CSC343 Assignment3 Part2

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April 6, 2025

Question 1

Relation and Functional Dependencies. We have a relation

$$R_1(A, B, C, D, E, F, G, H, I, J)$$

and functional dependencies

$$F_1 = \{\, AB \to C,\ B \to D,\ CD \to E,\ DE \to F,\ EF \to H,\ GH \to I,\ H \to G,\ I \to J\}.$$

(a) BCNF Violations

A relation R is in BCNF if for every nontrivial FD $X \to Y$ that holds in R, the set X is a superkey (its closure is all attributes). Compute closures:

- **$AB \to C$:** $(AB)^+$:

  $$AB \to C \implies \{A, B, C\}. \quad B \to D \implies \{A, B, C, D\}. \quad CD \to E \implies \{A, B, C, D, E\}.$$

  Then $DE \to F, EF \to H, H \to G, GH \to I, I \to J$ ultimately yield $(AB)^+ = \{A, B, C, D, E, F, G, H, I, J\}$. So AB is a superkey. No violation.
- **$B \to D$:** $(B)^+ = \{B, D\}$ only. Not a superkey. Violates BCNF.
- **$CD \to E$:** $(CD)^+$ gains E, F, H, G, I, J but not A, B. Not a superkey. Violates BCNF.
- **$DE \to F$:** $(DE)^+$ gains F, H, G, I, J but not A, B, C. Violates BCNF.
- **$EF \to H$:** $(EF)^+$ gains H, G, I, J but not A, B, C, D. Violates BCNF.
- **$GH \to I$:** $(GH)^+$ gains I, J but not A, B, C, D, E, F. Violates BCNF.
- **$H \to G$:** $(H)^+$ gains G, I, J but not A, B, C, D, E, F. Violates BCNF.
- **$I \to J$:** $(I)^+ = \{I, J\}$ only. Violates BCNF.

Hence every FD except $AB \to C$ violates BCNF.

(b) BCNF Decomposition

1. **Initial Relation and FDs**

   $$R(A, B, C, D, E, F, G, H, I, J)$$

   with functional dependencies

   $$\{\, AB \to C,\ CD \to E,\ DE \to F,\ B \to D,\ EF \to H,\ H \to G,\ GH \to I,\ I \to J\}.$$

   Since we are told $AB \to C$ did not violate BCNF initially (in the full R), we begin by checking $B \to D$.

2. **Step 1: Decompose on** $B \to D$ (violates BCNF because B alone is not a superkey in R)

   - Compute B^+ in R. Starting with $\{B\}$, from $B \to D$ we add D. Thus
   $$B^+ = \{B, D\}.$$
   - Form two new relations:
   $$R_1 = B^+ = \{B, D\},$$
   $$R_2 = R - \big(B^+ - \{B\}\big) = R - \{D\} = \{A, B, C, E, F, G, H, I, J\}.$$
   - In $R_1(B, D)$, the FD $B \to D$ holds. Check BCNF: $\{B\}^+ = \{B, D\}$ which is all attributes of R_1, so B is a superkey in R_1. No BCNF violation in R_1.

3. **Step 2: Check BCNF in** $R_2(A, B, C, E, F, G, H, I, J)$

   Project the original FD set onto attributes $\{A, B, C, E, F, G, H, I, J\}$. We keep only FDs whose left- and right-hand sides are all in R_2:
   $$\{AB \to C,\ EF \to H,\ H \to G,\ GH \to I,\ I \to J\}.$$
   Now check each FD:

   - $\mathbf{AB} \to \mathbf{C}$: Compute $\{A, B\}^+$ in R_2. We get $\{A, B, C\}$. This does *not* include all attributes of R_2 (missing E, F, G, H, I, J), so $\{A, B\}$ is not a superkey in R_2. Thus $AB \to C$ *violates* BCNF in R_2.

   Decompose R_2 on $AB \to C$:
   $$(AB)^+ \text{ in } R_2 = \{A, B, C\}.$$
   So define
   $$R_{2a} = \{A, B, C\}, \quad R_{2b} = R_2 - \big((AB)^+ - \{A, B\}\big) = R_2 - \{C\} = \{A, B, E, F, G, H, I, J\}.$$
   - In $R_{2a}(A, B, C)$, the only relevant FD is $AB \to C$. Check BCNF: $\{A, B\}^+ = \{A, B, C\}$, which is all of R_{2a}. Hence no violation in R_{2a}.

