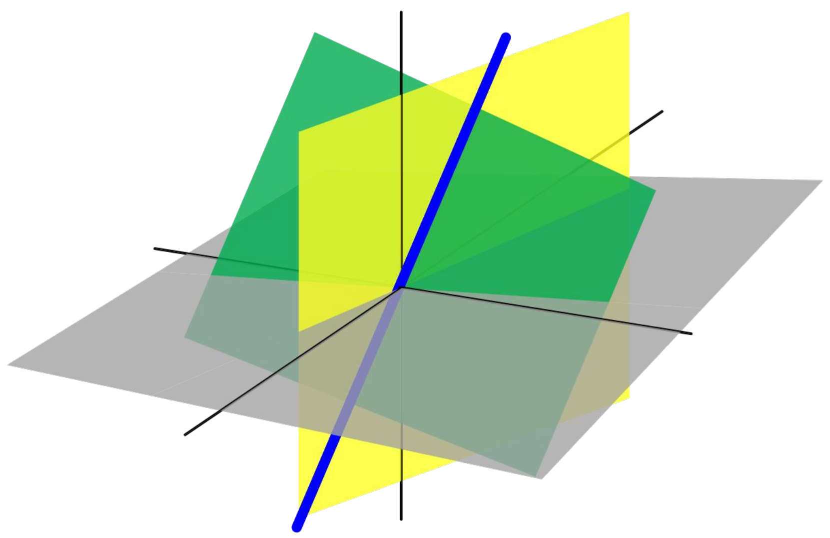

Linear Algebra Cheat Sheet

V2020.10.9

(Dr Yan Xu)

Matrix basic operation

- ➔ Scaling
- ➔ Matrix Addition (Subtraction)
- ➔ Matrix Multiplication

Dot products

$$\vec{u}\cdot\vec{v} = [u_1 \quad u_2]\cdot\begin{bmatrix}v_1\\ v_2\end{bmatrix} = u_1v_1 + u_2v_2$$

- ➔ Dot product measures how much two vectors move in the same direction

$$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$$
$$(c\vec{u})\cdot\vec{v} = c(\vec{v}\cdot\vec{u})$$
$$(\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w}$$
$$|\vec{u}\cdot\vec{v}| \le ||\vec{u}||\;||\vec{v}||$$
$$\vec{u}\cdot\vec{v} = ||\vec{u}||\,||\vec{v}||\cos\theta$$

Linear subspace

- ➔ Basis vectors, Span
- ➔ Linear Independence
- ➔ Closed under scaling and addition

Transformations

- ➔ Transformations are essentially functions on matrix (vector, subspace)
  $T:\ \mathbb{R}\to\mathbb{R}$
- ➔ Linear Transformation as Matrix
- ➔ Reflection as Matrix
- ➔ Rotations (counterclockwise) as Matrix

$$\text{Rot}_\theta = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

- ➔ Rotations (3D) as Matrix

$$\text{Rot}_{\theta \text{ around } x} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix}$$

$$\text{Rot}_{\theta \text{ around } y} = \begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$$

$$\text{Rot}_{\theta \text{ around } z} = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

Projections

- ➔ projection of a vector onto a line:

$$\text{Proj}_L(\vec{v}) = \left(\frac{\vec{v}\cdot\vec{x}}{\vec{x}\cdot\vec{x}}\right)\vec{x}$$

- ➔ projection of a vector onto a subspace (note: A may not be a square):

$$\text{Proj}_V\vec{x} = A(A^TA)^{-1}A^T\vec{x}$$

- ➔ The least square solution for **A**x = b:

$$A^TA\vec{x}^* = A^T\vec{b}$$

- ➔ Both projections are linear

Inverse $\mathbf{A}^{-1}$

$\mathbf{A}^{-1}$ exists,

- ➔ A^{-1} is unique
- ➔ Det(A) != 0
- ➔ Linearly independent row/col vectors
- ➔ Ax = O means x = O
- ➔ If A is sparse, A^{-1} could be dense

Det(A) = 0,

- ➔ $\mathbf{A}^{-1}$ does not exists

Transposes $\mathbf{A}^T$

- ➔ $(A+B)^T=B^T+A^T$
- ➔ $(A^T)^{-1} = (A^{-1})^T$
- ➔ $(AB)^T=B^TA^T$

Determinants

- ➔ $\text{Det}(A) = |A| = \begin{vmatrix}a & b\\ c & d\end{vmatrix} = ad - bc$
- ➔ If the determinant is 0, the matrix is not invertible, so there is no inverse.
- ➔ Switch any row in a matrix will change the determinant to negative
- ➔ Row operations don't change the determinant
- ➔ Geometric: parallelogram area

Eigenvalues and Eigenvectors

- ➔ $T(\vec{v}) = \lambda\vec{v}$
- ➔ If there are ***n*** eigenvectors, they are linearly independent
- ➔ Application: Markov chain, PCA