Seeking the Truth in a Decentralized Manner

Zhiying Xu

January 4, 2018

Outline

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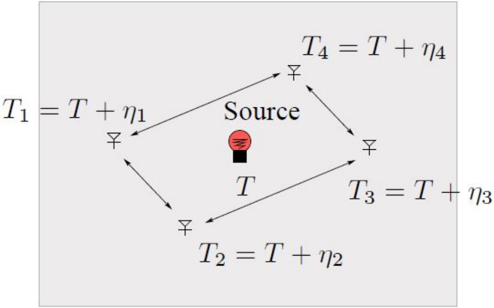
Gossip Algorithm

Gossip algorithms: schemes which distribute the computation burden and in which a node communicates with a randomly chosen neighbor.



Seeking the Truth

- Seeking the truth from noisy crowd sensing data
- Evaluate the reliability of sensors

Outline

A Real-life Case

Seeking the Truth in a Decentralized Manner

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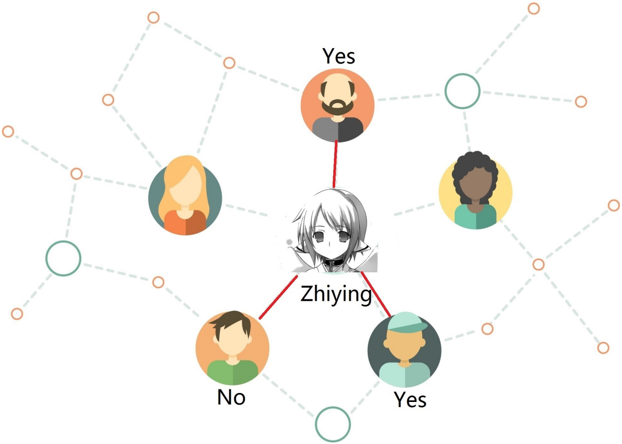
Whether the class is canceled?



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Model

Definition (Problem Formulation)

Given a crowd sensing model where n sensors make individual observations $\boldsymbol{SC}^{n \times m} \in \{0,1\}^{n \times m}$ about a set of m variables, determine the true value of these variables $\boldsymbol{t} \in \{0,1\}^m$ which satisfies

$$\langle \boldsymbol{t}, \boldsymbol{r} \rangle = \arg\max_{\langle \boldsymbol{t}, \boldsymbol{r} \rangle} p\left(\boldsymbol{SC} | \boldsymbol{t}, \boldsymbol{r}\right),$$

where $\boldsymbol{r} \in [0,1]^n$ is the reliability of sensors.

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- $\boldsymbol{t}, \boldsymbol{r}$ are related to each other.
- There is no need for iteration by EM algorithm.
- We denote the ith row of the $\boldsymbol{SC}$ as $\boldsymbol{s}_i$ and the jth column of the $\boldsymbol{SC}$ as $\boldsymbol{c}_j$

Objective Function

Using the relationship between $\boldsymbol{t}$ and $\boldsymbol{r}$, We can simplify the previous problem into the optimization of the objective function

$$\boldsymbol{t} = \arg\min_{\boldsymbol{t}} \sum_{i=1}^{n} f\left(\boldsymbol{s}_i, \boldsymbol{t}\right),$$

where $f\left(\boldsymbol{s}_i, \boldsymbol{t}\right)$ denotes
$-\frac{||\boldsymbol{s}_i - \boldsymbol{t}||}{m} \ln \frac{||\boldsymbol{s}_i - \boldsymbol{t}||}{m} - \left(1 - \frac{||\boldsymbol{s}_i - \boldsymbol{t}||}{m}\right) \ln \left(1 - \frac{||\boldsymbol{s}_i - \boldsymbol{t}||}{m}\right)$.

Truth Discovery Strategy

- All sensors are in a connected networks.
- Sensors have limited memory and can't store much data.
- Each sensor communicates with a randomly chosen neighbor asynchronously.

Outline

NP Hardness of the Decision Version

Given a matrix $\boldsymbol{SC}^{n \times m}$, is there a vector $\boldsymbol{t}$ which satisfies $\sum_{i=1}^{n} f\left(\boldsymbol{s}_i, \boldsymbol{t}\right) < C$?

Proof of NP Hardness

Theorem

Discovering the truth $\boldsymbol{t}$ *from the observed matrix* $\boldsymbol{SC}$ *is NP-hard.*

- By reduction from 3 exact cover problem.
- Inspired by Cardinal, J., Fiorini, S. and Joret, G., 2012. Minimum entropy combinatorial optimization problems. Theory of Computing Systems, 51(1), pp.4-21.