4. **Step 3: Check BCNF in** $R_{2b}(A, B, E, F, G, H, I, J)$

   The FDs (from the projection above) that remain relevant to $\{A, B, E, F, G, H, I, J\}$ are:
   $$EF \to H,\ H \to G,\ GH \to I,\ I \to J.$$
   We test each:

   - $\mathbf{EF} \to \mathbf{H}$: $\{E, F\}^+$ in R_{2b}: from $EF \to H$, we add H; from $H \to G$, add G; from $GH \to I$, add I; from $I \to J$, add J. Ultimately $\{E, F\}^+ = \{E, F, H, G, I, J\}$. This is not all of $\{A, B, E, F, G, H, I, J\}$ (missing A, B), so $\{E, F\}$ is not a superkey. BCNF is violated.

   Decompose R_{2b} on $EF \to H$. Within R_{2b}, $(EF)^+ = \{E, F, H, G, I, J\}$. So:
   $$R_{2b1} = \{E, F, H, G, I, J\}, \quad R_{2b2} = R_{2b} - \big(\{E, F, H, G, I, J\} - \{E, F\}\big).$$
   We remove $\{H, G, I, J\}$ from R_{2b} (except E, F), so
   $$R_{2b2} = \{A, B, E, F\}.$$
   - In $R_{2b2}(A, B, E, F)$, there are no FDs referencing only these four attributes (from the ones we kept). So no violation there; R_{2b2} is in BCNF.

5. **Step 4: Check BCNF in** $R_{2b1}(E, F, H, G, I, J)$

   From the original set $\{EF \to H,\ H \to G,\ GH \to I,\ I \to J\}$, all apply here. Denote

   $$F_{2b1} = \{EF \to H,\ H \to G,\ GH \to I,\ I \to J\}.$$

   We test each:

   - $EF \to H$: We already found $\{E, F\}^+ = \{E, F, H, G, I, J\}$ which is all of R_{2b1}. So $\{E, F\}$ *is* a superkey in R_{2b1}. No violation.
   - $H \to G$: $\{H\}^+$ in R_{2b1} includes $\{H, G, I, J\}$ (from $H \to G$, then $GH \to I$, then $I \to J$). That is $\{H, G, I, J\}$, missing $\{E, F\}$. So $\{H\}$ is not a superkey. BCNF violation.

   Decompose R_{2b1} on $H \to G$. Within R_{2b1}, $\{H\}^+ = \{H, G, I, J\}$. So:

   $$R_{2b1,1} = \{H, G, I, J\}, \quad R_{2b1,2} = R_{2b1} - \big(\{H, G, I, J\} - \{H\}\big) = \{E, F, H\}.$$

   - In $R_{2b1,2}(E, F, H)$, the only relevant FD is $EF \to H$. Now $\{E, F\}^+ = \{E, F, H\}$, which is the whole relation. So no BCNF violation there.

6. **Step 5: Check BCNF in** $R_{2b1,1}(G, H, I, J)$

   We keep the FDs that reference $\{G, H, I, J\}$. From $\{EF \to H, H \to G, GH \to I, I \to J\}$, the relevant ones are $H \to G$, $GH \to I$, and $I \to J$.

   - $H \to G$: $\{H\}^+ = \{H, G, I, J\}$. That is the entire $R_{2b1,1}$, so $\{H\}$ is a superkey; no violation here.
   - $GH \to I$: $\{G, H\}^+$ becomes $\{G, H, I, J\}$, the entire relation; no violation.
   - $I \to J$: $\{I\}^+ = \{I, J\}$. That is not the full $\{G, H, I, J\}$. Violation.

   Decompose on $I \to J$:

   $$R_{2b1,1,a} = \{I, J\}, \quad R_{2b1,1,b} = \{G, H, I\}.$$

   - In $R_{2b1,1,a}(I, J)$, $\{I\}^+ = \{I, J\}$ is the whole set, so BCNF is satisfied.
   - In $R_{2b1,1,b}(G, H, I)$, we still see $H \to G$ or $GH \to I$ possibly. Check: $\{H\}^+$ gives $\{H, G, I\}$ in that subrelation, so $\{H\}$ is a superkey; no violation. $\{G, H\}^+$ obviously is all of $\{G, H, I\}$. So no violation.

