Assignment2
MVE155 Statistical inference

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1. Question a

As figure 1 shown, the histogram of these 210 data appears to fitted to the Gamma plausible distribution.

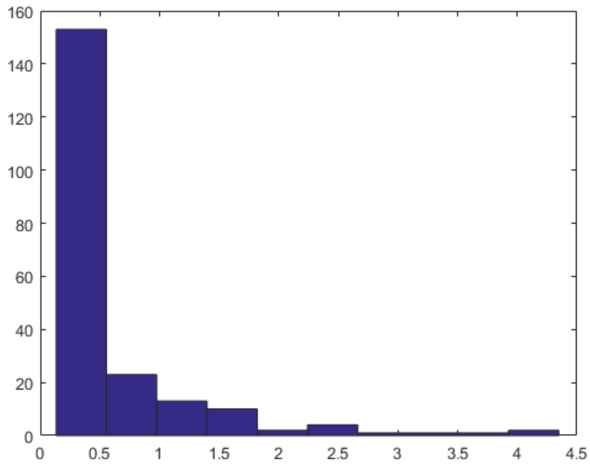

Figure 1. Histogram of data

2. Question b

Now, we need use the method of moments to fit the parameters of gamma distribution. The first two moments of the gamma distribution are:

$$\mu_1 = \frac{\alpha}{\lambda}$$

$$\mu_2 = \frac{\alpha(\alpha+1)}{\lambda^2}$$

We will figure out two parameters according to moments:

$$\hat{\lambda} = \frac{\overline{X}}{\hat{\sigma}^2} \qquad \hat{\alpha} = \frac{\overline{X}^2}{\hat{\sigma}^2}$$

In this case, I get the result: $\hat{\alpha} = 0.7954$, $\hat{\lambda} = 1.3125$.

3. Question c

Apart from method of moments, we could also use maximum likelihood method to fit parameters. Firstly, we could get likelihood function in terms of gamma distribution:

$$L(\alpha,\lambda)=\prod_{i=1}^{n}\frac{1}{\Gamma(\alpha)}\lambda^{\alpha}x_i^{\alpha-1}e^{-\lambda x_i}=\frac{\lambda^{n\alpha}}{\Gamma^n(\alpha)}(x_1\cdots x_n)^{\alpha-1}e^{-\lambda(x_1+\ldots+x_n)}$$

Then, we set two derivatives equal to zero:

$$\frac{\partial}{\partial\alpha}\log L(\alpha,\lambda)=n\log(\lambda)-n\frac{\Gamma'(\alpha)}{\Gamma(\alpha)}+\log t_2,$$
$$\frac{\partial}{\partial\lambda}\log L(\alpha,\lambda)=\frac{n\alpha}{\lambda}-t_1.$$

$$(t_1 = x_1 + \cdots + x_n \text{ and } t_2 = x_1 * \ldots * x_n)$$

In final, we could figure out parameters according to formulas as following:

$$\log(\hat{\alpha}/\bar{x})=-\frac{1}{n}\log t_2+\Gamma'(\hat{\alpha})/\Gamma(\hat{\alpha})$$

$$\hat{\lambda}=\hat{\alpha}/\bar{x}$$

In Matlab, a build-in function `gamfit()` can be used to find parameters directly, but we can also use `fzero()` function to figure out what we want. I use the command:

```
f_para1 = log(t2)/n - log(X_bar);
fun1 = @(x) log(x) + f_para1 - psi(x);
alpha2 = fzero(fun1, [0.5 3]);
lamda2 = alpha2 / X_bar;
```

to find parameters, and result is $\widehat{\alpha}=1.5954$, $\hat{\lambda}=2.6327$ which is the same as the results from `gamfit()`. The results got from maximum likelihood method are both larger than using method of moments.

4. Question d

Next, we could use the parameters from two different methods to draw gamma distribution via `gmapdf()` command in Matlab. However, there is an important different between Matlab and our theoretic expression. In Matlab, gamma function use $1/\lambda$ instead of λ. Therefore, we need set `gampdf()` and `gamrnd()` (used in bootstrap) input as $1/\lambda$ in order to get what we want.

Two gamma densities plot on top of the histogram is shown as figure 2. Two gamma densities are both reasonable, and maximum likelihood method looks fit better.

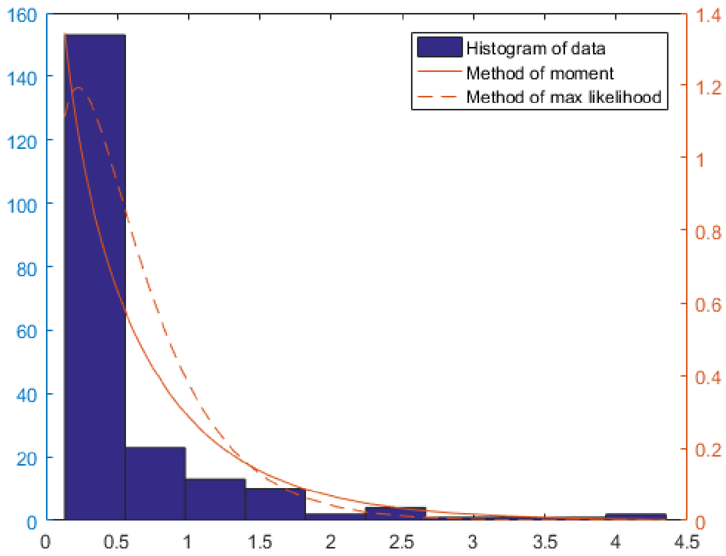

Figure 2. Two gamma densities plot on histogram

5. Question e and f

Now, we will use bootstrap method to estimate sampling distributions and the standard errors of the parameters by method of moments. I simulate B = 1000 samples of size n = 210 from Gamma (0.7954, 1.3125). According to two formulas:

$$\bar{\alpha} = \frac{1}{B}\sum_{j=1}^{B}\hat{\alpha}_j$$

$$s_{\hat{\alpha}}^2 = \frac{1}{B-1}\sum_{j=1}^{B}(\hat{\alpha}_j - \bar{\alpha})^2$$

Then, I can find $\bar{\alpha}_1 = 0.8158$, $s_{\hat{\alpha}_1} = 0.013$, and similar formula can be used to lambda, then $\overline{\lambda_1} = 1.3577$, $s_{\hat{\lambda}_1} = 0.047$.

In terms of using maximum likelihood method by bootstrap, I simulate same samples and size from Gamma (1.5954, 2.6327). Then, I find $\bar{\alpha}_2 = 1.6138$, $s_{\hat{\alpha}_2} = 0.021$, and $\overline{\lambda_2} = 2.676$, $s_{\hat{\lambda}_2} = 0.077$.

Firstly, the results by bootstrap are a little larger than input parameters both in method of moments and maximum likelihood. Then, we could find the standard errors in maximum likelihood are larger than, nearly twice as using method of moments.

Figure 3 and 4 shows the sampling distribution of two parameters in two method by bootstrap.

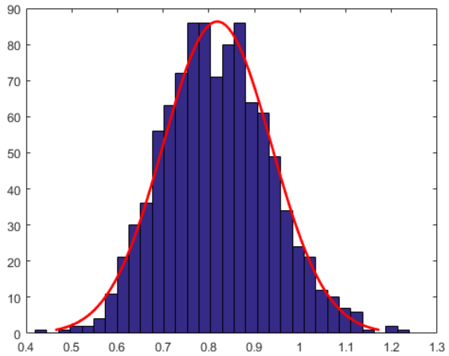

Figure 3.1 Alpha sampling distribution in method of moments by bootstrap

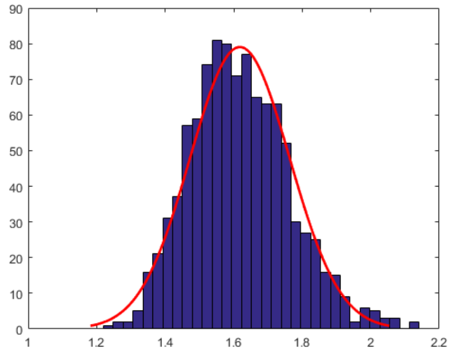

Figure 3.2 Alpha sampling distribution in maximum likelihood by bootstrap

Figure 3. Alpha sampling distribution in two method by bootstrap

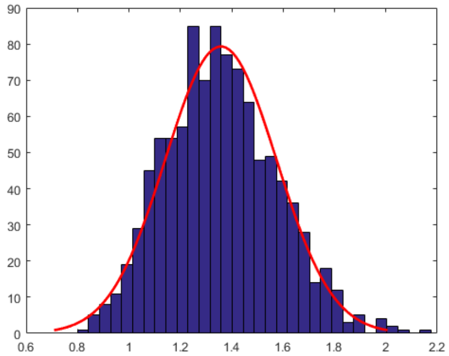

Figure 3.1 Lambda sampling distribution in method of moments by bootstrap

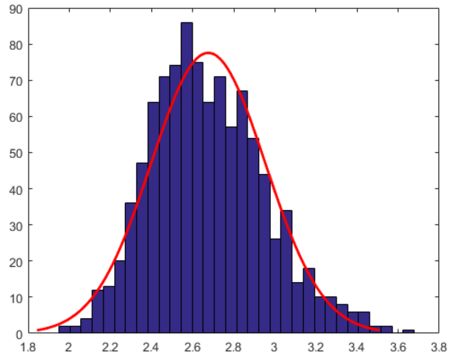

Figure 3.2 Lambda sampling distribution in maximum likelihood by bootstrap

Figure 4. Lambda sampling distribution in two method by bootstrap

6. Question g

To calculate the 95% confidence interval of parameters in maximum likelihood method, we should find quantile of parameters on 2.5% and 97.5% at first. Then we compute CI for α as $(2\hat{\alpha} - c_2,\ 2\hat{\alpha} - c_1)$ and for λ as $(2\hat{\lambda} - c_2,\ 2\hat{\lambda} - c_1)$. The result for α is (1.2898, 1.8603) and for λ is (2.0468, 3.1666).