Friendship Paradox in Envolving Networks

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December 8, 2017

Outline

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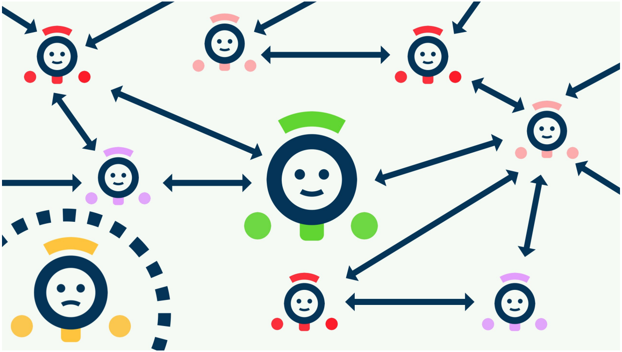
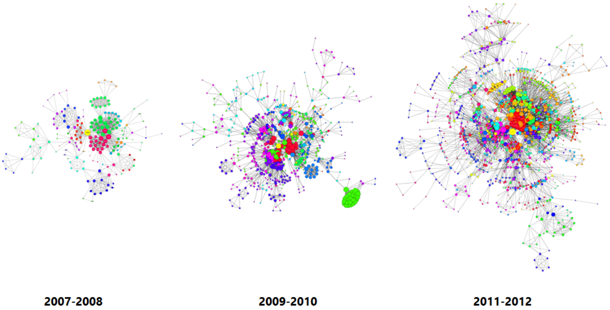
Friendship Paradox

Friendship Paradox: Most people have fewer friends than their friends have, on average.



Evolving Networks

Evolving Networks: New nodes are more likely to link popular old nodes of high degree.



Question

Does the topology of evolving networks make friendship paradox more common?

Outline

Evolving Networks

Preferential Attachment Graphs

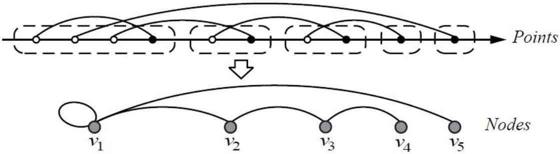
- The new node linked to old node i preferentially at each time slot

$$P(i = s) = \begin{cases} d_s\left(t\right) / \left(2t - 1\right) & 1 \leq s \leq t - 1, \\ 1 / \left(2t - 1\right) & s = t. \end{cases}$$

with $d_i\left(t\right)$ representing the degree of node v_i in time t

Evolving Networks

- The process can be interpreted from the aspect of graph formation based on the idea of linearized chord diagrams (LCDs).



Non-evolving Networks

E-R Random Graphs

- Constructed by connecting every node pair in n nodes with an equal probability p.
- We define $n = t$, $p = \frac{2m}{t-1}$ to both ensure graphs to be connected and better compare with evolving ones.

Outline

Single-step Friendship Paradox

Definition (Single-step Friendship Paradox)

We define a random nodes v^0 by picking v^0 uniformly from all nodes in G. Then single-step friendship paradox exists in G if the average degree of v^0's neighbors is larger than the degree of v^0.

Single-step Friendship Paradox

In an evolving network, a node chosen uniformly at random exhibits the phenomenon of single-step friendship paradox with a **higher** probability than that in a non-evolving network.

Single-step Friendship Paradox

In an evolving network, a node chosen uniformly at random exhibits the phenomenon of single-step friendship paradox with a **higher** probability than that in a non-evolving network.

- This benefits from the preferential attachment.

Outline

Multi-step Friendship Paradox

Definition (Single-step Friendship Paradox)

We define a random sequence of nodes denoted as $(v^0, v^1, v^2, ...)$ that results from neighboring node selection based a specified random walk at each step, with the original node v^0 uniformly picked from all the nodes in G. We say that multi-step friendship paradox exists if the sequence can reach the node of higher degree during the process.

Outline

Mean hitting rate of reaching the max degree node:

- Evolving networks: $\Omega\left(\frac{1}{\sqrt{t}\log^2 t}\right)$
- Non-evolving networks: $O\left(\frac{\log^2 t}{t}\right)$

Outline

Biased Random Walk

- Biased random walk which at each step prefers to select a neighbor with higher degree.
- Particularly, we let the neighboring node v_j of the current node v_i be chosen proportionally to its degree, i.e.,

$$P(v = v_j) = \begin{cases} \frac{d_j(t)}{\sum_{v_k \in N(v_i)} d_k}, & v_j \in N(v_i) \\ 0, & v_j \notin N(v_i), \end{cases} \tag{1}$$

Biased Random Walk

Mean hitting rate of reaching the max degree node:

- Evolving networks: $\Omega\left(\frac{\log\log t}{t^{1/3+\epsilon}\log^7 t}\right)$
- Non-evolving networks: $O\left(\frac{\log^3 t}{t}\right)$

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Thank You!