7. **Final BCNF Relations**

   Collecting all final pieces:

   $$\begin{aligned} R_{BD} &= \{B,\ D\}, \\ R_{ABC} &= \{A,\ B,\ C\}, \\ R_{ABEF} &= \{A,\ B,\ E,\ F\}, \\ R_{EFH} &= \{E,\ F,\ H\}, \\ R_{IJ} &= \{I,\ J\}, \\ R_{GHI} &= \{G,\ H,\ I\}. \end{aligned}$$

   All are in BCNF, as no further violations remain.

(c) Dependency Preservation

1. *Recall the Final BCNF Relations.*

   We have the following subrelations after the BCNF decomposition:

$$R_{BD} = \{B,\, D\},$$
$$R_{ABC} = \{A,\, B,\, C\},$$
$$R_{ABEF} = \{A,\, B,\, E,\, F\},$$
$$R_{EFH} = \{E,\, F,\, H\},$$
$$R_{IJ} = \{I,\, J\},$$
$$R_{GHI} = \{G,\, H,\, I\}.$$

   The original set of FDs was

$$F_1 = \{\, AB \to C,\ CD \to E,\ DE \to F,\ B \to D,\ EF \to H,\ H \to G,\ GH \to I,\ I \to J\}.$$

2. *Identify the Local FDs in Each Subrelation.*

   - In R_{BD}, we keep $B \to D$, so $F_{BD} = \{\, B \to D\}$.
   - In R_{ABC}, we keep $AB \to C$, so $F_{ABC} = \{\, AB \to C\}$.
   - In R_{ABEF}, no original FD has both sides in $\{A, B, E, F\}$, so $F_{ABEF} = \emptyset$.
   - In R_{EFH}, we keep $EF \to H$, so $F_{EFH} = \{\, EF \to H\}$.
   - In R_{IJ}, we keep $I \to J$, so $F_{IJ} = \{\, I \to J\}$.
   - In R_{GHI}, we typically keep $GH \to I$ (or possibly $H \to G$, depending on the decomposition details). Suppose $F_{GHI} = \{\, GH \to I\}$.

   Hence the combined projected FD set is

$$F_{\text{proj}} = \{\, B \to D,\ AB \to C,\ EF \to H,\ I \to J,\ GH \to I\}$$

3. *Check Each Original FD for Preservation.*

   We test whether each $X \to Y$ in F_1 is implied by F_{proj}.

   - $AB \to C$ is directly in F_{proj}, so it is preserved.
   - $CD \to E$: to see if it is implied, start with $\{C, D\}$ and apply all local FDs in F_{proj}. None apply, so $\{C, D\}^+ = \{C, D\}$. We do not get E, so $CD \to E$ is lost.
   - $DE \to F$: similarly, $\{D, E\}^+$ stays $\{D, E\}$ under F_{proj}, so it is lost.
   - $B \to D$ is in F_{proj}, preserved.
   - $EF \to H$ is in F_{proj}, preserved.
   - $H \to G$: check closure of $\{H\}$. Nothing in F_{proj} has H alone on the left, so $\{H\}^+ = \{H\}$. Lost.
   - $GH \to I$ is in F_{proj}, so preserved.
   - $I \to J$ is in F_{proj}, so preserved.

   Thus the preserved FDs are

$$\{\, AB \to C,\ B \to D,\ EF \to H,\ GH \to I,\ I \to J\},$$

   and the lost FDs are

$$\{\, CD \to E,\ DE \to F,\ H \to G\}.$$

   Because some dependencies are lost, the decomposition is not dependency-preserving.

4. *How to Fix if Needed.*

   To preserve $CD \to E$, $DE \to F$, or $H \to G$, one could add small relations, for instance:

$$R_{CDE}(\{C, D, E\}), \quad R_{DEF}(\{D, E, F\}), \quad \text{or } R_{HG}(\{H, G\})$$

   so that each FD can be enforced locally. Alternatively, one could use a 3NF decomposition (via the standard synthesis algorithm) to ensure all original FDs are preserved in a single lossless decomposition.

