Compressed Sensing MRI Reconstruction Algorithm Based on Contourlet Transform and Split Bregman Method

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***Abstract*—Compressed sensing (CS) based methods have recently been used to reconstruct magnetic resonance (MR) images from undersampled measurements, which is known as CS-MRI. In traditional CS-MRI, wavelet transform can hardly capture the information of image curves and edges. In this paper, we present a new CS-MRI reconstruction algorithm based on contourlet transform and split Bregman method. Contrast with wavelet based algorithms, the proposed method not only enforces the curve sparsity of MR images with fast computation, but also outperforms on reconstruction accuracy. Numerical results show the effectiveness of the proposed algorithm.**

***Keywords*-CS-MRI; Contourlet transform; Split bregman method.**

I. Introduction

CS is a new sampling and compression theory. It utilizes the sparseness of a signal in a particular domain and can reconstruct the signal from significantly fewer samples than Nyquist sampling, which has been the fundamental principle in signal processing for many years [1]–[3]. Due to the above advantages, CS has received considerable attentions in many areas, one of which is MRI reconstruction [4], [5]. MRI is safer,more frequent and accurate for clinical diagnosis. However, conventional MRI needs to spend much time scanning body regions, causing the expensive cost and the not idealized space resolution. In the meantime, the physiological property in the tested body will make the image blurry and distortional. Therefore, under the premise of guaranteeing the image quality, speeding up the MRI compression and reconstruction has been the powerful impetus to promote the development of MRI techniques.

For CS-MRI, there are two key points to further investigate. The first one is the sparse transform. In MRI reconstruction, the MR images themself are not sparse, but have sparse representations in some transform domains. In traditional CS-MRI, wavelet transform is commonly used as a sparse transform [6], [7]. However, as the limitations of direction, wavelet transform can hardly capture the information of image curves and edges fully and accurately. In contrast, curves and edges are mainly features of MR images. Therefore, more effective sparse transform should be considered for CS-MRI. Contourlet transform, also known as Pyramid Directional Filter Bank (PDFB),is put forward to make up for the inadequacy of the wavelet transform [8].Contourlet transform can describe the image's contour and directional texture information fully and accurately since it realizes any directional decomposition at each scale. Furthermore, contourlet is constructed directly in a discrete domain and has low computing complexity. Thus, contourlet transform can be easily implemented for MR images [9], [10].

The second part is the reconstruction algorithm. In recently years, a number of algorithms have been put forward for the signal reconstruction in CS. e.g. interior- point algorithm [11], iterative shrinkage thresholding algorithm [12], et. al. But not all of these algorithms are suitable for CS-MRI since the dimensions of the MR images are huge. Split Bregman method, motivated by the Bregman distance, has been successfully applied to a variety of convex optimization problems, including compressed sensing [13], [14], image processing [15], [16] and many other applications [17], [18]. Split Bregman method is able to solve large-scale CS problem since all the iterations of split Bregman iteration only contain the first-order information of the objective function, which has low computing complexity.

In this paper, we present a new CS-MRI reconstruction algorithm based on contourlet transform and split Bregman iteration. The proposed algorithm can recover the curves and edges of a MR image more precisely and suit large-scale MRI reconstruction.

Through the paper, we denote vectors by boldface lowercase letters, e.g., $\mathbf{x}$, matrices by boldface uppercase letters, e.g., $\mathbf{A}$. $\mathbf{E}$ is an identity matrix. $\|\mathbf{x}\|_0$, $\|\mathbf{x}\|_1$ and $\|\mathbf{x}\|_2$ denote the l_0, l_1 and l_2 norm of a vector $\mathbf{x}$, respectively. $\mathbb{R}$ denote the set of real numbers and the set of positive integers.

The organization of the rest of this paper is as follows: in section II, we introduce CS-MRI model briefly and then present our new algorithm. Numerical results demonstrate the effectiveness of the proposed algorithm in section III. Finally, we conclude the paper in section IV.