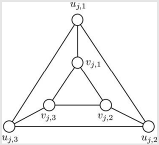
Proof of NP Hardness

Proof: Step 1

3 exact cover: Given a set system (U, S), decide whether U can be covered using pairwise disjoint sets from S

$$\Downarrow$$

minimum orientation defined on a concave function:

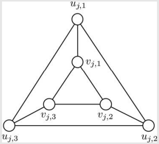


Construct a graph by creating a gadget for each element in U.
Given the graph, decide whether there is an orientation that satisfies $\sum_{i=1}^{|V|} g\left(\frac{d_i^+}{|E|}\right) < C_1$.
Here $g(\cdot)$ is a concave function and d_i^+ is the in-degree of the node i.

Proof of NP Hardness

Proof: Step 2

minimum orientation defined on a concave function:



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$$\Downarrow$$

Truth Discovery:

Convert the graph into a matrix $\boldsymbol{SC}$, decide whether there is a vector $\boldsymbol{t}$ which satisfies $\sum_{i=1}^{n} f(\boldsymbol{s}_i, \boldsymbol{t}) < C_2$.

$\boldsymbol{SC}$ = ancillary part + graph part.

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The Complexity of Exact Algorithm

Theorem

The complexity of the exact algorithm is $\Theta\left(mn2^{m'}\right)$.

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- m denotes the number of vectors in the largest set $\{\boldsymbol{c}_i'\} \subset \{\boldsymbol{c}_i\}$ with $\boldsymbol{c}_i' \neq \boldsymbol{c}_j'$, $\boldsymbol{c}_i' \neq \tilde{\boldsymbol{c}}_j'$, which can be obtained in polynomial time.
- The truth vector $\boldsymbol{t}$ is a combination of vectors in $\{\boldsymbol{c}_i'\}$.

Outline

Trade-off in Dencentralized Algorithm

- It's hard to calculate the base vectors in a decentralized ways, so we decide to try every vector in $\{0,1\}^m$
- There is a trade-off between run time and accuracy.

$$\text{run time: } \Theta\left(\frac{2^{m'}\left(m'+\log^{-1}\epsilon\right)\log n}{n\log\lambda_2\left(\boldsymbol{W}\right)^{-1}}\right)$$

$$\text{accuracy: } 1+\epsilon$$

Algorithm

Algorithm 1 Gossip-based Decentralized Exact Algorithm (GDEA)

Input: Observed matrix $\boldsymbol{SC}$

Output: Truth vector $\boldsymbol{t}^*$

Initialize: $\boldsymbol{t}^* = \boldsymbol{t} = 0, \boldsymbol{s}^* = \boldsymbol{0}, \min = 1, \max = 0$

for $j = 1$ to m **do**

  if the j^{th} element in $\boldsymbol{s}^*$ equals 0 **then**

    for $i = 1$ to n **do**

      if $S_iC_j = 0$ **then**

        Invert $\boldsymbol{s}_i$

    Implement $\mathcal{A}(P, \text{ave})$ to obtain $\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{s}_i$ and mark the elements in $\boldsymbol{s}^*$ as j whose corresponding elements in $\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{s}_i$ are 0s or 1s

for $j = 1$ to $2^{m'}$ **do**

  We obtain $\boldsymbol{s}_i'$ and $\boldsymbol{s}''$ from $\boldsymbol{s}^*$ and set $\boldsymbol{t} = \bigoplus_{i=1}^{m'} \boldsymbol{s}_i'''$, where $\boldsymbol{s}_i''' \in \{\boldsymbol{s}_i', \boldsymbol{s}_i''\}$. Implement $\mathcal{A}(P, \text{ave})$ to obtain $\frac{1}{n}\sum_{i=1}^{n} d(\boldsymbol{s}_i, \boldsymbol{t})$ and $\frac{1}{n}\sum_{i=1}^{n} r_i$

  if $\frac{1}{n}\sum_{i=1}^{n} d(\boldsymbol{s}_i, \boldsymbol{t}) < \min$ or $\frac{1}{n}\sum_{i=1}^{n} d(\boldsymbol{s}_i, \boldsymbol{t}) = \min$ and $\frac{1}{n}\sum_{i=1}^{n} r_i = \frac{1}{n}\sum_{i=1}^{n} \frac{\|\boldsymbol{t}^* - \boldsymbol{s}_i\|_1}{m} > \max$ **then**

    $\boldsymbol{t}^* = \boldsymbol{t}, \min = \frac{1}{n}\sum_{i=1}^{n} d(\boldsymbol{s}_i, \boldsymbol{t}), \max = \frac{1}{n}\sum_{i=1}^{n} r_i$

Acceleration

- Accelerated by optimizing the probability of gossiping with neighbors.
- Preferential Attachment Graphs: $\Theta\left(\frac{m'2^{m'}\log n}{n}\right)$
- Ramdom Geometric Graphs: $\Theta\left(\frac{m'2^{m'}\log n}{n\ell^2}\right)$

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The Complexity of Exact Algorithm

Theorem

The approximation algorithm achieves an approximation ratio of 1.64.