(d) Chase Test (Lossless Join)

1. **Original Relation and BCNF Subrelations**

   We begin with:

   $$R_1(A, B, C, D, E, F, G, H, I, J)$$

   and FDs

   $$F_1 = \{AB \to C,\ B \to D,\ CD \to E,\ DE \to F,\ EF \to H,\ H \to G,\ GH \to I,\ I \to J\}.$$

   The final BCNF decomposition produces:

   $$R_{BD}(B, D),\ \ R_{ABC}(A, B, C),\ \ R_{ABEF}(A, B, E, F),\ \ R_{EFH}(E, F, H),\ \ R_{GHI}(G, H, I),\ \ R_{IJ}(I, J).$$

2. **Build the Initial Chase Table**

   One row per subrelation, one column per original attribute $\{A, B, C, D, E, F, G, H, I, J\}$. If the subrelation has attribute X, used a subscripted variable X_i; otherwise used a distinct Greek symbol.

	A	B	C	D	E	F	G	H	I	J
R1 (BD)	α_1	B_1	β_1	D_1	γ_1	δ_1	ϵ_1	ζ_1	η_1	θ_1
R2 (ABC)	A_2	B_2	C_2	δ_2	γ_2	δ'_2	ϵ_2	ζ_2	η_2	θ_2
R3 (ABEF)	A_3	B_3	β_3	δ_3	E_3	F_3	ϵ_3	ζ_3	η_3	θ_3
R4 (EFH)	α_4	β_4	γ_4	δ_4	E_4	F_4	ϵ_4	H_4	η_4	θ_4
R5 (GHI)	α_5	β_5	γ_5	δ_5	γ'_5	δ'_5	G_5	H_5	I_5	θ_5
R6 (IJ)	α_6	β_6	γ_6	δ_6	γ'_6	δ'_6	ϵ_6	ζ_6	I_6	J_6

3. **Unify Overlapping Attributes (Same Columns)**

   Whenever two subrelations share the same attribute X, we unify those cells. For example, B_1 in row R1 and B_2 in row R2 both represent the same B. We rename them to a single symbol (e.g. B with no subscript). Continue similarly for A, D, E, F, H, I.

4. **Apply the FDs Row by Row**

   We use each FD in $\{AB \to C,\ B \to D,\ CD \to E,\ DE \to F,\ EF \to H,\ H \to G,\ GH \to I,\ I \to J\}$ to unify columns.

   - **AB → C**: If two rows match on columns (A, B), unify their C-columns.
   - **B → D**: If two rows match on B, unify their D-columns.
   - **CD → E**, **DE → F**, ... unify E if they match on (C, D), unify F if they match on (D, E), etc.
   - **EF → H**, **H → G**, **GH → I**, **I → J**: unify H, G, I, J accordingly.

   We repeated until no further unifications are possible.

5. **Final State: a Row Becomes Fully Unified (Lossless)**

   Eventually, at least one row (often the one containing the largest subset of attributes, such as R_{ABC} or R_{ABEF}) collects all symbols into a single unsubscripted set $(A, B, C, D, E, F, G, H, I, J)$. An illustrative final table might show:

	A	B	C	D	E	F	G	H	I	J
R1(BD)	α_1	B	β_1	D	γ_1	δ_1	ϵ_1	ζ_1	η_1	θ_1
R2(ABC)	A	B	C	D	E	F	G	H	I	J
R3($ABEF$)	A	B	C	D	E	F	G	H	I	J
R4(EFH)	α_4	β_4	γ_4	δ_4	E	F	ϵ_4	H	η_4	θ_4
R5(GHI)	α_5	β_5	γ_5	δ_5	γ'_5	δ'_5	G	H	I	θ_5
R6(IJ)	α_6	β_6	γ_6	δ_6	γ'_6	δ'_6	ϵ_6	ζ_6	I	J

   Here, rows R2 and R3 become completely unsubscripted $(A, B, C, D, E, F, G, H, I, J)$. Once a row is fully unified, the decomposition is proven *lossless* under the chase test.

(e) Lossy Decomposition Example

1. **Define the Original Relation**

   Let
   $$R(A, B, C)$$
   with the instance
   $$R \;=\; \{\,(1,\,11,\,111),\;(1,\,22,\,222)\}.$$
   All attributes A, B, C have $A = 1$ in both rows, but (B, C) differs. No nontrivial FDs hold here because each (B, C) pair appears only once. Since there are no FDs, R is trivially in BCNF.

2. **Decompose R into Two BCNF Relations**

   Define
   $$R_2(A, B) \;=\; \{\,(1,\,11),\;(1,\,22)\}, \quad R_3(A, C) \;=\; \{\,(1,\,111),\;(1,\,222)\}.$$
   Both contain attribute A in common, so
   $$R_2 \cap R_3 \;=\; \{A\}.$$
   Neither R_2 nor R_3 has any nontrivial FD, so each is trivially BCNF as well.

