The Application of Wavelet-Based Contourlet Transform on Compressed Sensing

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Abstract. Reasonable sparse representation of signals are one of the key factors to ensure the quality of compressed sampling, so a proper sparse representing methods should be selected to make the signals sparse to the greatest extent in the applications of compressed sensing. In this paper we adopted the framework of block compressed sensing to sample the images, used the iterative hard thresholding(IHT) algorithm to reconstruct the original images, and employed the wavelet-based contourlet transform, an improved contourlet transform, to decompose 2D images in IHT reconstruction process. Numerical experiments indicated that the runtime of the reconstruction algorithm adopting wavelet-based contourlet transform is the shortest compared to that adopting contourlet transform and that adopting wavelet transform; under low compression ratios, the quality of the reconstructed images using wavelet-based contourlet transform is superior to that using contourlet transform and that using traditional wavelet transform.

Keywords: Sparse Representation, Wavelet-Based Contourlet Transform, Block Compressed Sensing, Iterative Hard Thresholding Algorithm.

1 Introduction

Recently an innovative idea named Compressed Sensing appeared in signal processing area[1,2]. The central idea of compressed sensing(CS) is that, when dealing with signals which are highly compressible in some transform domain, one can take much less samples than traditional ones which are contributing of the whole data stream. There are some key factors in CS and selecting a proper sparse basis is one

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of these factors. These factors have no exceptions when CS is applied to 2D signals, so in this paper we lay stress on the sparse representation of 2D images.

Recent years multiscale geometric analysis(MGA) technologies[3,4] suitable to 2D images have developed rapidly. These MGA transforms feature a highly directional decomposition and have brought hope to overcome deficiencies of traditional wavelet transforms in several application areas. One of the MGA transforms, contourlet[5,6], has shown high performance in nonlinear approximation of 2D images. The contourlet has a property of non-redundancy, which attracts many researchers to make various improvements based on contourlet. In this paper we apply these reformative contourlet transforms to CS in order to improve the quality of the reconstructed images.

2 The WBCT Construction

Contourlet transform, a recently introduced directional transform, is referred to as a 'true' two dimensional image-representing method. Contourlet transform breakdowns the transform task into two parts — multiscale analysis and directional decomposition. Multiscale analysis is first obtained by employing Laplacian pyramid (LP) which can capture the discontinuous points. Directional decomposition is then acquired by applying a directional filter bank(DFB) to the result of multiscale analysis. Thus LP and DFB together compose the Contourlet transform structure. This structure can decompose an image into many directional subbands at differential scales. Fig.1(a) illustrates a schematic plot of the contourlet transform using 3 LP levels and 8-4-4directional levels (8 directions at the finest level).

From Fig.1(a) and (b) we can see that, Laplacian pyramid has not down-sampled at parts of high frequency, which results in an additional 1/3 size compared to mallat pyramid decomposition of wavelet. Although representing an image by contourlet transform can preserve better details and contours in comparison to a wavelet transform, it may not be the most excellent choice. This observation motivated researchers to investigate for a new multiresolution decomposition mechanism to replace Laplacian pyramid. As for multiresolution analysis, the most famous one is mallat pyramid decomposition of wavelet. Compared to Laplacian pyramid decomposition, the mallat pyramid decomposition has the feature of non-redundancy in addition to the feature of multiresolution. Consequently, we introduced the wavelet-based contourlet transform (WBCT) [7-10] that is a non-redundant version of the contourlet transform.

The basic idea of WBCT is that first to substitute mallat pyramid decomposition for Laplacian pyramid, then to convolute the data of each high frequency subbands of mallat decomposition by a directional filter bank. The principle of WBCT is illustrated in Fig. 1(b) in which a 3-level wavelet decomposition was used, 16 directions were set in each high frequency subbands of the first wavelet level, and both 8 directions was set in the second and the third wavelet levels. Because of the non-redundancy, WBCT shows promise over contourlet decomposition on image compression.

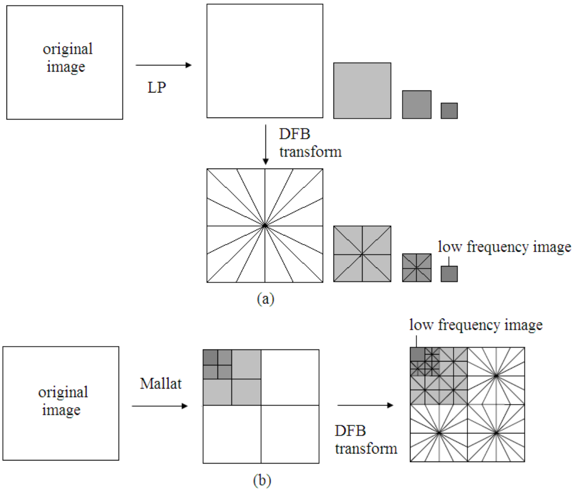

Fig. 1. schematic plots of contourlet transform and WBCT （a） a schematic plot of the contourlet transform using 3 LP levels and 4-3-3directional levels. （b） A schematic plot of the WBCT using 3 dyadic wavelet levels and 4-3-3directional levels.

