Compressed Sensing MRI based on Nonsubsampled Contourlet Transform

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Abstract

How to reduce acquisition time is very important in magnetic resonance imaging (MRI). Compressed sensing MRI emerges recently to suppress the aliasing when undersampling k-space data is employed. However, typical sparse transform for compressed sensing MRI ever used is wavelet, which only captures limited directional information with decay rate M^{1}. *In this paper, we introduce contourlet into compressed sensing to obtain a sparse expansion for smooth contours with decay rate* $C(\log M)^3 M^{2}$ *and employ nonsubsampled contourlet to increase the redundancy of basis for magnetic resonance images. We propose compressed sensing MRI based on nonsubsampled contourlet transform (NSCT). Experimental results demonstrate that NSCT outperforms wavelet on suppressing the aliasing and improves the visual appearance of magnetic resonance images.*

1. Introduction

How to reduce acquisition time is an important problem in magnetic resonance imaging (MRI) since the limitation of physical constraints. Some researchers try to exploit the redundancy in k-space data by using partial-Fourier [1] or exploit temporal redundancy of 3-D resonance imaging movies [2]. However, undersampling k-space samples will result in aliasing artifacts presented in the reconstructed magnetic resonance images.

Recently, compressed sensing is an emerging field in signal processing community [3,4] and attracts a lot of attention. It combines the sampling and compressing together to avoid throwing away the samples obtained according to Nyquist sampling rule. Under the assumption that the acquired data is sparse or compressible in a certain sparse transform domain, compressed sensing needs fewer measurements than those are traditionally thought necessary. It successfully recover original signal from a small number of linear projections without losing information or with little loss of information. Thus compressed sensing is employed to undersample the k-space to reduce the scanning time in MRI [5,6].

However, efficiency of compressed sensing relies on the sparsity of sparse transform. There are three ways to increase the sparsity [7,8]. First, increase the redundancy of the basis. For example, the orthogonal wavelet lacks shift-invariance and cycle spinning wavelet is shift-invariant by replicating spatial basis functions [9]. Second, one can adopt non-adaptive or adaptive sparse transform to better present signal. For images including magnetic resonance images, the non-adaptive transform could be contourlet [10,11] while the adaptive transform could be bandelet [12,13].

Since two-dimensional (2D) separable wavelet transform adopted in [5] is obtained by a separable extension from one-dimensional wavelet basis. It is expensive for wavelet to represent sharp image transitions such as edges [12]. Furthermore, wavelet will not "see" the smoothness along the contours and separable wavelets can only capture limited directional information [10]. In this paper, contourlet [10] is introduced to compressed sensing MRI and a redundant form of contourlet, named nonsubsampled contourlet transform (NSCT) [14], is employed to increase the redundancy of basis for magnetic resonance images.

The outline of this paper is as follows: Section 2 briefly introduces compressed sensing theory and its application in MRI. Section 3 proposes compressed sensing MRI based on NSCT. Section 4 shows the experimental results and section 5 gives discussion and conclusion.

2. Compressed sensing theory in MRI

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Compressed sensing MRI utilizes the sparse representation of magnetic resonance images in a transform domain to sample in advance only a subset of the signal and reconstruct the original image from the small set of acquired k-space samples [3,4,14]. Or it tries to recover the original image from partial k-space samples via nonlinear reconstruction enforcing the sparsity in a known transform domain [5].

Suppose m denote a complex vector, ψ denote the linear operator that transforms from pixel representation into a sparse representation, F_u denote the undersampled fourier transform, and y is the measured k-space samples from the MRI scanner and ε controls the fidelity of the reconstruction to the measured data. The reconstruction is obtained by solving the constrained optimization problem [5]:

$$\begin{aligned} &minimize \quad \|\psi m\|_1 \\ &s.t. \quad \|F_u m - y\|_2 < \varepsilon \end{aligned} \tag{1}$$

where ψ could be sparse transform, e.g. wavelet, and $\|\bullet\|_1$ is the l_1 norm.