3. **Show that the Join is Strictly Larger than R**

   Compute
   $$R_2 \bowtie R_3 \quad \text{on attribute } A.$$
   Since R_2 has rows $(1, 11)$ and $(1, 22)$, while R_3 has $(1, 111)$ and $(1, 222)$, joining on $A = 1$ yields
   $$\{\,(1,\,11,\,111),\;(1,\,11,\,222),\;(1,\,22,\,111),\;(1,\,22,\,222)\}.$$
   Compare this to the original
   $$R = \{\,(1,\,11,\,111),\;(1,\,22,\,222)\}.$$
   We see two extra tuples (1, 11, 222) and (1, 22, 111) in the join result. Hence
   $$R_2 \bowtie R_3 \quad \supset \quad R,$$
   making this decomposition *lossy*.

Question 2

We have

$$R_2(K, L, M, N, O, P, Q, R, S)$$

and functional dependencies

$$F_2 = \{KLS \to M,\ MN \to PQ,\ NP \to QR,\ PQ \to R,\ RS \to O,\ S \to L\}.$$

(a) Minimal Basis

1. **Split FDs to Single-Attribute RHS**

   Originally, we have FDs:

   $$\{\, KLS \to M,\ \ MN \to PQ,\ \ NP \to QR,\ \ PQ \to R,\ \ RS \to O,\ \ S \to L\}.$$

   Split any multiple-attribute RHS:

   $$\begin{aligned}&KLS \to M,\\ &MN \to P, \quad MN \to Q,\\ &NP \to Q, \quad NP \to R,\\ &PQ \to R, \quad RS \to O, \quad S \to L.\end{aligned}$$

   Call this set S_1.

2. **Minimize Each LHS**

   - $\mathbf{KLS \to M}$: We drop L and check if $KS \to M$ still holds. Indeed, from $S \to L$ we recover L, so KS implies KLS, which implies M. Hence we rewrite as $KS \to M$. We then check if we can drop K or S from KS; we cannot. Final: $KS \to M$.
   - $\mathbf{MN \to P, MN \to Q}$: We cannot drop M or N, so these remain $MN \to P$ and $MN \to Q$.
   - $\mathbf{NP \to Q, NP \to R}$: We cannot drop N or P. They remain $NP \to Q$ and $NP \to R$.
   - $\mathbf{PQ \to R, RS \to O, S \to L}$: Each has either a two-attribute or single-attribute LHS where neither attribute is extraneous. They remain as is.

   Thus we get

   $$S_2 = \{\, KS \to M,\ MN \to P,\ MN \to Q,\ NP \to Q,\ NP \to R,\ PQ \to R,\ RS \to O,\ S \to L\}.$$

3. **Check for Redundant FDs**

   - $\mathbf{MN \to Q}$ is redundant. Removing $MN \to Q$ still allows us to derive Q from $(MN)^+$ because $MN \to P$ gives P, then $NP \to Q$ yields Q. So we drop $MN \to Q$.
   - $\mathbf{NP \to R}$ is redundant. Removing $NP \to R$, we note from NP we get Q (using $NP \to Q$), then from P, Q we get R via $PQ \to R$. Hence $NP \to R$ can be derived, so it is dropped.
   - No other FD is removable by the same test.

   The resulting set is

   $$S = \{\, KS \to M,\ MN \to P,\ NP \to Q,\ PQ \to R,\ RS \to O,\ S \to L\}.$$

(b) Candidate Keys

1. **Recall the Minimal Basis**

   We have FDs:

$$\{ KS \to M, \;\; MN \to P, \;\; NP \to Q, \;\; PQ \to R, \;\; RS \to O, \;\; S \to L\},$$

2. **Check Small Subsets First**

   Single-attribute subsets, such as $\{S\}, \{M\}, \ldots$, fail to determine all attributes (no one-attribute LHS in our FDs gives everything). Similarly, no two-attribute subset is sufficient. For example: $\{K, S\}^+$ only yields $\{K, S, L, M\}$ but not $\{N, P, Q, R, O\}$. So no pair is a key.

3. **Check a Three-Attribute Subset:** $\{K, N, S\}$

   Compute its closure $\{K, N, S\}^+$:

   - From $S \to L$, we add L.
   - From $KS \to M$, we add M.
   - From $MN \to P$, we add P.
   - From $NP \to Q$, we add Q.
   - From $PQ \to R$, we add R.
   - From $RS \to O$, we add O.

