Strategy Arrangement of Road Cycling Individual Time Trial Based on Topology Optimization Algorithm

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For more efficient training, it is of great significance to know the position-power relationship of riders of different types and genders on different venues for guiding riders' training.

A number of previous studies have used pacing strategies to examine the impact of different venues on rider decision-making. Firstly, according to the test data of different athletes, the OmPD model is used to establish the rider's own power profile. Through force analysis, after discretization, the relationship between power and position is numerically simulated.

In addition, the limitation of anaerobic work ability to decision-making is added. In order to facilitate the calculation, the two-dimensional situation is considered first, and then the three-dimensional situation of the turning is corrected separately. For different regions and courses, after determining the local environmental parameters according to the data, the optimization goal is to take the shortest time after spline interpolation. Anaerobic working capacity and maximum power are the constraints. The optimal numerical solution is carried out by using Method of Moving Asymptotes.

The 2020 Olympic Games and the 2021 UCI World Championship are simulated, and the power-position curves are obtained.

Additional Key Words and Phrases: Numerical optimization; Road cycling; Simulation

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1 INTRODUCTION

Road cycling is a sport that challenges speed and endurance. The environment and scenery along the way are very beautiful, and it also brings warmth and comfort to this extremely difficult sport. The 120-kilometer bicycle race from Paris to Lyon in 1869 was the earliest road bicycle race. In the 1st Olympic Games in 1896, the bicycle project was included in the official competition.

The individual time trial (ITT) is an event in road cycling. There is a 1-2 minute interval between each driver's departure time, usually based on previous race rankings, and the top-ranked driver will start last. Riders are prohibited from helping each other. The player with the least amount of time wins.

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Fig. 1. Four-time U.S. National Road Race Champion Freddie Rodriguez.

2 PREPARATION OF THE MODELS

The primary notations used in this paper are listed in Table 1.

Table 1. Notations

Symbol	Definition
AAW	Available Anaerobic Work
W'	Maximum Available Anaerobic Work
CP	Critical Power
$P_{\max}$	Maximum power output of rider
α	Grade of hill
S	Distance
$\dot{S}$	Instantaneous speed
$\ddot{S}$	Instantaneous acceleration
m_s	Mass of rider-bike system
M_s	Inertia of rider-bike system
g	Gravitational acceleration constant

3 DEFINE THE POWER PROFILES

Background information shows that in addition to time trial specialists, riders participating in time trials also include climbers, sprinters, rouleurs and puncheurs.

Climbers, like Nairo Quintana, can only have minimal muscles. In order to fight against gravity, light weight is definitely the key to victory in hill climbing competitions. From the body, equipment to the body shape, light weight is the priority. Therefore, lightness, slenderness and petiteness are common characteristics of climbers.

Marcel Kittel and Mark Cavendish are definitely the representatives of standard **sprinters**. On a flat road, in order to fight against the wind resistance, a stronger force is needed to break the wind resistance, and the faster the breaking speed, the more advantages. Therefore, sprinters are usually strong and have more muscle volume. Large muscles can enable them to generate powerful explosive power in a short time and have faster sprinting power. But the relative disadvantage is that on hillside terrain, the gravity generated by gravity will make them miserable on climbing.

Rouleurs belong to the category of all-rounders, with good abilities in all aspects, but they are slightly inferior to the all-rounders of time trial specialists. Rouleurs usually play the role of assistant to the leader of the auxiliary team. They must not only have high stability, strong endurance, but also be good at breaking through sieges when necessary, and actively and proactively launch attacks.

For **puncheurs**, short-range explosiveness is their forte, and their slender stature allows them to climb steep hills quickly. But the disadvantage is that the stamina is poor, and coupled with the limited body, the explosive power of sprinting is difficult to beat sprinters. Therefore, the main battlefield of puncheurs is the one-day match. Unlike pure climbers, puncheurs specialize in rolling hilly terrain, as well as short and steep hills rather than long ones.

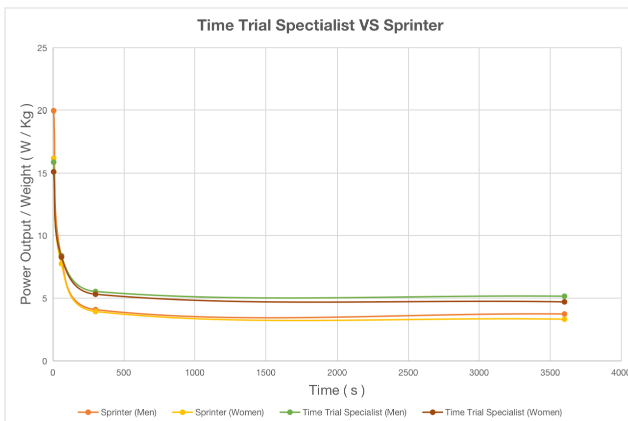

Fig. 2. Power Profiles of Time Trial Specialists and Sprinters.