Eq. 1 finds a solution which is compressible by transform thus enforces data consistency. However, when a certain sparse transform is used, the objective of Eq. 1 is often extended by some kind of quality weighting by adding penalties to present some priori knowledge about the true object [5,6]. Under the assumption that the object consists of constant or mildly varying areas, total variation (TV) is often used as the priori knowledge for MRI [5,6]. TV helps to suppress gibbs ringing artifacts and noises, which preserves edges of images. Now, Eq. 1 could be formulated as

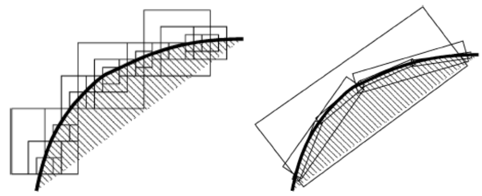
$$\begin{aligned} &minimize \quad \|\psi m\|_1 + \alpha TV(m) \\ &s.t. \quad \|F_u m - y\|_2 < \varepsilon \end{aligned} \tag{2}$$

in which $TV(m) = \sum_i |D_x(m_i)| + |D_y(m_i)|$ and α trades ψ sparsity with certain transform while D_x and D_y denote the derivatives in the *x* and *y* directions, respectively.

However, it is important to note that total variation has first and second order derivatives forms or the combination forms. In addition, any priori knowledge that can present the true object characteristic can be added to Eq. 1 to improve the image quality. For simplicity, only the first order derivative is employed in this paper.

3. Compressed sensing MRI based on nonsubsampled contourlet transform

Because 2-D wavelets are constructed from tensor products of one-dimensional wavelets, 2-D wavelets are limited in representing the contours by using square-shaped brush strokes and many fine "dots" are needed to capture the contour. In order to improve the performance of image approximation, M N Do and M Vetterli proposed contourlet to obtain a sparse expansion for smooth contours [10], which gathers the nearby basis functions at the same scale into linear. It provides different and flexible number of directions at each scale and offers high degree of directionality and anisotropy than wavelet transform. The comparison on wavelet-based and contourlet-based representation of contour is shown in Figure 1.

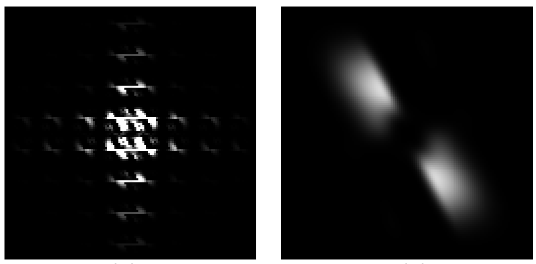


(a) Wavelet (b) Contourlet

Figure 1. Comparison on contour representation with wavelet and contourlet.

For a function f that is C^2 away from a C^2 discontinuity curve, the M -term approximation by contourlet achieves $\left\|f - \hat{f}_M\right\|_2^2 \le C(\log M)^3 M^{-2}$ while for wavelet the decay rate is only M^{-1}.

Unfortunately, the original contourlet is shift-invariant due to downsamplers and upsamplers and easily cause fuzzy artifacts along the main image ridges because of non-ideal filter. Nonsubsampled contourlet transform (NSCT) is an overcomplete form of contourlet, it ensures good frequency location, shown in Figure 2, and shift-invariance. For compressed sensing MRI, NSCT increase the redundancy of basis for magnetic resonance images.



(a) (b)

(c) d)

Figure 2. Basis images of original contourlet and NSCT. (a) and (b) are basis images of original contourlet and NSCT in frequency domain. (c) and (d) are basis images of the two transforms in spatial domain.

So, in this paper, we take NSCT as the sparse transform to provide overcomplete contourlet basis with good frequency location. The framework of compressed sensing MRI based on NSCT is shown in Figure 3. Suppose Φ stand for NSCT and X^* denote the transpose of transform X, m is calculated by Backtracking line-search algorithm as follows

$$m_{k+1} = m_k + t\Delta m_k \qquad (3)$$

where $\Delta m_k = -2F_u^*\left(F_u m_{k-1} - y\right) + \lambda \Phi^* W^{-1} \Phi m_{k-1} + \gamma \Delta m_{k-1}$.

For more information, please check the Ref. [5] for details.

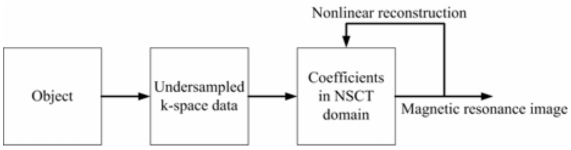


Figure 3. Framework of compressed sensing MRI.

4. Experimental results

Our experiment is based on the sparse MRI toolbox generously shared by Lustig on his personal website [15]. All the sparse transforms are performed on 3 decomposition level and NSCT are set as *{2, 3, 4}* in the decomposition stage of directional filter banks and other parameters of compressed sensing are set as default values in Lustig's toolbox.