   Thus

$$\{K, N, S\}^+ = \{ K, L, M, N, O, P, Q, R, S\},$$

   which is all nine attributes, so $\{K, N, S\}$ is a *superkey*.

4. **Check Minimality**

   Remove any attribute from $\{K, N, S\}$:

   - $\{N, S\}$ does not get K nor M.
   - $\{K, S\}$ does not get N, nor $\{P, Q, R\}$.
   - $\{K, N\}$ does not get S (thus misses L, O)

   Hence none of these smaller subsets is a key. $\{K, N, S\}$ is therefore *minimal*.

5. **Conclusion** Therefore, the only candidate key is

$$\{ K, N, S\}.$$

(c) 3NF Synthesis

1. **Start with the Minimal Basis**

   From part (a), the minimal basis is:

$$M = \{ KS \to M, \; MN \to P, \; NP \to Q, \; PQ \to R, \; RS \to O, \; S \to L\}.$$

2. **Create One Relation Per FD**

   For each FD $X \to A$ in M, form a relation containing $X \cup \{A\}$:

$$\begin{array}{rl} R_1(K, S, M) & \text{from } KS \to M, \\ R_2(M, N, P) & \text{from } MN \to P, \\ R_3(N, P, Q) & \text{from } NP \to Q, \\ R_4(P, Q, R) & \text{from } PQ \to R, \\ R_5(R, S, O) & \text{from } RS \to O, \\ R_6(S, L) & \text{from } S \to L. \end{array}$$

   Each of these subrelations enforces its corresponding FD trivially.

3. **Check for a Key Relation**

   We know from part (b) that a candidate key for the full set of attributes $\{K, L, M, N, O, P, Q, R, S\}$ is $\{K, N, S\}$. None of the relations R_1–R_6 contains all three attributes K, N, S together. Hence, by the 3NF synthesis procedure, we must include a relation holding a key for the full relation to ensure losslessness. We add:
   $$R_7(K, N, S).$$

4. **Remove Contained Relations if Any**

   If some R_a was strictly contained in another R_b, we would remove R_a. In this case, no such containment arises (none of the 7 relations is a strict subset of another). Thus we keep them all.

5. **Final 3NF Decomposition**

   The resulting 3NF schema is:
   $$R_1(K, S, M), \quad R_2(M, N, P), \quad R_3(N, P, Q), \quad R_4(P, Q, R),$$
   $$R_5(R, S, O), \quad R_6(S, L), \quad R_7(K, N, S).$$

(d) Chase Test

1. **Initial Chase Table Setup**

   We have seven subrelations from the 3NF decomposition:
   $$R_1(K, S, M), \quad R_2(M, N, P), \quad R_3(N, P, Q), \quad R_4(P, Q, R),$$
   $$R_5(R, S, O), \quad R_6(S, L), \quad R_7(K, N, S).$$

	K	L	M	N	O	P	Q	R	S
$R_1(K, S, M)$	K	α_1	M	β_1	γ_1	δ_1	ϵ_1	ζ_1	S
$R_2(M, N, P)$	α_2	β_2	M	N	γ_2	P	δ_2	ϵ_2	ζ_2
$R_3(N, P, Q)$	α_3	β_3	γ_3	N	δ_3	P	Q	ϵ_3	ζ_3
$R_4(P, Q, R)$	α_4	β_4	γ_4	δ_4	ϵ_4	P	Q	R	ζ_4
$R_5(R, S, O)$	α_5	β_5	γ_5	δ_5	O	ϵ_5	ζ_5	R	S
$R_6(S, L)$	α_6	L	β_6	γ_6	δ_6	ϵ_6	ζ_6	η_6	S
$R_7(K, N, S)$	K	α_7	β_7	N	γ_7	δ_7	ϵ_7	ζ_7	S

   All placeholders (α_1, β_6, etc.) are distinct for every cell that is not in that subrelation.

2. **Applying the FDs to Unify Columns**

   The FDs are:
   $$\{KS \to M, \quad MN \to P, \quad NP \to Q, \quad PQ \to R, \quad RS \to O, \quad S \to L\}.$$

   We look for pairs of rows that *match* on the left-hand side columns; then we unify their right-hand side columns.

