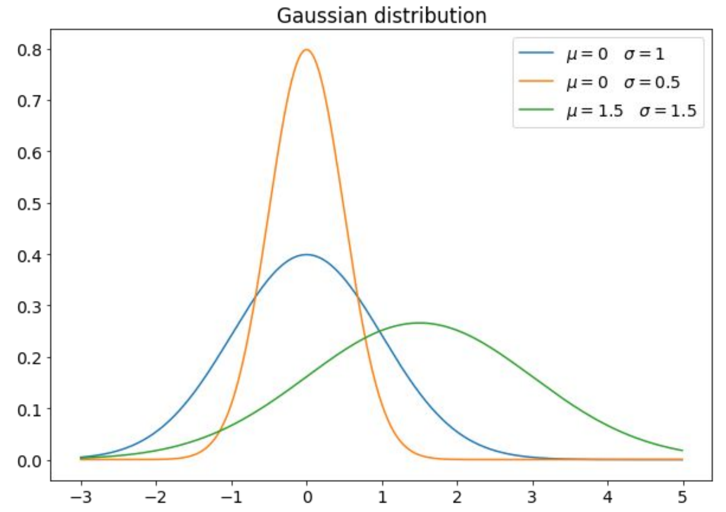


Common Distributions Cheat Sheet

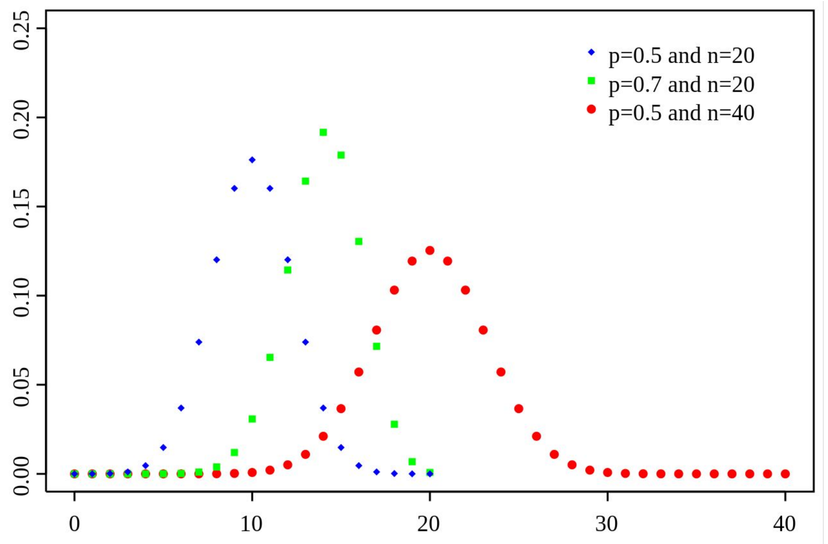
V2020.10.15

(Dr Yan Xu)

Discrete: **Bernoulli** (*p*)

➔ Variable space: {0, 1}
➔ Pr(***X***=1) = ***p*** = 1 - Pr(***X***=0)
➔ hyperparameters: ***p***
➔ Mean: ***p***
➔ Variance: ***p* * (1 - *p*)**

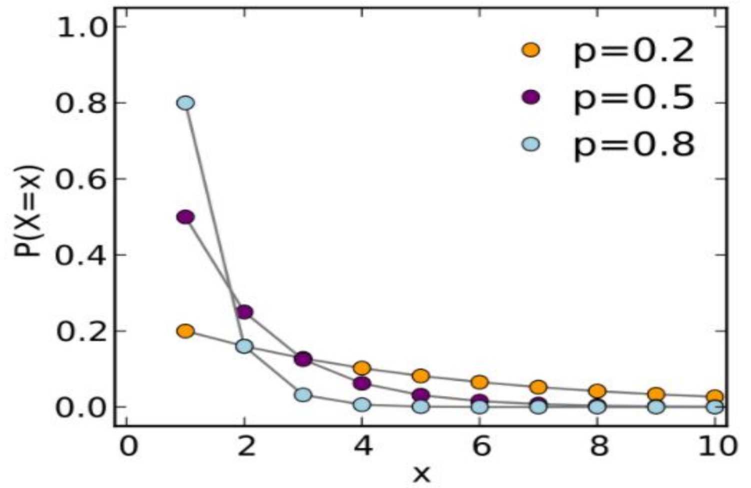
Discrete: **Binomial *B(n, p)***



➔ Variable space: ***k*** = {1, 2, 3, … n}
➔ Pmf: $\binom{n}{k} p^k q^{n-k}$
➔ Hyperparameters: ***n, p***
➔ Mean: ***n*p***
➔ Variance: ***n * p * (1 - p)***