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II. Split Bregman for Contourlet-based CS-MRI

A. Contourlet-based CS-MRI Model

The basic problem of CS is to recover a signal $\mathbf{x}$ from underdetermined linear measurement $\mathbf{y} = \mathbf{\Phi x}$ where , $\mathbf{y} \in \mathbb{R}^m, \mathbf{x} \in \mathbb{R}^n, \mathbf{\Phi} \in \mathbb{R}^{m \times n}, m < n$.This underdetermined linear system has infinite solutions when seen from the aspect of algebra. However, according to the CS theory, under the assumption that is sparse, can be reconstructed by the following optimization problem:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad \text{s.t.}\, \mathbf{y} = \mathbf{\Phi x}, \tag{1}$$

where $\|\mathbf{x}\|_0$ is the ℓ_0 norm which means the nonzero numbers of $\|\mathbf{x}\|$.

Problem (1) is difficult to solve since it is NP-hard. It can be relaxed as the following convex problem:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad \text{s.t.}\, \mathbf{y} = \mathbf{\Phi x}, \tag{2}$$

where $\|\mathbf{x}\|_1$ is the sum of absolute values of $\|\mathbf{x}\|$.

Problem (2) is a convex optimization problem and can be solved by many algorithms. Problem (1) and (2) are under the assumption that $\|\mathbf{x}\|$ is sparse. However, in many applications, the signal itself is not sparse, but has a sparse representation in some transform domains. e.g. the formula $\mathbf{x} = \mathbf{\Psi s}$, where $\|\mathbf{x}\|$ is the original signal which is not sparse, $\|\mathbf{s}\|$ is the sparse coefficient with respect to the sparse transform matrix $\mathbf{\Psi}$. In this case, CS model should be $\mathbf{y} = \mathbf{\Phi s} = \mathbf{\Phi \Psi^* x} = \mathbf{Ax}$, where $\mathbf{\Psi}^*$ denotes the inverse of $\mathbf{\Psi}$. The optimization problem (2) should be changed as follows:

$$\min_{\mathbf{x}} \|\mathbf{\Psi}^*\mathbf{x}\|_1 \quad \text{s.t.}\, \mathbf{y} = \mathbf{Ax}. \tag{3}$$

B. Split Bregman for Contourlet-based CS-MRI

In this subsection, we solve (3) by split Bregman method. Based on variable splitting strategy, we first introduce a new auxiliary variable $\boldsymbol{\theta} = \mathbf{\Psi}^*\mathbf{x}$ and transform (3) into an equivalent problem:

$$\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_1 \quad \text{s.t.}\, \mathbf{y} = \mathbf{Ax}, \boldsymbol{\theta} = \mathbf{\Psi}^*\mathbf{x}. \tag{4}$$

The unconstrained form of (4) is given by

$$\min_{\mathbf{x},\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_1 + \frac{\lambda}{2}\|\mathbf{Ax}-\mathbf{y}\|_2^2 + \frac{\mu}{2}\|\boldsymbol{\theta} - \mathbf{\Psi}^*\mathbf{x}\|_2^2, \tag{5}$$

where $\{\lambda, \mu : |\lambda \mathbf{A}^T\mathbf{A} + \mu \mathbf{E}| \neq 0\}$, $\mathbf{E}$ is the unit matrix.

According to the Bregman iteration theory [14], problem (5) can be reduced to a sequence of unconstrained problems as follows:

$$\begin{cases} (\mathbf{x}^{k+1}, \boldsymbol{\theta}^{k+1}) = \arg\min_{\mathbf{x},\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_1 + \frac{\lambda}{2}\|\mathbf{Ax}-\mathbf{y}\|_2^2 \\ +\frac{\mu}{2}\|\boldsymbol{\theta} - \mathbf{\Psi}^*\mathbf{x} - \mathbf{b}^k\|_2^2, \\ \mathbf{b}^{k+1} = \mathbf{b}^k + \mathbf{\Psi}^*\mathbf{x}^{k+1} - \mathbf{z}^{k+1}). \end{cases} \tag{6}$$

