Proof For Prop 1 and 2 in GraphWave

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1 note for prop 1

1.1 equation 1

$$\sum_{m=1}^{N} \left(\Psi_{ma}^{(s)}\right)^2 = \Psi_{aa}^{(2s)}$$

because left side equal $\mathbf{1_a^T} U g(s\Lambda) g(s\Lambda) U^T \mathbf{1_a} = \Psi_{aa}^{(2s)}$

1.2 equation 2

$$\sum_{m=1}^{N} \Psi_{ma}^{(s)} = 1 \quad s \; for \; any \; value$$

because 1 is the eigenvalue of $\Psi^{(s)}$ for eigenvector $\mathbf{1}$

2 note for prop 2

Ψ^s eigenvector for eigenvalue 1

$$U_{.1} = \frac{1}{\sqrt{N}}$$

so

$$\left|\Psi_{aa}^{(s+1)} - \frac{1}{N}\right| = \left|\sum_{j=1}^{N} e^{-\lambda_j (s+1)} U_{ja}^2 - \frac{1}{N}\right| = \left|\sum_{j=2}^{N} e^{-\lambda_j (s+1)} U_{ja}^2\right| \leq \left|\sum_{j=2}^{N} e^{\lambda_2 s} e^{-\lambda_j s} U_{ja}^2\right| = e^{-\lambda_2} \left|\Psi_{aa}^{(s)} - \frac{1}{N}\right|$$