3 The Reconstruction Method of Block Compressed Sensing Using WBCT

As applied to 2D images, CS faces two main difficulties: one is enormous computation encountered in reconstruction procedure; the other is huge storage requirement in order to store the random sampling operator. In this paper, we adopt the framework of block compressed sensing(BCS)[11]. In this framework, the original image is divided into a certain number of blocks depending on rows and columns of the image and each block is sampled independently using the same measurement operator matching the size of the blocks. This method provides comparable performances compared to existing CS strategies with much lower implementation cost and makes the real time image processing possible.

In BCS scheme first an image with $N = I_r \times I_c$ pixels is divided into blocks of size B×B each. We can assign B to 16 or 32. Suppose that x_i is a vector representing block i. Sample x_i with the same measurement operator $\boldsymbol{\Phi_B}$. Then the corresponding y_i is $\boldsymbol{y_i = \Phi_B x_i}$ where $\boldsymbol{\Phi_B}$ is an $M_B \times \mathrm{B}^2$ orthonormal measurement matrix with $\boldsymbol{M_B} = \lfloor \frac{\boldsymbol{M}}{\boldsymbol{N}} \boldsymbol{B}^2 \rfloor$. Here M is the times of CS measurement and the size of M should meet the principle of Restricted Isometry Property[1,2]. We get M CS values after BCS operation.

Because of the low efficiency of the traditional reconstruction approaches such as basis pursuit, Orthogonal Matching Pursuit, we employ the iterative hard thresholding (IHT) algorithm in reconstruction process[12]. In addition wiener filtering is incorporated into IHT in order to eliminate the blocking artifacts inducted by BCS. Finally, as illustrated above, our schema can be represented as Fig.2.

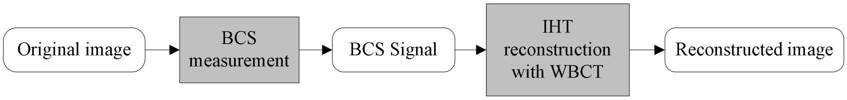

Fig. 2. BCS reconstruction process based on WBCT

4 Numerical Experiments

We now present several experiments employing BCS sampling scheme and IHT reconstruction algorithm by WBCT, contourlet transform and 2-D discrete wavelet transform(DWT), and compare the results by WBCT with the ones by the other two transforms. First, Fig. 3. presents an example of WBCT decomposition on Barbara image which uses 4 decomposition levels and 1,1,1,2,3 DFB decomposition levels — 3 at the finest scale. The following experiments use 4 decomposition levels and the number of DFB decomposition levels is 3,3,3,3 at each scale. In these experiments, for a given value R, we select R percent of the most significant coefficients in each transform domain, and then evaluate the reconstructed images from these sets of R percent of the coefficients. Fig. 4 shows the reconstruction results of 10 or 20 percent of total coefficients of a part of the Barbara,Baboon,Boat, Goldhill, Lena and Peppers images by WBCT, contourlet and DWT. The detailed imges and the PSNR values show that WBCT capture finer contours than contourlet and DWT do.

