
    # Construct the design matrix: the first column is 1 (intercept term), the␣
↪second column is x
    X = np.column_stack((np.ones(m), x))
    theta = np.zeros(2)  # Initialize the parameters to zero
    losses = []
```

```
    for i in range(num_iterations):
        # Compute prediction
        # y_pred = theta * x
        y_pred = X.dot(theta)
        error = y_pred - y
        # Mean Squared Error cost
        cost = (1 / (2 * m)) * np.sum(error ** 2)
        losses.append(cost)
        # Compute gradient
        gradient = (1 / m) * X.T.dot(error)
        # Update theta
        theta -= learning_rate * gradient

    return theta, losses
```

Run Experiments for Each Explanatory Variable

[ ]:
```
# Extract explanatory variables and target variable
x1 = data.iloc[:, 0].values
x2 = data.iloc[:, 1].values
x3 = data.iloc[:, 2].values
y = data.iloc[:, 3].values

# Define the learning rates to compare
learning_rates = [0.1, 0.05, 0.01]

# num_iterations = 1000
num_iterations = 2000

# For storing results for each variable
results = {}

# Process each variable separately
for var_name, x in zip(['x1', 'x2', 'x3'], [x1, x2, x3]):
    results[var_name] = {}
    for lr in learning_rates:
        theta, losses = gradient_descent_single(x, y, learning_rate=lr,␣
    ↪num_iterations=num_iterations)
        results[var_name][lr] = {'theta': theta, 'losses': losses}
