Using the Physics-Informed Neural Network (PINN) method to solve the highly nonlinear Korteweg-de Vries (KdV) equation

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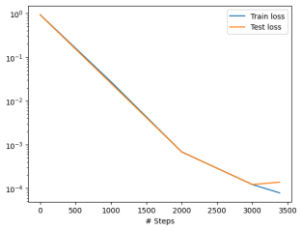
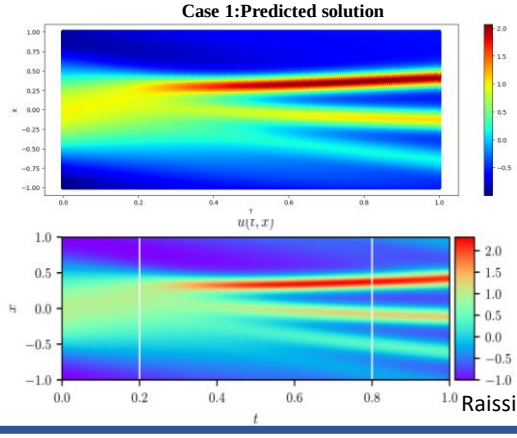
$$u_t + uu_x + 0.025u_{xxx} = 0, x \in [-1,1], t \in [0,1]$$
$$u(t_0, x) = \cos(\pi x),$$
$$u(t,-1) = u(t,1),$$



Best model at step 3385 train loss: 7.90e-05
test loss: 1.39e-04 'train' took 111.37 s

Case 1:Predicted solution



Raissi et al., 2019

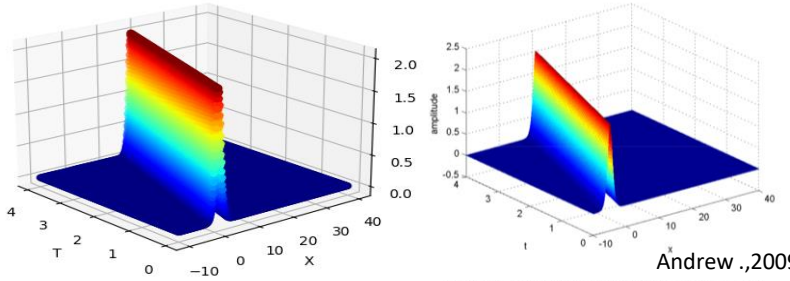
Case 2 shows the one soliton solution on the region $[-10, 40] \times [0, 4]$ with η=1,x_0=5 of KdV Equation.

$$u_t + 6uu_x + u_{xxx} = 0. \quad x \in [-10,40], t \in [0,4]$$
$$u(t,-10) = u(t,40),$$
$$u(t_0,x) = u_0(x),$$
$$u(x,t) = 2\eta^2 \operatorname{sech}^2\eta(x - x_0 - 4\eta^2 t).$$

$$c = 4\eta^2 \quad \xi_0 = x_0,$$

$$u(x,t) = \frac{c}{2}\operatorname{sech}^2\left[\frac{\sqrt{c}}{2}(x - ct - \xi_0)\right].$$

Case 2:Predicted solution



Andrew .,2009

Figure 1: The one soliton solution with $\eta = 1$ and $x_0 = 5.0$.

Best model at step 2349 train loss: 5.93e-07 test loss: 9.48e-07 'train' took 26.26 s

Case 3 shows an example of the two soliton solution on the domain $[-10, 10]\times[0, 1]$ with p = 2, q = 1.5, and t_0 = 0.5.

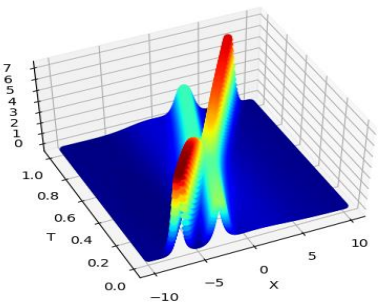
When two solitons collide, they interact elastically. The exact solution for the two soliton equation is given by:

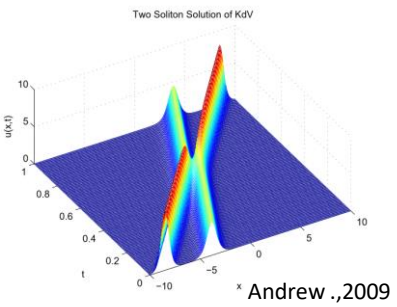
$$u(x,t) = \frac{2(p^2 - q^2)(p^2 + q^2\operatorname{sech}^2\chi(x,t)\sinh^2\theta(x,t))}{(p\cosh\theta(x,t) - q\tanh\chi(x,t)\sinh\theta(x,t))^2}$$

$$\theta(x,t) = px - 4p^3(t - t_0)$$

$$\chi(x,t) = qx - 4q^3(t - t_0).$$

Case 3:Predicted solution

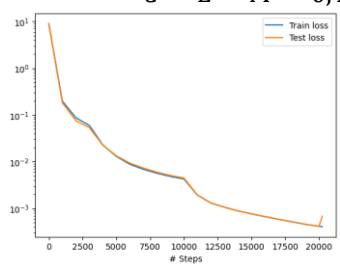




Andrew .,2009

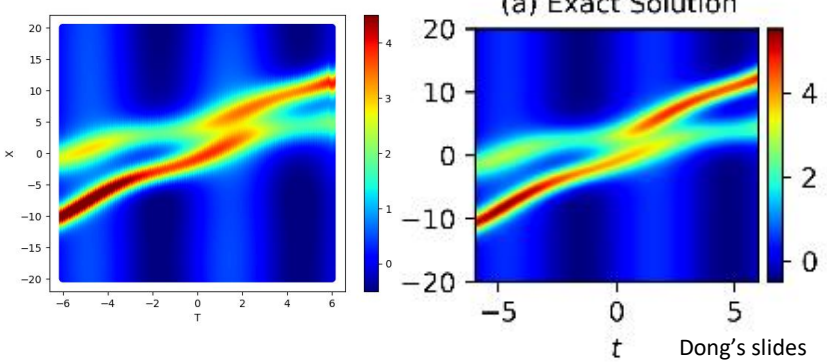
Figure 2: 3D plot of the two soliton solution.

$$u_t + \alpha uu_x + \beta u_{xxx} = R(t), x \in [-20,20], t \in [-6,6]$$
$$u(t_0,x) = u_0(x),$$
$$u(t,-20) = u(t,20)$$
$$a_1 = a_2 = 1, m_1 = 1.3, m_2 = 0.9, \alpha = 1, \beta = 1,$$
$$C = L = M = 0, R(t) = 0.5\cos t$$



Best model at step 20247
train loss: 4.04e-04
test loss: 6.78e-04
'train' took 307.232 s

Case 4:Predicted solution



Dong's slides

All code and reference are available on GitHub at https://github.com/xuyings/KdV_TF_PINN