Split Bregman technique in [13], [15] shows that the first minimization of (6) can be solved efficiently by minimizing with respect to $\mathbf{x}$ and $\boldsymbol{\theta}$ separately as follows:

$$\mathbf{x}^{k+1} = \arg\min_{\mathbf{x}} \frac{\lambda}{2}\|\mathbf{Ax}-\mathbf{y}\|_2^2 + \frac{\mu}{2}\|\boldsymbol{\theta} - \mathbf{\Psi}^*\mathbf{x} - \mathbf{b}^k\|_2^2, \tag{7}$$

$$\boldsymbol{\theta}^{k+1} = \arg\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_1 + \frac{\mu}{2}\|\boldsymbol{\theta} - \mathbf{\Psi}^*\mathbf{x} - \mathbf{b}^k\|_2^2. \tag{8}$$

(7) is convex and differentiable, we can solve with optimization condition: $0 \in \partial[\frac{\lambda}{2}\|\mathbf{Ax}-\mathbf{y}\|_2^2 + \frac{\mu}{2}\|\boldsymbol{\theta} - \mathbf{\Psi}^*\mathbf{x} - \mathbf{b}^k\|_2^2]$ and then get

$$\mathbf{x}^{k+1} = (\lambda\mathbf{A}^T\mathbf{A} + \mu\mathbf{E})^{-1}[\lambda\mathbf{A}^T\mathbf{y} + \mu\mathbf{\Psi}(\boldsymbol{\theta}^k - \mathbf{b}^k)]. \tag{9}$$

(8) has a close form solution by shrinkage (or soft thresholding) formula [12] :

$$\boldsymbol{\theta}^{k+1} = shrink(\mathbf{\Psi}^*\mathbf{x}^{k+1} + \mathbf{b}^k), 1/\mu), \tag{10}$$

where $shrink(\alpha, \beta) = \frac{\alpha}{\|\alpha\|}\max\{\|\alpha\| - \beta\}$.

Now, we present split Bregman algotithm for problem (3) as Algorithm 1.

Algorithm 1

Require: $tol, \lambda, \mu, \mathbf{A}, \mathbf{\Psi}, \mathbf{y}, \mathbf{b}^0, \boldsymbol{\theta}^0$.
Ensure: $\mathbf{x}$.
  while $\|\mathbf{x}^k - \mathbf{x}^{k-1}\|_2 > tol$ **do**
    $\mathbf{x}^{k+1} = (\lambda\mathbf{A}^T\mathbf{A} + \mu\mathbf{E})^{-1}[\lambda\mathbf{A}^T\mathbf{y} + \mu\mathbf{\Psi}(\boldsymbol{\theta}^k - \mathbf{b}^k)]$;
    $\boldsymbol{\theta}^{k+1} = shrink(\mathbf{\Psi}^*\mathbf{x}^{k+1} + \mathbf{b}^k), 1/\mu)$;
    $\mathbf{b}^{k+1} = \mathbf{b}^k + \mathbf{\Psi}^*\mathbf{x}^{k+1} - \mathbf{z}^{k+1})$.
  end while

III. Numerical Experiments

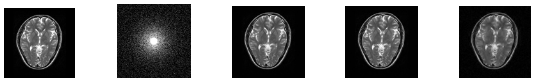
In this section, we present numerical results to illustrate the performance of the proposed algorithm for MRI reconstruction. All experiments are made by using MATLAB 7.8.0 on the PC with Intel Core 3.4 GHz and 4 G memory.

We compare the proposed algorithm 1 with the algorithm in paper [9], which is based on contourlet transform and iterative thresholding algorithm (ICOTA). We also compare the reconstruction performance of contourlet and wavelet. We quantify the reconstruction performance by peak signal to noise ratio (PSNR) and CPU time. PSNR is defined as

$$\text{PSNR} = 10 \cdot \log_{10}\left(\frac{255^2}{\text{MSE}}\right) \tag{11}$$

where $\text{MSE} = \frac{1}{mn}\sum_{i=1}^{m-1}\sum_{j=1}^{n-1}[\mathbf{I}_{ori}(i,j) - \mathbf{I}_{rec}(i,j)]^2$, $\mathbf{I}_{ori}$ and $\mathbf{I}_{rec}(i,j)$ are original image and reconstructed image, respectively, m, n are the size of the images.