        print(f"Variable {var_name}, Learning Rate {lr}: theta0 = {theta[0]:.4f},␣
    ↪theta1 = {theta[1]:.4f}, final loss = {losses[-1]:.6f}")
```

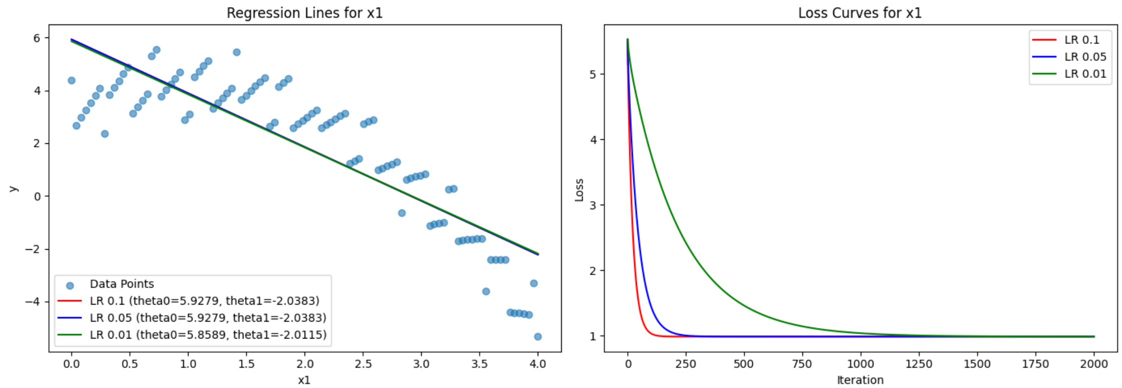
```
Variable x1, Learning Rate 0.1: theta0 = 5.9279, theta1 = -2.0383, final loss =
0.984993
Variable x1, Learning Rate 0.05: theta0 = 5.9279, theta1 = -2.0383, final loss =
0.984993
Variable x1, Learning Rate 0.01: theta0 = 5.8589, theta1 = -2.0115, final loss =
```

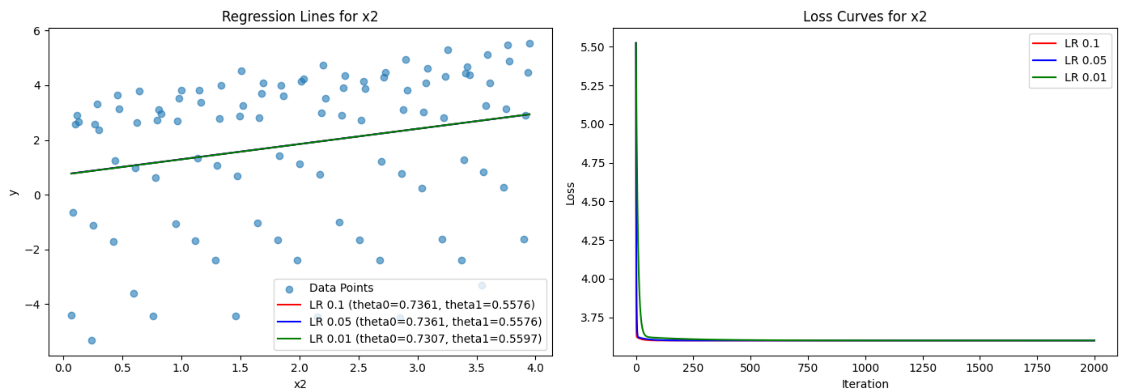
```
0.985605
Variable x2, Learning Rate 0.1: theta0 = 0.7361, theta1 = 0.5576, final loss =
3.599366
Variable x2, Learning Rate 0.05: theta0 = 0.7361, theta1 = 0.5576, final loss =
3.599366
Variable x2, Learning Rate 0.01: theta0 = 0.7307, theta1 = 0.5597, final loss =
3.599370
Variable x3, Learning Rate 0.1: theta0 = 2.8714, theta1 = -0.5205, final loss =
3.629451
Variable x3, Learning Rate 0.05: theta0 = 2.8714, theta1 = -0.5205, final loss =
3.629451
Variable x3, Learning Rate 0.01: theta0 = 2.8419, theta1 = -0.5088, final loss =
3.629565
```

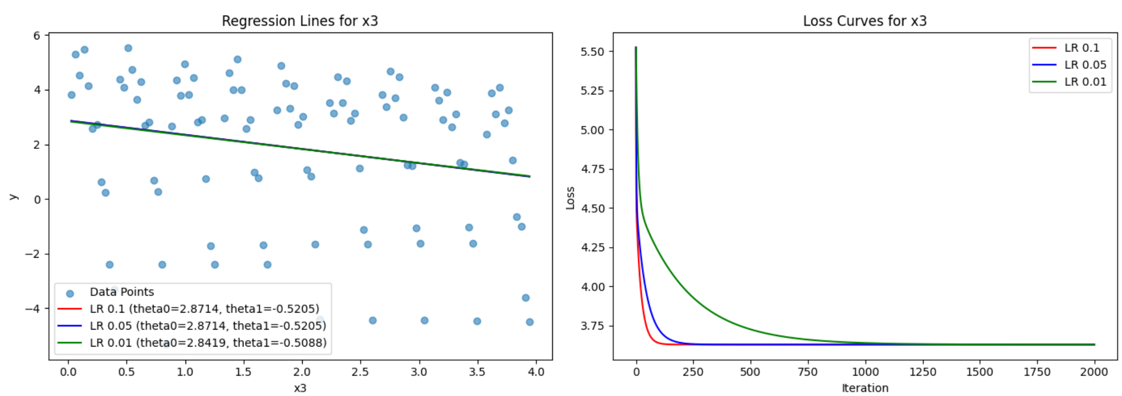
Plot Regression Lines and Loss Curves for Each Variable

[ ]:
```
# Define colors for different learning rates
colors = {0.1: 'red', 0.05: 'blue', 0.01: 'green'}

# Loop through each variable to create plots
for var_name, x in zip(['x1', 'x2', 'x3'], [x1, x2, x3]):
  plt.figure(figsize=(14, 5))

