IPEC 2020 PACE Solver Description

Computing Exact Treedepth via Minimal Separators

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Computing Treedepth via Minimal Separators

Notation

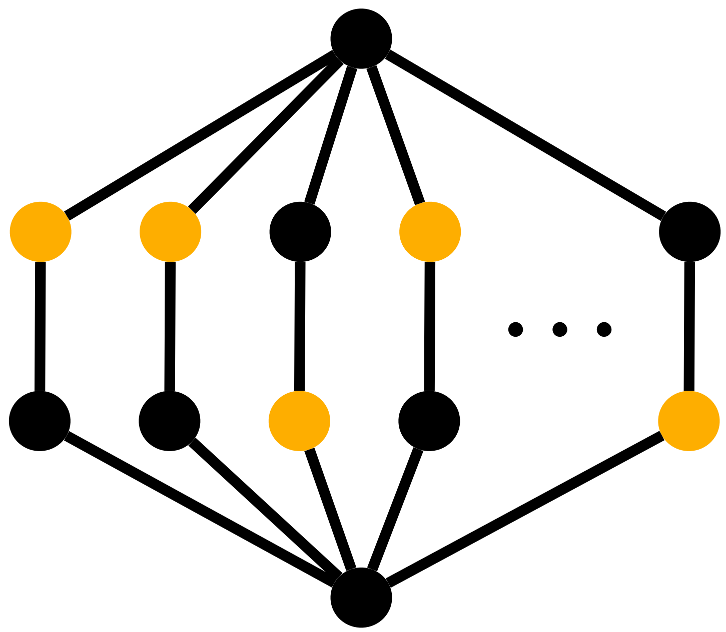
- G : undirected graph
- Δ_G : set of minimal separators of G
- $G \backslash S$: G separated by S
- $\mathscr{C}(G \backslash S)$: connected components of $G \backslash S$

[Deogun et al. '99]

$$td(G) = \begin{cases} |V| & \text{if } G \text{ is complete} \\ \min\limits_{S \in \Delta_G} \left(|S| + \max\limits_{C \in \mathscr{C}(G \backslash S)} td(G[C]) \right) & \text{otherwise} \end{cases}$$

Bottleneck of Algorithm

The number of minimal separators can be exponential



Left graph has $\Omega(2^{(n-2)/2})$ minimal separators

[Deogun et al. '99]

$$td(G) = \begin{cases} |V| & \text{if } G \text{ is complete} \\ \min\limits_{S \in \Delta_G} \left(|S| + \max\limits_{C \in \mathscr{C}(G \setminus S)} td(G[C]) \right) & \text{otherwise} \end{cases}$$

Bottleneck!

Main Algorithm

Main Algorithm based on Conjecture

$$td(G) = \begin{cases} |V| & \text{if } G \text{ is complete} \\ \min\limits_{S \in \Delta_G^{(tw(G))}} \left(|S| + \max\limits_{C \in \mathscr{C}(G \setminus S)} td(G[C]) \right) & \text{otherwise} \end{cases}$$

- $\Delta_G^{(tw(G))}$: set of minimal separators S such that $|S| \leq tw(G)$
- An upper bound of $tw(G)$ is computed by MinDegree and MinFill heuristic in solver

- The conjecture is solved after PACE (see P. 7)
- This algorithms is proved to be inexact

Pruning Techniques

- **Degeneracy Bound**
  - Degeneracy (deg) can be computed in linear time and $td \geq tw + 1 \geq deg + 1$
- **Path-length Bound**
  - If G contains P_{2^k}, $td \geq k + 1$
- **Pruning by Blocks**
  - In preprocessing, sample many small subgraphs (blocks) and compute their exact treedepth
  - If G contains a block B, $td(G) \geq td(B)$
  - Solved almost 20 more public instances

Pruning by Blocks

We combined following partitioning heuristics to recursively decompose input graph

1. Select two random vertices p and q.
   $V_1 = \{v \mid \mathrm{dist}(v, p) < \mathrm{dist}(v, q)\},\ V_2 = V \backslash V_1$

2. Compute small α-vertex separator S [Althoby et al. '20] and obtain $G \backslash S$
   - α-vertex separator: every component C of $G \backslash S$ has size $|C| \leq \alpha |V(G)|$

10 minutes is spent for preprocessing (partitioning + treedepth computation)

Theoretical Study (After PACE)

On the Size of Minimal Separators for Treedepth Decomposition: https://arxiv.org/abs/2008.09822

Definition

$S \in \Delta_G$ is an optimal top separator if $td(G) = |S| + td(G \backslash S)$

Conjecture (P. 4) → Wrong!

For all G, there exists an optimal top separator S s.t. $|S| \leq tw$

Theorem 1

For all G, there exists an optimal top separator S s.t. $|S| \leq 2tw$

Theorem 2

For all $c < 2$, there exists a G s.t. for any optimal top separator S of G, $|S| > c \cdot tw$