The Complexity of Exact Algorithm

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The approximation algorithm achieves an approximation ratio of 1.64.

- The approximation ratio is an upper bound, for the worst case is difficult to find.
- Use triangle inequality $f(\boldsymbol{x}, \boldsymbol{y}) + f(\boldsymbol{y}, \boldsymbol{z}) > f(\boldsymbol{x}, \boldsymbol{z})$.

Outline

Trade-off in Dencentralized Algorithm

- Use triangle inequality in metric space and Chebyshev Inequility.
- There is a trade-off between run time and accuracy.

$$\text{run time: } \Theta\left(\frac{\log n\left(\log n+\log \epsilon_1^{-1}\right)}{\epsilon_2^2\left(1-\lambda_{\max}\left(\boldsymbol{W}_1\right)\right)}+\frac{\log n}{n\Phi\left(\boldsymbol{P}_2\right)}+\frac{\log n+\log m}{n\log\lambda_2\left(\boldsymbol{W}_3\right)}\right)$$

$$\text{accuracy: } 1+\epsilon_1+\epsilon_2$$

Algorithm

Algorithm 2 Gossip-based Decentralized Approximation Algorithm (GDAA)

Input: Observation matrix $\boldsymbol{SC}$
Output: Truth vector $\boldsymbol{t}^*$

Initialize: $t_1 = t_2 = 0$
for $i = 1$ to k_2 **do**
  Implement $\mathcal{A}(\boldsymbol{P}_1, \text{mix})$ and sample every k_1 ticks to estimate $\sum_{i=1}^{n} d(\boldsymbol{s}_i, \boldsymbol{s}_k)$ in participant S_k
Implement $\mathcal{A}(\boldsymbol{P}_2, \text{min})$ to find 1-median point $\boldsymbol{t}_1$
if $||\boldsymbol{s}_i - \boldsymbol{t}_1||_1/m > \frac{2}{3}$ **then**
  Invert $\boldsymbol{s}_i$
else if $\frac{1}{3} \leq ||\boldsymbol{s}_i - \boldsymbol{t}_1||_1/m \leq \frac{2}{3}$ **then**
  $\boldsymbol{s}_i = \frac{\boldsymbol{1}}{\boldsymbol{2}}$
Implement $\mathcal{A}(\boldsymbol{P}_3, \text{ave})$ to find $\boldsymbol{t}_2 = round\left(\frac{1}{n}\sum_{i=1}^{n} \boldsymbol{s}_i\right)$
Implement $\mathcal{A}(\boldsymbol{P}_3, \text{ave})$ to calculate $\frac{1}{n}\sum_{i=1}^{n} d(\boldsymbol{s}_i, \boldsymbol{t}_1)$, $\frac{1}{n}\sum_{i=1}^{n} d(\boldsymbol{s}_i, \boldsymbol{t}_2)$, $\frac{1}{n}\sum_{i=1}^{n} \frac{||\boldsymbol{s}_i - \boldsymbol{t}_1||_1}{m}$ and $\frac{1}{n}\sum_{i=1}^{n} \frac{||\boldsymbol{s}_i - \boldsymbol{t}_2||_2}{m}$, and choose the one with the larger average value of distances between $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$ as $\boldsymbol{t}^*$
if $\frac{1}{n}\sum_{i=1}^{n} r_i < 0.5$ **then**
  Inverse $\boldsymbol{t}^*$

Acceleration

- Accelerated by optimizing the probability of gossiping with neighbors.
- Preferential Attachment Graphs: $\Theta\left(\frac{\log^2 n}{\ell^2}\right)$
- Ramdom Geometric Graphs: $\Theta\left(\log^2 n\right)$

Boyd, S., Ghosh, A., Prabhakar, B. and Shah, D., 2005, March. Gossip algorithms: Design, analysis and applications. In INFOCOM 2005. 24th Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings IEEE (Vol. 3, pp. 1653-1664). IEEE.

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Thank You!