Figure 4(a) shows the noisy magnetic resonance images and its fully sampled k-space data is shown in Figure 4 (b). Since the central k-space data affects the approximation information of image, a variable sampling density, which samples more central k-space data than other regions as shown in Figure 4(c), will be more reasonable than uniform sampling. The variable density k-space data is shown in Figure 4(d).

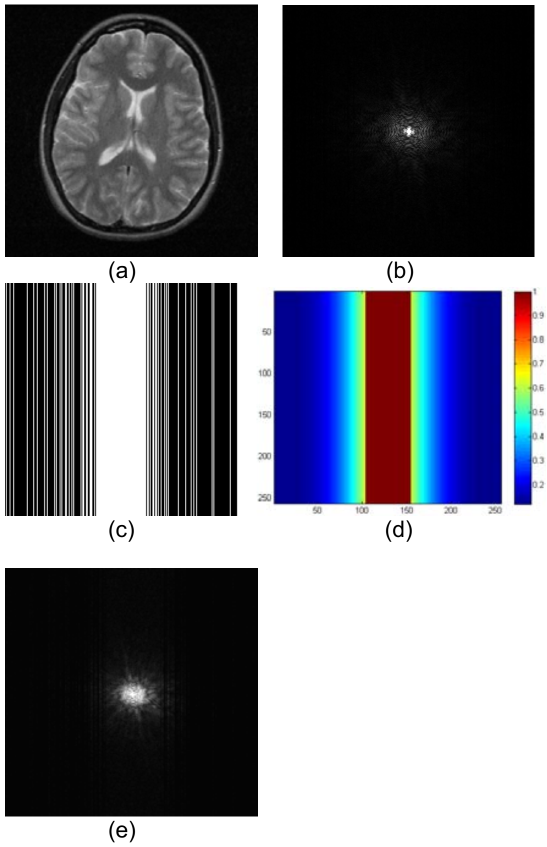


Figure 4. Variable density undersampling of k-space data. (a) noisy magnetic resonance images, (b) k-space data of (a), (c) sampling mask, (d) variable undersampling density, and (e) undersampled k-space data.

Figure 5 shows the reconstructed images from zero-filling k-space data and undersampled k-space data using compressed sensing reconstruction with wavelet and NSCT. One can see that, when undersampling is applied and zeros are filled into the absent in k-space data, aliasing will present on the reconstructed image. Compared with zero filling method, compressed sensing performs effectively to suppress the noise-like aliasing. Furthermore, NSCT outperforms wavelet on the visual appearance and the reconstructed image of NSCT is acceptable.

However, we should note that the complexity of NSCT is higher than wavelet because of the nonsubsampling in the directional filter banks and laplacian pyramid decomposition of NSCT. In addition, though the visual appearance of compressed sensing MRI is acceptable, the magnitude of images is lower than the optimal image reconstructed by fully sampling k-space data.

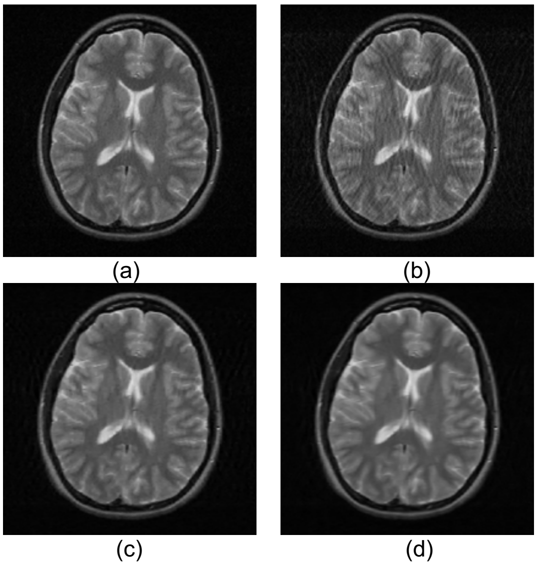


Figure 5. Compressed sensing magnetic resonance images using wavelet and NSCT. (a) reconstructed image from fully k-space data, (b) reconstructed image from zero-filling k-space data, (c) compressed sensing reconstruction with 2D wavelet, and (d) compressed sensing reconstruction with NSCT.

5. Discussion and conclusion

In this paper, we present a compressed sensing MRI using overcomplete contourlet. Nonsubsampled contourlet transform outperforms wavelet for compressed sensing MRI on visual appearance of reconstructed image. However, how to reduce the complexity of NSCT maintaining the good frequency location and providing overcomplete basis for images is a good future work. Moreover, improving the magnitude of magnetic resonance images may be another issue for compressed sensing MRI.

Acknowledgments

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