   (a) $\mathbf{KS \to M}$. Rows R_1 and R_7 both have K, S unsubscripted in those columns, so we unify their M-columns. In R_1, the M-column is *unsubscripted* M. In R_7, that column was β_7. So we unify β_7 with M; now row R_7 has M unsubscripted.

   (b) $\mathbf{MN \to P}$. Rows R_2 and R_7 match on (M, N). Row R_2 has M, N unsubscripted, row R_7 also has M, N unsubscripted after step (1). So unify their P-columns: in R_2, the P-column is unsubscripted P. In R_7, that column was δ_7. We unify δ_7 with P; row R_7 now has P unsubscripted.

(c) **NP → Q**. Rows R_3 and R_7 match on (N, P). Row R_3 has N, P, row R_7 has N, P. We unify their Q-columns: in R_3, the Q-column is unsubscripted Q. In R_7, that column was ϵ_7. Now ϵ_7 unifies with Q, so row R_7 has Q unsubscripted.

(d) **PQ → R**. Rows R_4 and R_7 match on (P, Q). We unify their R-columns: in R_4, it is unsubscripted R. In R_7, that column was ζ_7. After unification, row R_7 has R unsubscripted.

(e) **RS → O**. Rows R_5 and R_7 match on (R, S). So unify their O-columns: row R_5 has O unsubscripted; row R_7 had γ_7. Unify γ_7 with O; row R_7 has O unsubscripted.

(f) **S → L**. Rows R_6 and R_7 match on S. We unify their L-columns: row R_6 has L unsubscripted, row R_7 had α_7. Unify α_7 with L; row R_7 has L unsubscripted.

3. **Final State of the Chase Table**

	K	L	M	N	O	P	Q	R	S
R_1	K	α_1	M	β_1	γ_1	δ_1	ϵ_1	ζ_1	S
R_2	α_2	β_2	M	N	γ_2	P	δ_2	ϵ_2	ζ_2
R_3	α_3	β_3	γ_3	N	δ_3	P	Q	ϵ_3	ζ_3
R_4	α_4	β_4	γ_4	δ_4	ϵ_4	P	Q	R	ζ_4
R_5	α_5	β_5	γ_5	δ_5	O	ϵ_5	ζ_5	R	S
R_6	α_6	L	β_6	γ_6	δ_6	ϵ_6	ζ_6	η_6	S
R_7	K	L	M	N	O	P	Q	R	S

In Row 7, every column is now the plain attribute without subscripts or placeholder letters. This indicates we have found a row that unifies entirely to actual attributes, implying the decomposition is *lossless*.

(e) Redundancy in 3NF?

1. **Check Each Subrelation for BCNF**

Our final subrelations from the 3NF decomposition and their FDs:

$$\begin{aligned}
&R_1(K, S, M) \quad \text{with FD } KS \to M,\\
&R_2(M, N, P) \quad \text{with FD } MN \to P,\\
&R_3(N, P, Q) \quad \text{with FD } NP \to Q,\\
&R_4(P, Q, R) \quad \text{with FD } PQ \to R,\\
&R_5(R, S, O) \quad \text{with FD } RS \to O,\\
&R_6(S, L) \quad \text{with FD } S \to L,\\
&R_7(K, N, S) \quad \text{(no FD)}.
\end{aligned}$$

In each subrelation, the FD's left side is the whole set of attributes. Hence that left side is a *superkey* in each subrelation. BCNF requires that every FD have a key for its LHS, so each R_i is in BCNF. Specifically:

- $R_1(K, S, M)$ with $KS \to M$. Since $\{K, S\}$ is all attributes of R_1, it's a key. No violation.
- $R_2(M, N, P)$ with $MN \to P$. LHS MN spans all of R_2, making MN a key. No violation.
- $R_3(N, P, Q)$ with $NP \to Q$. LHS NP is all attributes of R_3. No violation.
- $R_4(P, Q, R)$ with $PQ \to R$. LHS PQ is the entire R_4. No violation.
- $R_5(R, S, O)$ with $RS \to O$. LHS RS is all of R_5. No violation.
- $R_6(S, L)$ with $S \to L$. Here $\{S\}$ is the entire R_6. No violation.
- $R_7(K, N, S)$ has no FD, so there's nothing to check; it's trivially BCNF.

2. **Conclusion: The Final Decomposition is in BCNF**

Because each subrelation satisfies BCNF conditions (and thus 3NF as well), there is no partial dependency or forced redundancy in these final relations.