Fast iterative contourlet thresholding for compressed sensing MRI

Wangli Hao, Jianwu Li, Xiaobo Qu and Zhengchao Dong

Proposed is the use of the contourlet as a sparse transform which is combined with the fast iterative shrinkage/threshold algorithm (FISTA) for compressed sensing magnetic resonance imaging reconstruction. The proposed method not only inherits the simplicity and effectiveness of the original FISTA but also has the sparse curve representation ability of the contourlet. Simulation results validate the superior performance of the proposed method in terms of reconstruction accuracy and computation time.

Introduction: Computational efficiency and image accuracy are two fundamental goals in image processing in general, and in the reconstruction of compressed sensing magnetic resonance imaging (CS-MRI) specifically. The fast iterative shrinkage/threshold algorithm (FISTA) has been widely used in recent years for linear inverse problems including image denoising [1, 2], image deblurring [1, 2], CS-MRI reconstruction [3, 4] and CS remote sensing imaging [5]. The FISTA is an efficient algorithm that has a faster convergence speed than some traditional methods such as the iterative shrinkage/threshold algorithm ISTA and two-step ISTA [6]. Meanwhile, the FISTA is able to obtain better results in terms of accuracy when applied to simple regularisation problems such as image denoising and deblurring [1, 2] than that obtained by traditional iterative TAs [1, 2]. When FISTA and composite splitting techniques [3] are combined, they can be used to solve the composite total variance (TV) and wavelet sparsity regularisation problems such as the reconstruction of CS-MRI [3]. Nevertheless, TV may result in loss of texture [7], and the wavelet may fail to represent image curves though it can enforce point singularities and isotropic features of images [4, 5]. The curvelet, on the other hand, outperforms the wavelet in representing curve-like features of images and was used to replace wavelet sparse transformation in the native FISTA to improve the effectiveness of remote sensing imaging reconstruction [5]. A clear drawback of the curvelet sparse transform is the long computation time due to its high redundancy. The contourlet, another image geometric transform akin to the curvelet, has much less redundancy but with efficient sparse representation of curves and therefore has been utilised for CS-MRI [4]. Although the use of the contourlet in [4] is simple and effective, the iterative threshold reconstruction algorithm in [4] is slow. This Letter proposes using the contourlet as a sparse transform of FISTA for CS-MRI, aimed at improving the quality of the reconstructed image and the computational efficiency.

Theory: The CS-MRI reconstruction problem in this Letter can be formulated as

$$\hat{x} = \arg\min_{x} \left\{ \frac{1}{2} \|Rx - b\|^2 + \alpha \|\Phi x\|_1 \right\} \tag{1}$$

where x is the underlying MR image to be reconstructed, R is the partial Fourier transform, b is the undersampled measurement of k-space data, Φ is the contourlet transform and α is a trade-off parameter to tune the data fidelity term and sparsity regularisation term in (1).

The proposed fast iterative contourlet thresholding algorithm (dubbed as FICOTA) is outlined as algorithm 1 below and is utilised to solve (1).

In algorithm 1, $f(r^k) = \frac{1}{2}\|Rr^k - b\|^2$, $\nabla f(r^k)$ describes the gradient of the function f at point r^k, L_f and ρ are two positive constant scalars,

$$\text{project}(x, [l, u]) = \begin{cases} x & l \le x \le u \\ l & x \le l \\ u & x \ge u \end{cases} \tag{2}$$

Pixel values of the image are normalised to the range $[l, u]$ via the 'project' function, where $u > l \ge 0$. Otherwise, the reconstructed image will have artefacts because of the appearance of the negative pixel values after sparsity transformation:

$$\text{prox}_\rho(g)(x) := \arg\min_{u} \{ g(u) + (1/2\rho)\|u - x\|^2 \} \tag{3}$$

Results: The fast composite splitting algorithm (FCSA) [3], the contourlet based iterative thresholding method [4] and two FISTA-based methods using wavelet and curvelet regularisation terms [2, 5], respectively, are utilised to compare the performance of FICOTA. The conventional four methods are simply described as FCSA, ICOTA, FIWTA and FICTA, respectively. For fair comparison, the coefficients normalising process is also added to other methods. All the algorithms are coded using MATLAB 2009b on a Dell PC T1500.