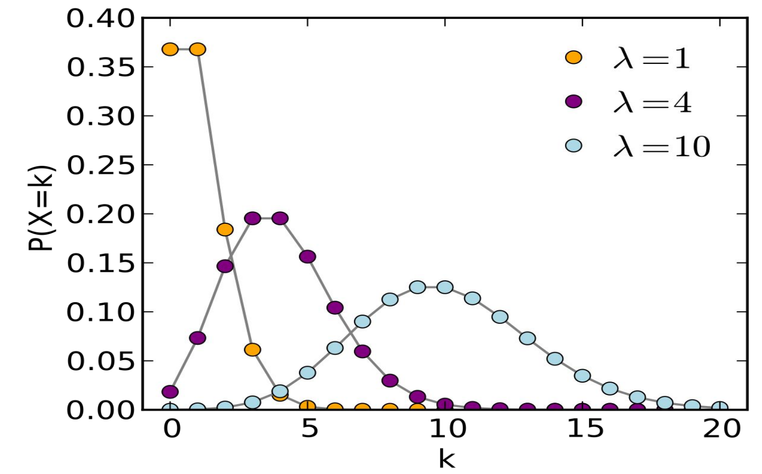
1: If n is large enough, => ***Normal(np, np(1 - p))***
2: If n is large enough and p small enough, => ***Poisson (λ = np)***, e.g. n ≥ 20 and p ≤ 0.05
3: **Beta distribution** provides conjugate prior for **p**.

Discrete: **Geometric** (*p*)



➔ the number of Bernoulli trials needed to get one success
➔ Variable space: {1, 2, 3, … }
➔ Pmf: $(1-p)^{k-1}p$
➔ Hyperparameters: ***p***
➔ Mean: **1.0 / *p***
➔ Variance: ***(1 - p)* / *p*²**

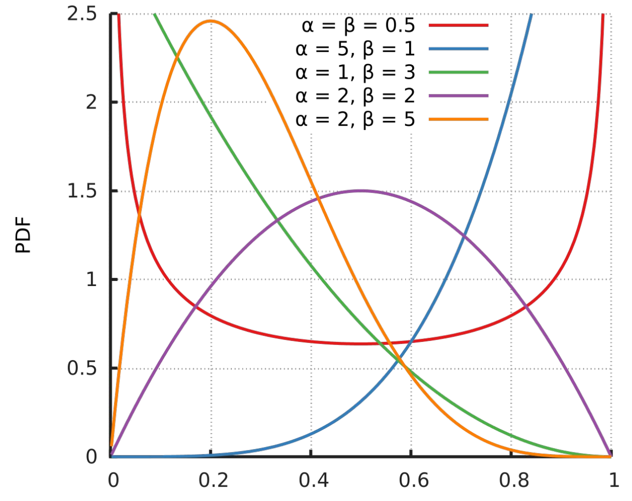
Discrete: **Poisson** (*λ*)



➔ the number of independent events occurring in a fixed interval
➔ Variable space: {0, 1, 2, … }
➔ Pmf: $\frac{\lambda^k e^{-\lambda}}{k!}$
➔ Hyperparameters: **λ**
➔ Mean: **λ**
➔ Variance: **λ**

1: If **λ** is large enough, => ***Normal*(λ, λ)**
2: inter-arrival times **=> *Exponential*(1/λ)**
3: **Gamma distribution** provides conjugate prior for λ.

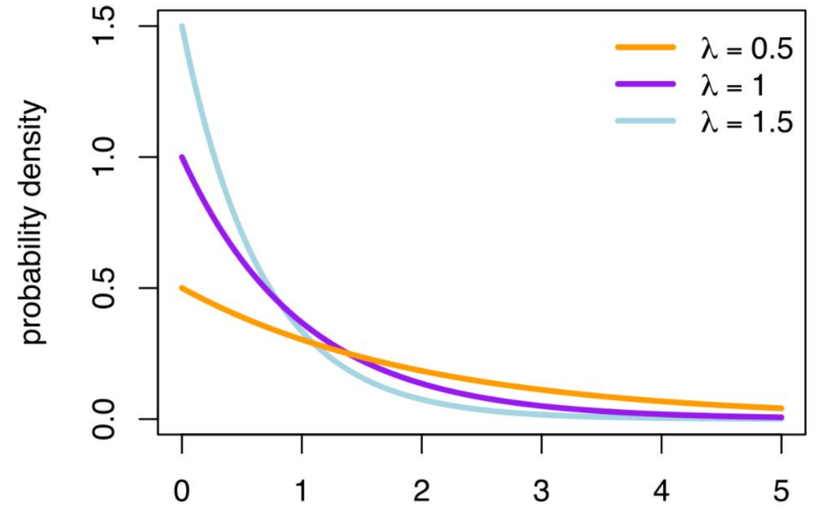
Continuous: **Beta** (*α, β*)