  # Subplot 1: Scatter plot and regression lines
  plt.subplot(1, 2, 1)
  plt.scatter(x, y, label='Data Points', alpha=0.6)
  x_line = np.linspace(min(x), max(x), 100)
  for lr in learning_rates:
    theta = results[var_name][lr]['theta']
    # model is theta[0] + theta[1]*x_line
    plt.plot(x_line, theta[0] + theta[1]*x_line, color=colors[lr], label=f'LR␣
↪{lr} (theta0={theta[0]:.4f}, theta1={theta[1]:.4f})')
  plt.xlabel(var_name)
  plt.ylabel('y')
  plt.title(f'Regression Lines for {var_name}')
  plt.legend()

  # Subplot 2: Loss curves over iterations
  plt.subplot(1, 2, 2)
  for lr in learning_rates:
    losses = results[var_name][lr]['losses']
    plt.plot(range(num_iterations), losses, color=colors[lr], label=f'LR {lr}')
  plt.xlabel('Iteration')
  plt.ylabel('Loss')
  plt.title(f'Loss Curves for {var_name}')
  plt.legend()

  plt.tight_layout()
```

```python
plt.savefig(f"/content/drive/MyDrive/ecge5105/hw/1/p1_{var_name}_plot.png")
plt.show()
```







1.1.1 Problem 1 Analysis and Conclusions

Problem 1 Analysis:

1) Comparing final losses for each single feature:

   - x1 final loss is around ~0.985 (lowest)
   - x2 final loss is around ~3.599
   - x3 final loss is around ~3.629 Therefore, x1 is the best single predictor for y (lowest loss).

2) Effect of different learning rates:

   - For x1, both 0.1 and 0.05 yield nearly identical final losses and converge quickly, while 0.01 converges more slowly but still reaches a similar final loss.
   - For x2 and x3, all three learning rates converge to roughly the same final loss, but 0.1 and 0.05 generally converge faster than 0.01.

Conclusion for Problem 1:

- x1 is the best explanatory variable in isolation (lowest loss ~0.985).
- Larger learning rates (0.1, 0.05) help faster convergence here without instability.
- A smaller learning rate (0.01) converges more slowly but can still reach a similar final loss.

1.2 Problem 2: Multi-Feature Linear Regression

For Problem 2, we run linear regression using all three features: x1, x2, x3. I again compare learning rates [0.1, 0.05, 0.01], initialize theta to zero, observe the loss curves, and pick the best model based on the final loss. I also compute R^2 (as an "accuracy") and track the evolution of each theta parameter.

Define Gradient Descent for Multi-Feature Regression

[ ]:
```
def gradient_descent_multi(X, y, learning_rate=0.1, num_iterations=1000):
    """
    Performs gradient descent for multi-feature linear regression.

    Parameters:
      X -- numpy array of input features (shape: [m, n])
      y -- numpy array of target values (shape: [m,])
      learning_rate -- learning rate for gradient descent
      num_iterations -- number of iterations

    Returns:
      theta -- final learned parameters of shape [n+1,] (theta0, theta1, theta2,␣
↪theta3)
      losses -- list of loss values over iterations
      r2_scores -- list of R² values over iterations
      theta_history -- list of theta vectors at each iteration
    """
    m = len(y)
    X_new = np.column_stack((np.ones(m), X))
    n = X_new.shape[1]
```

```
  theta = np.zeros(n)
  theta = np.zeros(n)  # Initialize parameters to zeros
  losses = []
  r2_scores = []
  theta_history = []

  for i in range(num_iterations):
    y_pred = X_new.dot(theta)
    error = y_pred - y
    cost = (1 / (2 * m)) * np.sum(error ** 2)
    losses.append(cost)