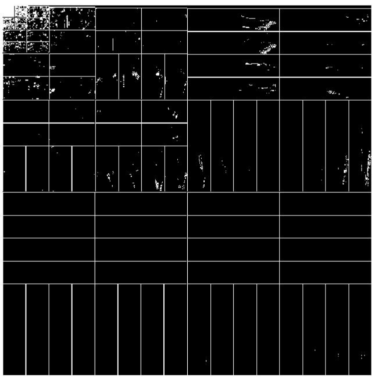

Fig. 3. An example of WBCT decomposition

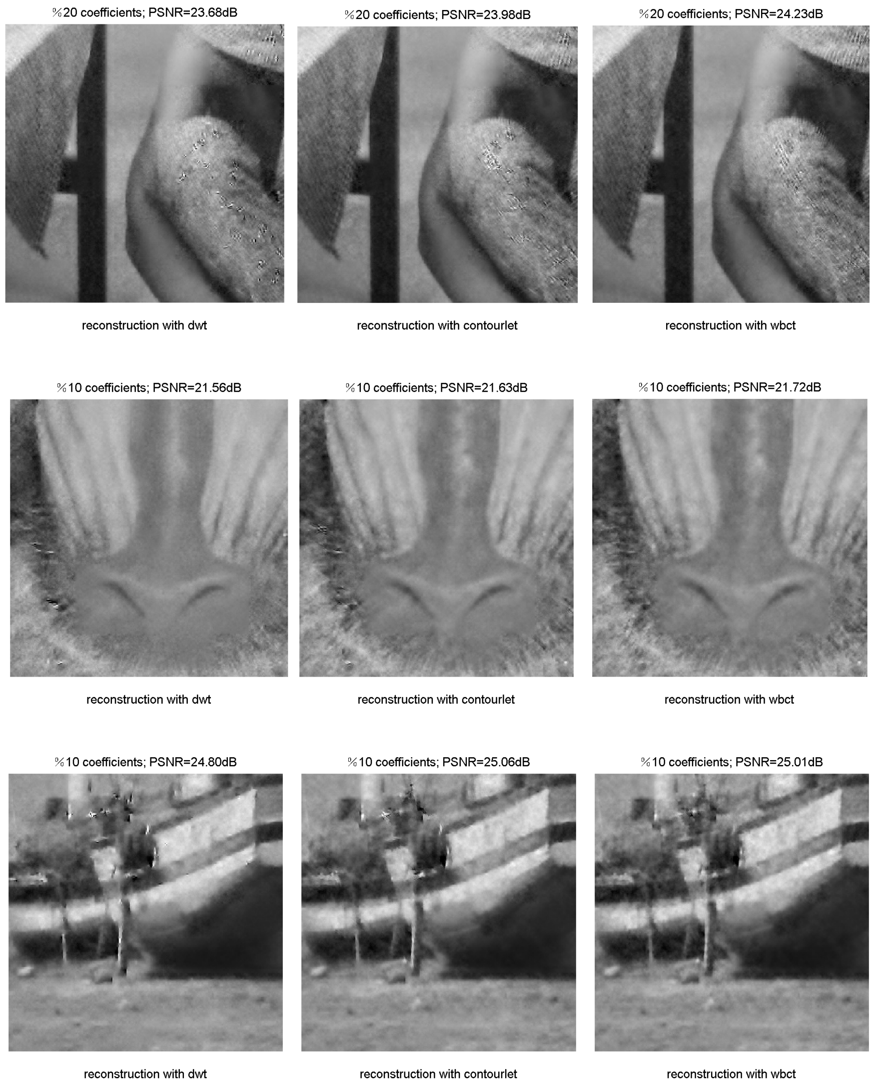

Fig. 4. Reconstructed results by WBCT, contourlet and DWT of different percent of coefficients. (a) Reconstructed results of dwts. (b) Reconstructed results of contourlets. (c) Reconstructed results of WBCTs.

(a) (b) (c)

Fig. 4. (*continued*)

Fig. 5. and Fig. 6. present some curves obtained by selecting different transform domain coefficients from 10 percent to 90 percent when reconstructing Barbara. Fig. 5. shows that the PSNR values of WBCTs are comparable to contourlets and are larger than DWTs when the coefficients are under 50 percent. From Fig. 6. we can see that the reconstruction time of WBCTs is a bit shorter than that of contourlets and much shorter than that of DWTs at different percent of coefficients. These experiments prove that WBCT is a bit superior compared with contourlet and DWT in

capturing better contours. The similar comparison results are obtained when reconstructing other images. We truncates a part of performance comparison values obtained from reconstructed images using 10 percent of transform domain coefficients and show them in Table 1.. The PSNR and time values in Tab. 1. indicates the trend that on the whole, WBCT is more excellent than the other two transforms.

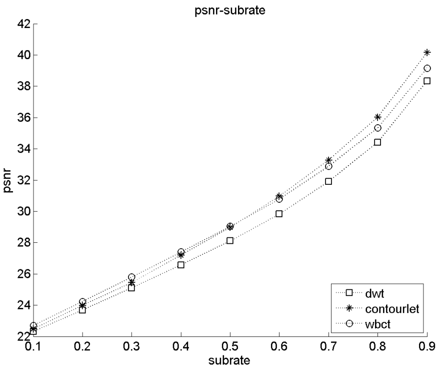

Fig. 5. The PSNR-coefficient curves for the reconstructed results

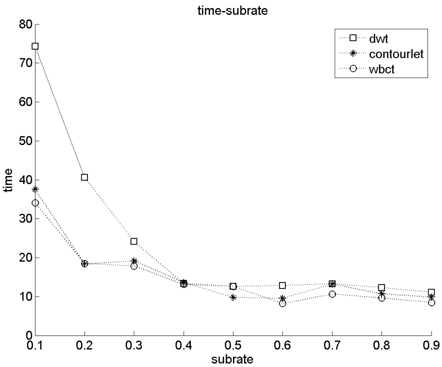

Fig. 6. The time-subrate curves for the reconstructed results

Table 1. Performance comparison (coefficient ratio=10%)

Item / Image	PSNR(dB)			Time(sec)		
	WBCT	Contourlet	DWT	WBCT	Contourlet	DWT
barbara	22.69	22.46	22.33	34.10	37.53	74.33
baboon	21.72	21.63	21.56	25.58	38.93	89.23
boat	25.01	25.06	24.80	32.49	35.52	59.65
goldhill	26.77	23.48	26.83	56.84	42.99	60.59
lena	28.05	27.46	27.59	46.03	52.20	60.28
peppers	27.03	26.25	27.20	108.58	106.12	130.81

5 Conclusion

This paper has proposed a scheme of compressed sensing employing the sparse representation of WBCT. Sparse representation is one of the key elements of CS, so we should select proper basis to preserve details and contours of images. Having the advantage of non-redundancy and being capable of capturing finer contours, WBCT is relatively a perfect sparse transform. In our experiments we have adopted the general paradigm of BCS coupled with an IHT reconstruction using WBCT sparse basis promoting not only sparsity but also smoothness of the reconstruction.

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