According to previous research, we choose the power curve formula fitted by OmPD.

$$
\begin{aligned}
P_{(t)} &= \frac{W'}{t} \times \left(1 - e^{-t \times \frac{P_{\max} - CP}{W'}}\right) + CP, \quad t \leq CP_{TTF} \\
P_{(t)} &= \frac{W'}{t} \times \left(1 - e^{-t \times \frac{P_{\max} - CP}{W'}}\right) + CP - A \times Log\left(\frac{t}{CP_{TTF}}\right), \quad t > CP_{TTF}
\end{aligned}
\tag{1}
$$

where A is fixed constant for the decline and CP_{TTF} is time to task failure at critical power.

Due to the limited space, only two types of riders (time trial specialists and sprinters) were selected to draw power profiles as shown in the figure 2.

In Figure 2, it is evident that sprinters are better at short-term sprints, while time trial specialists are more sustainable. It follows that sprinters can finish the race by accelerating quickly to high speed, while time trial specialists can finish the race by maintaining high speed for a long time.

Of course, riders of different genders have different power profiles, as shown in Figure 3. Figure 3 shows male power profiles on the left and power profiles on the right. It can be seen that they have the same shape but different values. This is due to the difference in the physiological structure of men and women. Generally speaking, the power of women is only about 90% of that of men.

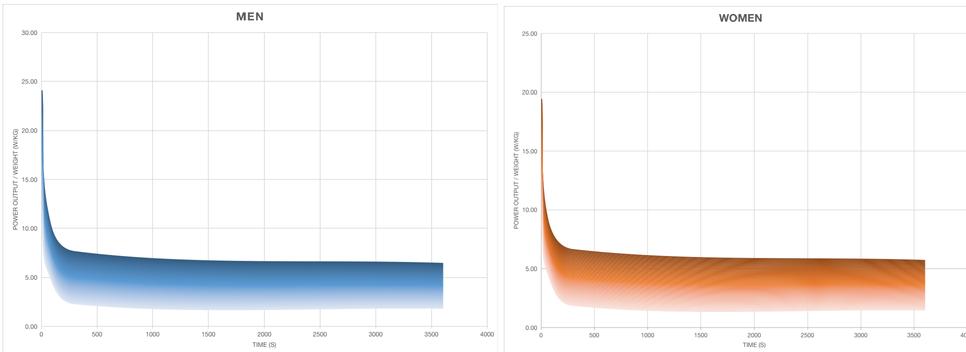

Fig. 3. Power Profiles of Men and Women.

4 ESTABLISHMENT AND APPLICATION OF SINGLE SYSTEM MODEL

4.1 Establishment a Single System Model

In order to build a model that is convenient for numerical simulation, we first consider only two dimensions, that is, do not consider any inertial forces in the third direction. The two-dimensional assumption enables us to use interpolation, which makes the computation easy. The model is based on the kinematics of moving particles, which means that the acceleration of the rider-bike system is generated by the resultant force acting on the system. The force diagram is in figure 4.

The model considers riders of different genders and ages. Assuming that the rider's weight is m, and the bicycle self-weight m_{clc}, then the rider-bike system weight $m_s = m + m_{clc}$. Consider the inertia generated by the wheel in motion $m_w = \frac{I_w}{r_w^2}$, where r_w is the radius of the wheel. The total inertia of the system is $M_s = m_s + m_w$.

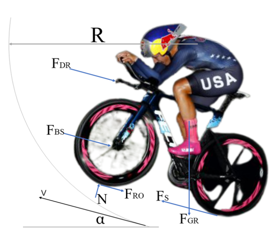

Fig. 4. Force Diagram.

The meanings in figure 4 are as follows.

Let S be the total distance, then the propulsive force F_S can be obtained as the (2):

$$F_S = P_{clc} \cdot \eta \cdot \dot{S}^{-1} = P_{\text{legs}} \, \dot{S}^{-1} \tag{2}$$

where P_{clc} is the power output at the power profile , η is the mechanical transmission efficiency.

The rolling friction force F_{RO} can be expressed as:

$$F_{RO} = C_{RR} \cdot N \tag{3}$$

where C_{RR} is rolling resistance coefficient of the tires and N is pressure.

F_{BS} is the positive resistance:

$$F_{BS} = b_1 + b_2 \cdot \dot{S} \tag{4}$$

where b_1, b_2 are the coefficients for wheel bearing resistance. F_{GR} is the rider-bike system gravity:

$$F_{GR} = m_s g \tag{5}$$

And F_{DR} is the wind stagnation resistance:

$$F_{DR} = \frac{1}{2} \cdot C_d A \cdot \rho \cdot (\dot{S})^2 \tag{6}$$

where A is frontal area, C_d is drag coefficient and ρ is air density.