Algorithm 1. FICOTA

Input: $\rho = 1/L_f$, α, $t^1 = 1$, $r^1 = x^0$
for $k = 1$ to K do

$x_g = r^k - \rho \nabla f(r^k)$

$x^k = prox_\rho(2\alpha\|\Phi x\|_1)(x_g)$

$x^k = project(x^k, [l, u])$

$t^{k+1} = \left(1 + \sqrt{1 + 4(t^k)^2}\right)/2$

$r^{k+1} = x^k + (t^k - 1/t^{k+1})(x^k - x^{k-1})$

end for

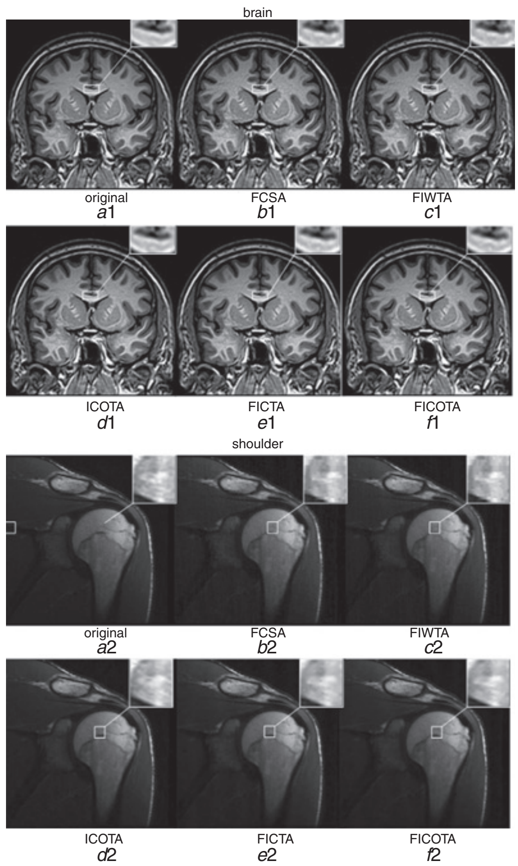

Fig. 1 *Original and reconstructed images*

*a*1, *a*2 Original MR images
*b*1–*f*1 Reconstructed brain images of different algorithms
*b*2–*f*2 Reconstructed shoulder images of different algorithms

Two MR images, brain and shoulder, are presented in Fig. 1 (*a*1 and *a*2) and were used in the experiments. For convenience, the MR images were resized to the same size 256 × 256 and the sampling ratio was set to be approximately 20%. We used the Daubechies wavelet with two decomposition levels for FIWTA, the wrapping version of the second generation curvelet [8] for FICTA, and the redundant sharp frequency localisation contourlet [9] with 2^5, 2^4, 2^3, 2^2 and 2^1 directional subbands from coarse to fine scales for ICOTA and FICOTA. Gaussian white noise with standard deviation 0.01 was added to the k-space measurements b. The regulation parameter α was assigned a value 0.075, and the maximum iteration number of each algorithm was set as 50. The tolerance of the residue parameter of ICOTA was set as 1×10^{-3}.

To compare the performance of different algorithms, some objective criteria including signal-to-noise ratio (SNR), peak SNR (PSNR) [4], transferred edge information (TEI) [4] and $L2$ norm error were also adopted.

Fig. 1 shows the reconstructed images using different algorithms. According to Fig. 1, FICOTA, ICOTA and FICTA are more effective in suppressing noise and representing edges when compared with FIWTA and FCSA.

Tables 1 and 2 summarise the comparisons of different algorithms based on the objective criteria. The proposed FICOTA outperforms FCSA and FIWTA in term of these criteria, although its running time is slower. In addition, FICTA and ICOTA can acquire comparable reconstruction quality as FICOTA, but their computational time costs are higher than the proposed FICOTA.

Table 1: Comparisons of different algorithms based on brain

Brain	FCSA	FIWTA	ICOTA	FICTA	FICOTA
SNR	19.80	18.62	22.10	22.16	22.51
PSNR	31.73	30.56	34.08	34.09	34.44
TEI	0.809	0.792	0.851	0.858	0.865
*L*2 norm error	0.070	0.080	0.054	0.053	0.051
CPU time (s)	2.48	1.56	39.41	72.39	12.11

Table 2: Comparisons of different algorithms based on shoulder

Shoulder	FCSA	FIWTA	ICOTA	FICTA	FICOTA
SNR	21.58	21.47	24.45	23.51	24.75
PSNR	40.38	40.28	43.29	42.31	43.55
TEI	0.691	0.692	0.753	0.751	0.769
*L*2 error	0.064	0.065	0.046	0.051	0.044
CPU time (s)	2.31	1.78	20.56	67.86	12.66

Conclusion: This Letter proposes to combine FISTA with the contourlet transform to enforce the curve sparsity of MRIs with fast computation. The experimental results show that the proposed method, FICOTA, significantly improves the quality of the reconstructed images, with slightly compromised computation time compared with FISTA-based methods with wavelet regularisation constraints. Compared with FICTA and ICOTA, FICOTA can reach comparable reconstruction effectiveness, but uses much less computation time. Future work may combine the more efficient FISTA-based methods and the more sparse transforms to improve the effectiveness and the efficiency of CS-MRI.

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