    # Calculate R² points
    ss_tot = np.sum((y - np.mean(y))**2)
    ss_res = np.sum((y - y_pred)**2)
    r2 = 1 - (ss_res / ss_tot)
    r2_scores.append(r2)

    gradient = (1 / m) * X_new.T.dot(error)
    theta -= learning_rate * gradient

    theta_history.append(theta.copy())


  return theta, losses, r2_scores, theta_history
```

Run Multi-Feature Experiments for Different Learning Rates

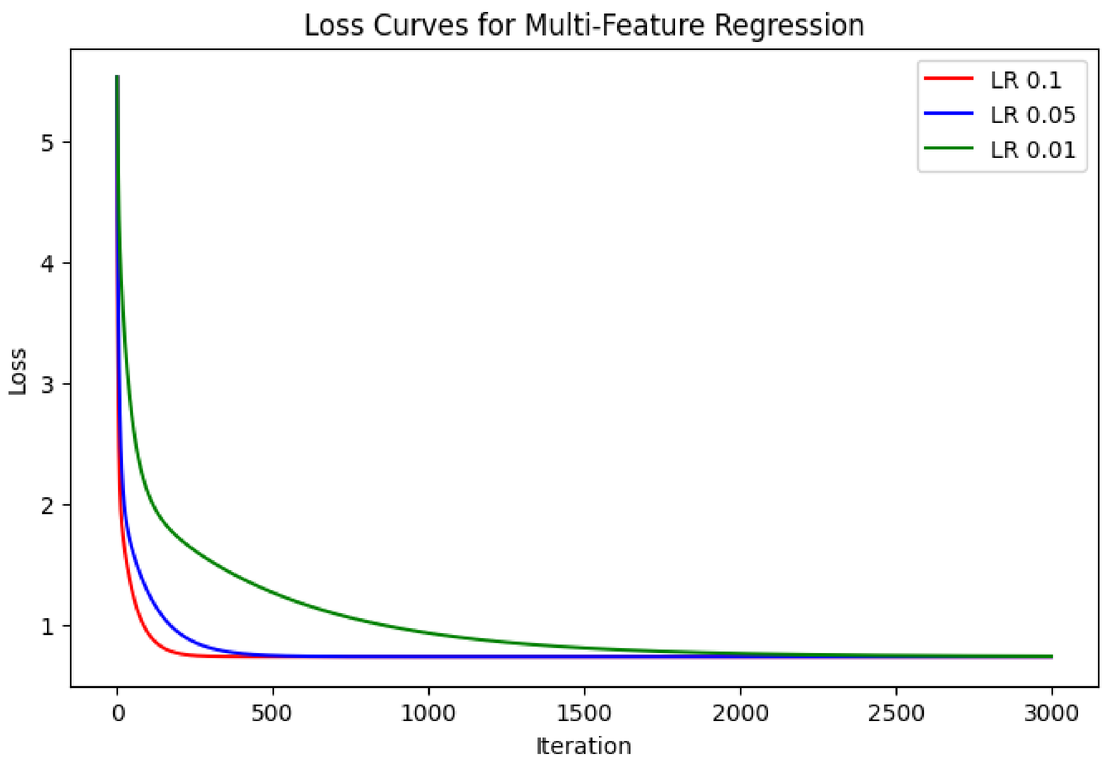
[ ]:
```
# Combine all three explanatory variables into a feature matrix X
X = np.column_stack((x1, x2, x3))