According to the force analysis, the accelerations of the rider-bike system in the two dimensions of x and y are obtained as the (7):

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix} \cdot \begin{bmatrix} (F_S - F_{DR} - F_{RO} - F_{BS} - F_{GR}\sin\alpha)/M_s \\ (N - F_{GR}\cos\alpha)/m_s \end{bmatrix} \tag{7}$$

Let the step size be dt, and the initial conditions of the speed in the x and y directions are both 0. According to the speed relationship, the recurrence relationship in the x and y directions is obtained, and then the resultant speed $\dot{S}$ is obtained:

$$\begin{aligned} &\dot{S} = \sqrt{\dot{x}^2 + \dot{y}^2} \\ &\begin{cases} \dot{x}_i = \dot{x}_{i-1} + \ddot{x}\Delta t, & x_0 = 0 \\ \dot{y}_j = \dot{y}_{j-1} + \ddot{y}\Delta t, & y_0 = 0 \end{cases} \end{aligned} \tag{8}$$

In this way, the relationship between speed and power at different positions is obtained:

$$(\dot{S}_i)^2 = \left((\dot{x}_{i-1})^2 + (\dot{y}_{j-1})^2\right) + 2\left(\dot{x}_{i-1}\ddot{x} + \dot{y}_{i-1}\ddot{y}\right)\Delta t + \left((\ddot{x}_{i-1})^2 + (\ddot{y}_{j-1})^2\right)\Delta t^2 \tag{9}$$

If the starting distance is $S_0 = 0$, the distance under step i can be obtained as S_i:

$$S_i = S_{i-1} + \dot{S}_i\Delta t; \quad S_0 = 0 \tag{10}$$

In order to optimize the physical configuration in the competition, an optimization simulation model is established. The optimization target total time T is limited by the output of the minimum power P_{min} and the maximum power P_{max} as well as the rider's threshold power. The model contains two parameters, the critical power CP for continuous aerobic work and the available anaerobic work AAW for anaerobic work, whose formula is 11.

$$AAW_i = AAW_{i-1} + \left[CP - \frac{1}{2}\left(P_j + P_{j-1}\right)\right] \cdot \Delta t \tag{11}$$

where Δt is the time step, P_i is the rider's leg power.

The optimization problem can be expressed as:

$$\begin{aligned} &\min \quad T = \sum_{i=1}^{n} \Delta t \\ &s.b. \begin{cases} 0 \leqslant AAW_i \leqslant W', & i = 1 \cdots m \\ P_{\min} \leqslant P_j \leqslant P_{\max}, & j = 1 \cdots n \\ S_k = S_{k-1} + \dot{S}_k\Delta t, & k = 1 \cdots p \\ S_0 = 0, S_p = S_{max} \end{cases} \end{aligned} \tag{12}$$

where T is the time to complete the game, Δt is the time step, P_{min}, P_{max} are the lower and upper limits of the variable P_i, respectively. These power output limits depend on the rider's own power curve (1) and are biologically meaningful.

4.2 Application of the Single System Model - 2021 Olympic Time Trial course

According to the official website of Union Cycliste Internationale, the official course maps of the 2021 Olympic Time Trial course in Tokyo, Japan are as shown in the figure 5. Among them, the left side is the men's individual time trial map, and the right side is the women's individual time trial map.

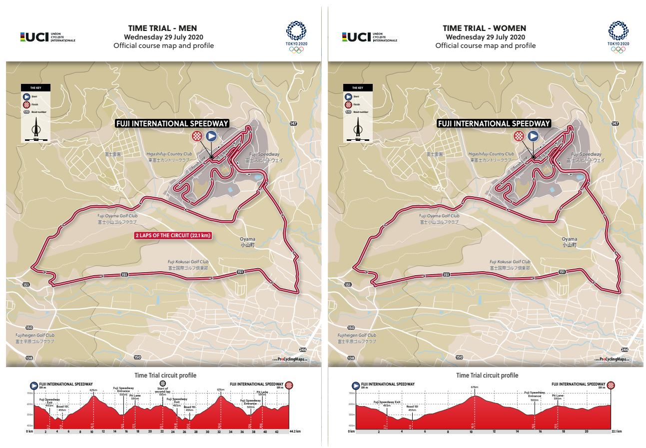

Fig. 5. Official Course Maps of the 2021 Olympic Time Trial course in Tokyo, Japan.

The 2021 Olympic Time Trial course in Tokyo, Japan all have Mount Fuji as the backdrop, starting and returning from the Fuji International Circuit. The men's and women's races were identical, but the men ran two laps of the 22.1km track while the women only ran one lap.