➔ Variable space: [0, 1]
➔ Pdf: $\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha,\beta)}$
➔ Hyperparameters: ***α, β***
➔ Mean: **α / (α+ *β*)**
➔ Variance: **α * *β* / {(α+ *β*)²*(α+ *β*+1)}**

The generalization of Beta to multiple variables is called a **Dirichlet distribution**.

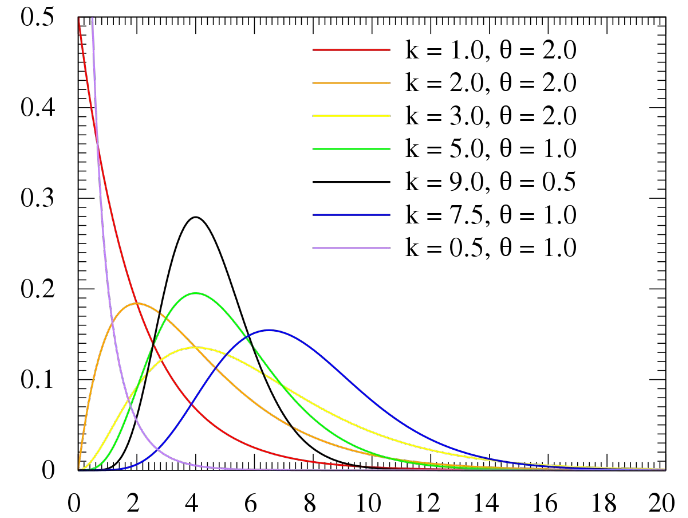
Continuous: **Exponential** (*λ*)



➔ Variable space: [0, ∞)
➔ Pdf: $\lambda e^{-\lambda x}$
➔ Hyperparameters: ***λ***
➔ Mean: **1.0 / *λ***
➔ Variance: **1.0 / *λ*²**
➔ Memorylessness: $\Pr(T > s + t \mid T > s) = \Pr(T > t)$

1: Exponential family: Exponential, Gamma, Normal, Chi-square, Beta, Dirichlet, Bernoulli, Possion, Geometric, etc.
2: Exponential families have conjugate priors, an important property in Bayesian statistics.

Continuous: **Gamma** (*α, β*)

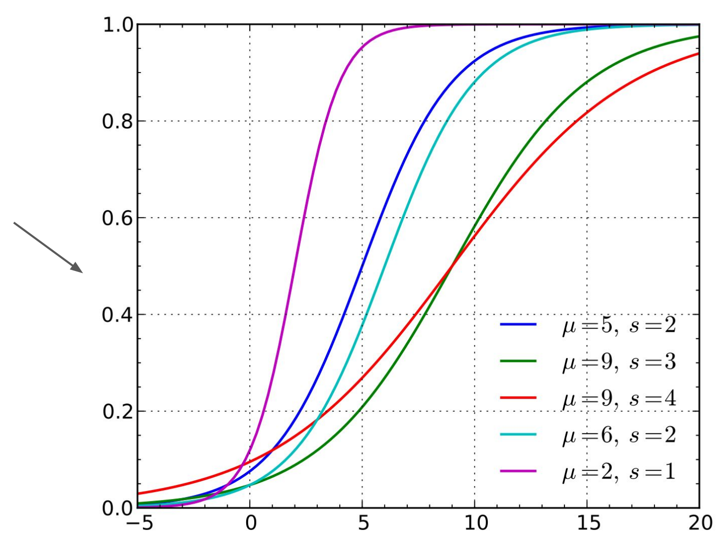


➔ Variable space: [0, ∞)
➔ Pdf: $f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$
➔ Hyperparameters: **α** (shape), **β** (inverse scale)
➔ Mean: **α / β**
➔ Variance: **α / β²**