# Dictionary to store multi-feature regression results for each learning rate
multi_results = {}

# num_iterations = 300
# num_iterations = 1000
num_iterations = 3000
# num_iterations = 3500

# Iterate over the learning rates
for lr in learning_rates:
  theta_multi, losses_multi, r2_scores_multi, theta_history_multi =␣
 ↪gradient_descent_multi(X, y, learning_rate=lr, num_iterations=num_iterations)
  multi_results[lr] = {
    'theta': theta_multi,
    'losses': losses_multi,
    'r2_scores': r2_scores_multi,
```

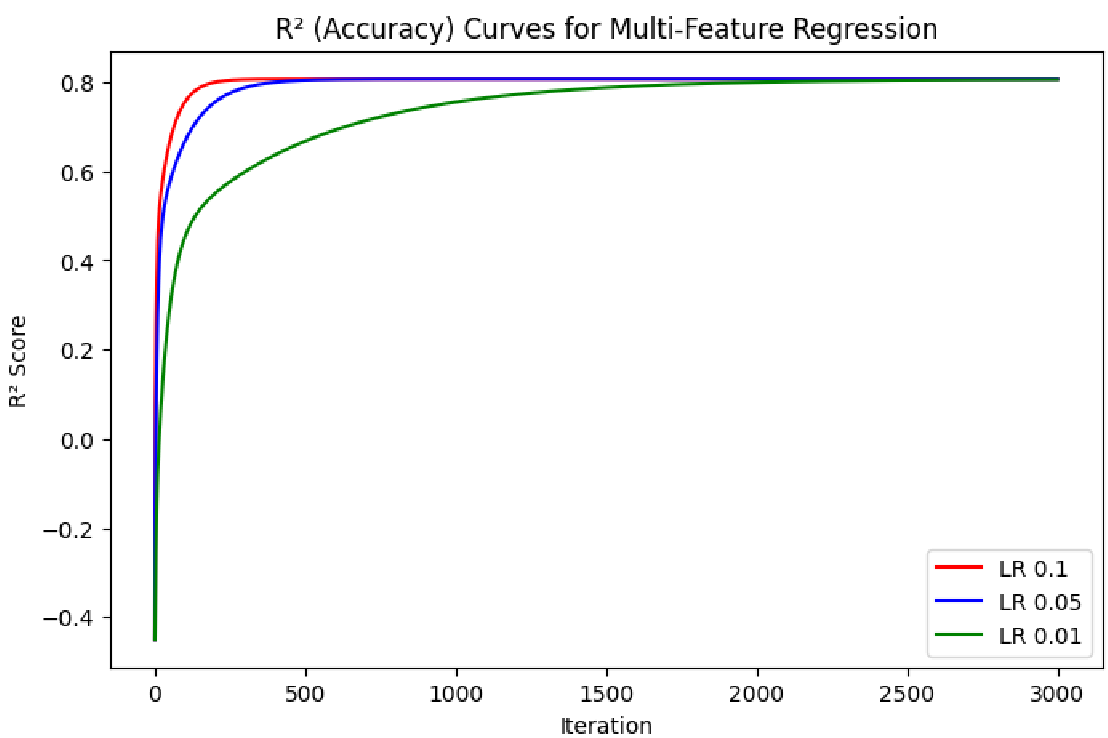
```
    'theta_history': theta_history_multi
  }
  print(f"Learning Rate {lr}: theta = {theta_multi}, final loss =␣
↪{losses_multi[-1]:.6f}")
```

