1 Discrete Random Variables and Expectation

A *(real-valued) random variable* X on a sample space Ω is a *measurable function* $X : \Omega \to \mathbb{R}$, and a *discrete random variable* is one which may take on only a countable number of distinct values. "$X = a$" represents the set $\{s \in \Omega \mid X(s) = a\}$, and we denote the probability of that event by $\mathbb{P}(X = a) = \sum_{s \in \Omega : X(s) = a} \mathbb{P}(s)$.

Random variables $X_1, X_2, \cdots, X_n$ are *mutually independent* (simply called *independent* when $k = 2$) if and only if, for any subset $I \subseteq \{1, 2, \cdots, k\}$ and any values x_i $(i \in I)$, $\mathbb{P}(\bigcap_{i \in I}(X_i = x_i)) = \prod_{i \in I} \mathbb{P}(X_i = x_i)$.

The *expectation* of a discrete random variable X, denoted by $\mathbb{E}[X]$, is given by $\mathbb{E}[X] = \sum_i i \cdot \mathbb{P}(X = i)$. Note that the infinite series needs to be *absolutely convergent* (i.e. rearrangements do not change the value of the sum).

Theorem 1.1 (Linearity of expectation) *For discrete random variables $X_1, X_2, \cdots, X_n$ with finite expectations and any contants $c_1, c_2, \cdots, c_n$, $\mathbb{E}[\sum_{i=1}^n c_i X_i] = \sum_{i=1}^n c_i \mathbb{E}[X_i]$.*