1: Exponential distribution (***α=1***), Erlang distribution (***α=INT***), and Chi-squared distribution (only for hypothesis testing) are special cases of the gamma distribution.
2: If k is large enough, => ***Normal*(α/β, α/β²)**
3: gamma distribution is widely used as a conjugate prior in Bayesian statistics

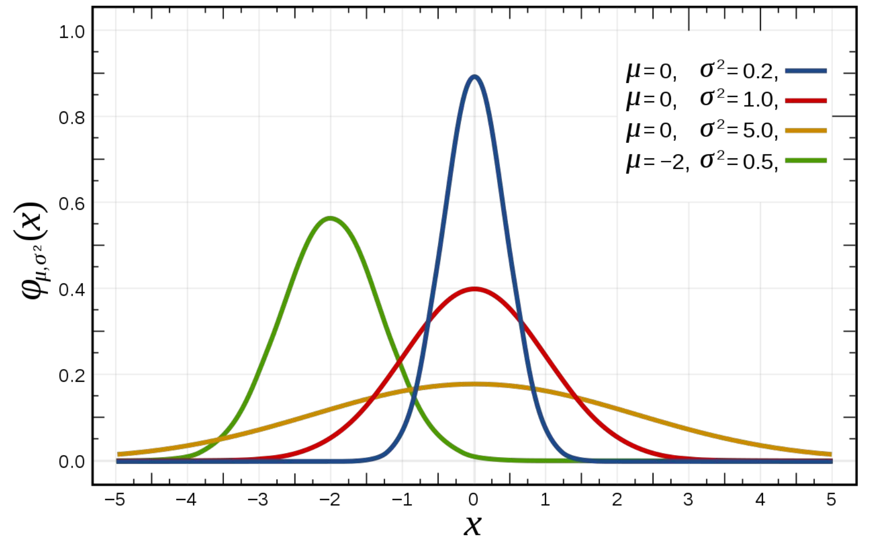
Continuous: **Logistic** (*μ, s*)

CDF



➔ Variable space: (-∞, ∞)
➔ Pdf: $\frac{e^{-(x-\mu)/s}}{s\left(1+e^{-(x-\mu)/s}\right)^2}$
➔ Cdf: $\frac{1}{1+e^{-(x-\mu)/s}}$
➔ Applications: Logistic regression

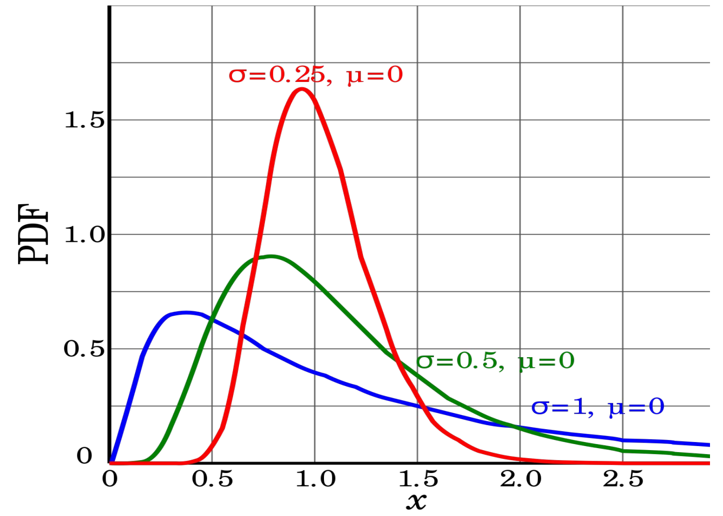
Continuous: **Normal** (*μ,σ²*) (or Gaussian)



➔ Variable space: (-∞, ∞)
➔ Pdf: $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$
➔ Mean: **μ**
➔ Variance: ***σ*2**
➔ Sample variance: **n / (n - 1) * *σ*2**

1: Central limit theorem: the sum of many random variables will have an approximately normal distribution. It is the key reason why Normal is widely used in various observations.
2: If X, Y are independent Normal with means u_1, u_2 and standard deviation σ_1, σ_2, then X + Y will be normally distributed, with mean ($u_1 + u_2$) and variance ($\sigma_1^2 + \sigma_2^2$)
3: Bayesian analysis of normally distributed data is complicated

Continuous: **Log-normal** (*μ,σ²*)



➔ Variable space: (0, ∞)
➔ Pdf: $\frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x-\mu)^2}{2\sigma^2}\right)$
➔ if X has a normal distribution, then the exponential function of exp(X), has a log-normal distribution.