```
Learning Rate 0.1: theta = [ 5.31416717 -2.00371927  0.53256334 -0.26560187],
final loss = 0.738464
Learning Rate 0.05: theta = [ 5.31416557 -2.00371904  0.5325636  -0.26560163],
final loss = 0.738464
Learning Rate 0.01: theta = [ 5.05362928 -1.96691398  0.57561561 -0.22741231],
final loss = 0.742087
```

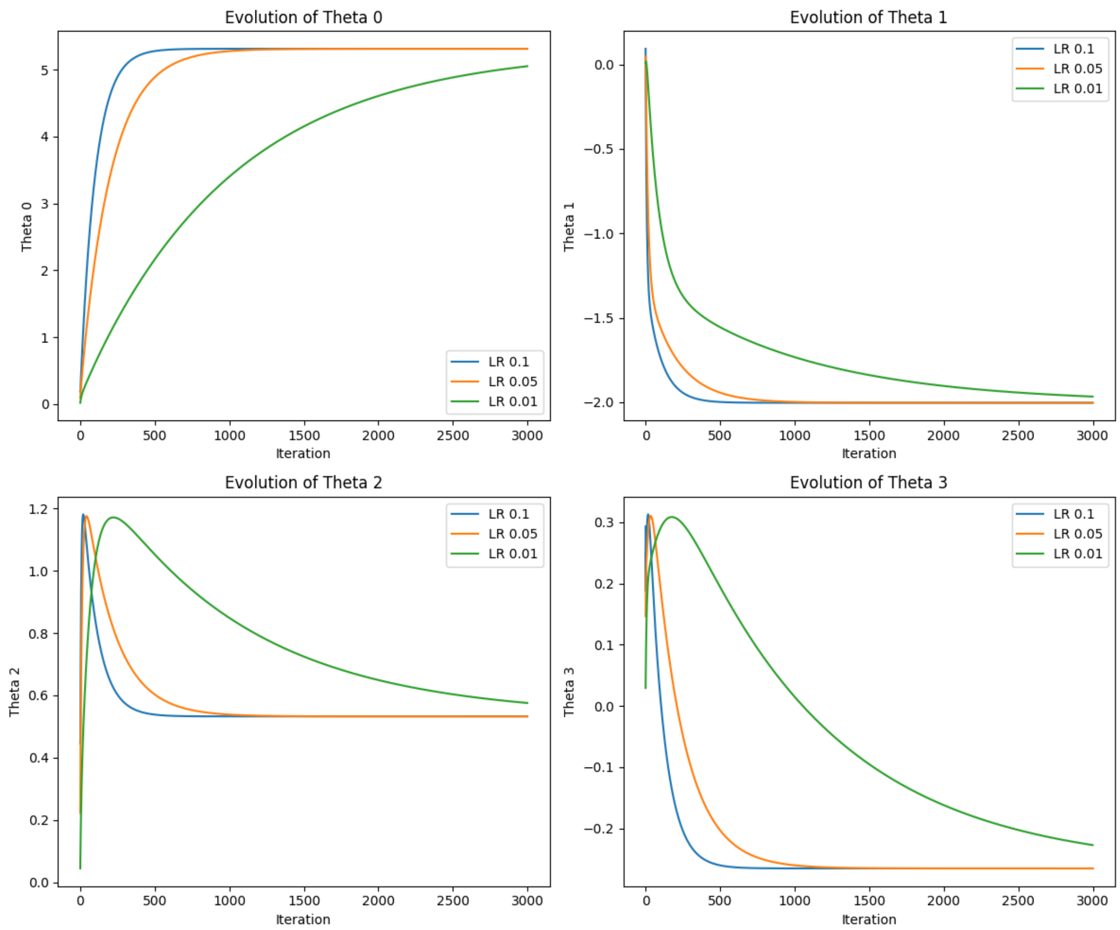
Plot Loss Curves for Multi-Feature Regression

[ ]:
```
# Plotting Loss curve
plt.figure(figsize=(8, 5))
for lr in learning_rates:
    losses = multi_results[lr]['losses']
    plt.plot(range(num_iterations), losses, color=colors[lr], label=f'LR {lr}')
plt.xlabel('Iteration')
plt.ylabel('Loss')
plt.title('Loss Curves for Multi-Feature Regression')
plt.legend()
plt.savefig(f"/content/drive/MyDrive/ecge5105/hw/1/p2_Loss_Curves_plot.png")
plt.show()