The men's road race is 234km long and requires nearly 5,000m of vertical climbing. The women's road race is much shorter, at just 137km, but still requires a lot of climbing with around 2,700m of ascent.

Riders have to fight not only the race but also the weather. The area is currently around 30 degrees at noon and humidity is 77%, which has a big impact on air resistance and the rider's performance.

The parameter values are shown in Table 2.

Considering the frontal force area of the system, according to the scaling laws of Heil, $C_dA = 0.54 \times 0.64$.

Consider a simulation of a world road cyclist, for the cyclist , anaerobic work capacity was calculated to 8400J.

Based on the actual local environment in Tokyo, consider an atmospheric density of $\rho = 1.2041kg \cdot m^{-3}$ and standard atmospheric pressure of 101.35 kPa. The acceleration of gravity was set to 9.81ms^{-2}.

The Method of Moving Asymptotes (MMA) is an optimization strategy that is primarily used in structural optimization. We can get the result by MMA as shown in the figure 6.

4.3 Application of the Single System Model - 2021 UCI World Championship Time Trial Course

According to the official website of Union Cycliste Internationale, the official course maps of the 2021 UCI World Championship time trial course in Flanders, Belgium are as shown in the figure 7.

Table 2. Parameter Values

Classification	Meaning	Symbol	Numerical Value
System	The rider-bike system weight	m_s	$85kg$
	The moment of inertia and the incremental drag area of the wheels	I_w	$0.14kg \cdot m^2$
	The wheel radius	r_w	$0.337m$
Environment	Gross efficiency	η	0.239
	The bearing resistance 1	b_1	0.089
	The bearing resistance 2	b_2	0.0084
	The rolling resistance coefficient of the tires	C_{RR}	0.0042
Energy	The critical power	CP	$470W$

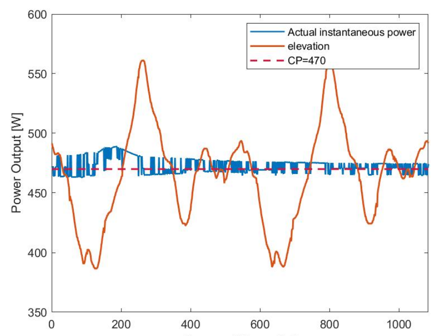

Fig. 6. Power Profile of 2021 Olympic Time Trial course in Tokyo, Japan.

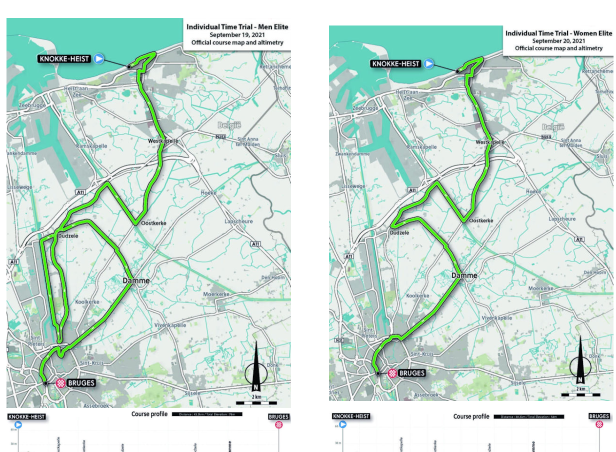

Fig. 7. Official Course Maps of 2021 UCI World Championship time trial course in Flanders, Belgium.

Among them, the left side is the men's individual time trial map, and the right side is the women's individual time trial map.

Unlike the 2021 Olympic Time Trial course in Tokyo, Japan, the 2021 UCI World Championship time trial course in Flanders, Belgium takes place on almost flat roads with a total length of 43.3 kilometers. Riders sprinted down the ramp from the beach start and continued along the North Sea for 1.5km before taking a sharp turn inland. After crossing Knokke-Heist, ride a long, flat straight to Westkapelle, Oostkerke and Dudzele. Another long straight along the Boudewijnkanaal leads to Bruges. The riders were in the city before the U-turn took them back to Dudzele. The riders make their way back to Bruges via the Damme and Damse Vaart passages, finally arriving at the finish at 't Zand square.

We can get the power profile by MMA as shown in the figure 8.

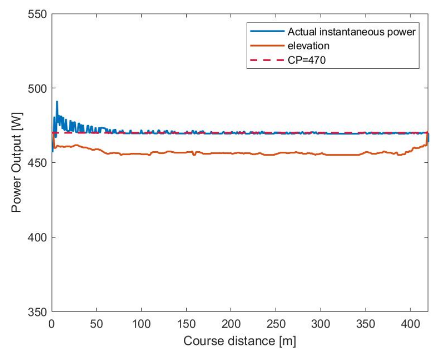

Fig. 8. Power Profile of 2021 UCI World Championship time trial course in Flanders, Belgium.

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