In the first experiment, we use the MR image as Fig. 1(a) shows, and the variable density sampling pattern as Fig. 1(b) shows. For wavelet transform, we use Daubechies wavelet with 4 vanishing moments, and contourlet transform with decomposition [5,4,4,3], just as the same as paper [9].



(a) (b) (c) (d) (e)

Figure 1: Comparison of CS-MRI reconstruction results obtained from different algorithms. (a) Original MR image. (b) Variable density sampling pattern with sampling rate 0.2. (c) Reconstructed result from Algorithm 1 with contourlet transform. (d) Reconstructed result from ICOTA in [9]. (e) Reconstructed result from Algorithm 1 with wavelet transform.

Table I: Comparisons of different algorithms

Algorithm	PSNR[dB]	CPU time (seconds)
Algorithm 1with contourlet	46.23	5.15
ICOTA in [9]	45.10	16.02
Algorithm 1 with wavelet	41.84	4.97

Fig. 1 shows the reconstructed images using different algorithms. Table 1 summarises the comparisons of different algorithms based on PSNR and CPU time. From Table 1 we can see that Algorithm 1 with contourlet transform outperforms Algorithm 1 with wavelet transform in term of PSNR, although its running time is slightly slower. Algorithm 1 with contourlet transform outperforms ICOTA in [9] in terms of both PSNR and CPU time.

In the second experiment, we illustrate the reconstruction performance of each algorithm as the sampling rate varies from 0.1 to 0.9. Fig. 2 shows the variations of PSNR and CPU time of CS-MRI reconstruction versus sampling rates for different algorithms. Fig. 2(a) shows that PSNR of Algorithm 1 with contourlet is better than Algorithm 1 with wavelet and ICOTA in [9] with sampling growing. Furthermore, Fig. 2(a) also shows that the advantage of contourlet becomes less obvious with the increase of sampling rate. Fig. 2(b) indicates Algorithm 1 with contourlet is slightly slower than Algorithm 1 with wavelet, but is much faster than ICOTA in [9].

IV. Conclusion

In this paper, we propose a novel algorithm based on contourlet transform and the classic split Bregman method to solve CS-MRI problem. The proposed algorithm has low computational complexity, and is suitable for large scale problem. Our numerical results show that the proposed algorithm compare favorably with these algorithms referred to in terms of PSNR and CPU time.

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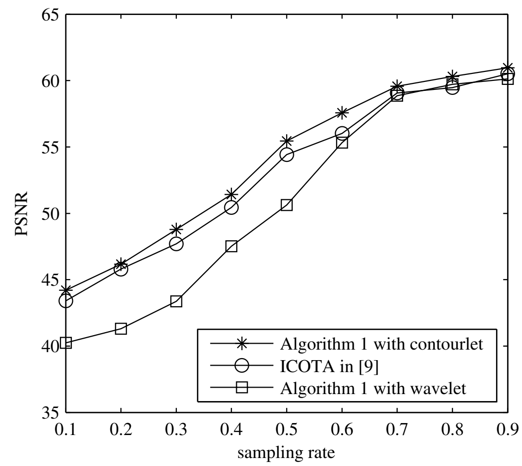
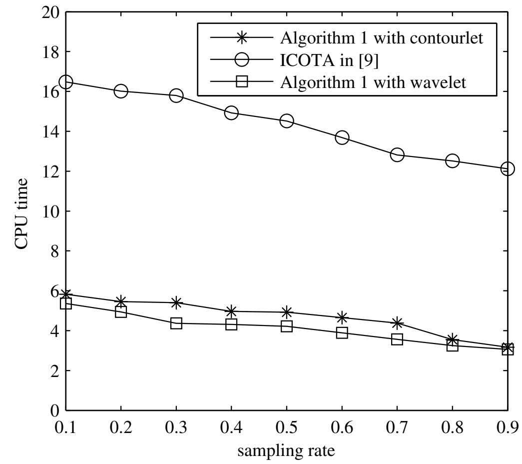


(a)



(b)

Figure 2: Comparison of CS-MRI reconstruction results obtained from different algorithms with different sampling rates. The results are average of 50 runs. (a) PSNR versus sampling rate. (b) CPU time versus sampling rate.

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