# Plotting R² curve
plt.figure(figsize=(8, 5))
for lr in learning_rates:
    r2_scores = multi_results[lr]['r2_scores']
    plt.plot(range(num_iterations), r2_scores, color=colors[lr], label=f'LR␣
↪{lr}')
plt.xlabel('Iteration')
plt.ylabel('R² Score')
plt.title('R² (Accuracy) Curves for Multi-Feature Regression')
plt.legend()
plt.savefig(f"/content/drive/MyDrive/ecge5105/hw/1/p2_R2_Curves_plot.png")
plt.show()
```


Loss Curves for Multi-Feature Regression
LR 0.1
LR 0.05
LR 0.01
Loss
Iteration
0
500
1000
1500
2000
2500
3000
1
2
3
4
5


R² (Accuracy) Curves for Multi-Feature Regression
LR 0.1
LR 0.05
LR 0.01
R² Score
Iteration
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
0
500
1000
1500
2000
2500
3000

[ ]:
```
plt.figure(figsize=(12, 10))
# theta0, theta1, theta2, theta3
for i in range(4):
  plt.subplot(2, 2, i+1)
  for lr in learning_rates:
    theta_history = np.array(multi_results[lr]['theta_history'])  # shape:␣
↪(num_iterations, 4)
    plt.plot(range(num_iterations), theta_history[:, i], label=f'LR {lr}')
  plt.xlabel("Iteration")
  plt.ylabel(f'Theta {i}')
  plt.title(f'Evolution of Theta {i}')
  plt.legend()
plt.tight_layout()
plt.savefig(f"/content/drive/MyDrive/ecge5105/hw/1/p2_Theta_Evolution.png")
plt.show()
```



Prediction with the Best Multi-Feature Model

[ ]:
```python
# Determine the best learning rate based on the lowest final loss
best_lr = min(learning_rates, key=lambda lr: multi_results[lr]['losses'][-1])
best_theta = multi_results[best_lr]['theta']
print(f"Best learning rate for multi-feature regression: {best_lr}")
print(f"Final model coefficients: theta0 = {best_theta[0]:.4f}, theta1 = {best_theta[1]:.4f}, theta2 = {best_theta[2]:.4f}, theta3 = {best_theta[3]:.4f}")

# New input data for prediction: (1,1,1), (2,0,4), (3,2,1)
new_data = np.array([
    [1, 1, 1],
    [2, 0, 4],
    [3, 2, 1]
])

new_data_with_intercept = np.column_stack((np.ones(new_data.shape[0]), new_data))
predictions = np.dot(new_data_with_intercept, best_theta)
for i, pred in enumerate(predictions):
  print(f"Input {new_data[i]}: predicted y = {pred:.4f}")
```

```
Best learning rate for multi-feature regression: 0.1
Final model coefficients: theta0 = 5.3142, theta1 = -2.0037, theta2 = 0.5326,
theta3 = -0.2656
Input [1 1 1]: predicted y = 3.5774
Input [2 0 4]: predicted y = 0.2443
Input [3 2 1]: predicted y = 0.1025
```

1.2.1 Problem 2 Analysis and Conclusions

Problem 2 Analysis:

1) Final Loss Comparison:

   - Learning Rate 0.1 => final loss ~0.738464
   - Learning Rate 0.05 => final loss ~0.738464
   - Learning Rate 0.01 => final loss ~0.742087 The best final loss is ~0.738464 (for LR=0.1 or LR=0.05).

2) Model Coefficients:

   For the best learning rate (chosen as 0.1), the final parameters are: theta0 ~ 5.3142 theta1 ~ -2.0037 theta2 ~ 0.5326 theta3 ~ -0.2656

3) Effect of Learning Rate:

   - LR=0.1 or 0.05 converges faster and reaches the lowest final loss quickly.
   - LR=0.01 converges more slowly and ends with a slightly higher loss (~0.7421).

4) R^2 and Theta Evolution:

   - R^2 curves show that the model can explain ~80% of the variance in y once converged.

- Theta evolution plots show each parameter stabilizes after sufficient iterations, with higher learning rates converging in fewer steps.

5) Predictions on new data:

    - For (1,1,1): y_pred ~ 3.5774
    - For (2,0,4): y_pred ~ 0.2443
    - For (3,2,1): y_pred ~ 0.1025

Conclusion for Problem 2:

- Including all three features yields a lower loss (~0.738) than any single feature alone.
- A learning rate of 0.1 or 0.05 is optimal, balancing speed of convergence and stability.
- The final model effectively predicts new inputs, confirming the benefit of multi-feature regression.