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(A) For the case where $z = 0$, result will be certain

Otherwize, if $z < |x-y|$ result will be certain

otherwise uncertain

If the result is certain, simply compare x & y and output the result

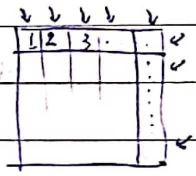
(B) For piece 1 and piece n we have: $|r_1 - r_n| + |c_1 - c_n| \geq n-1$

For a board of size m, the maximum manhattan distance between any two cells is $2m-2$ (cells on opposite ends of longest diagonal)

$\Rightarrow 2m-2 \geq n-1$

$m \geq \frac{n+1}{2} \quad \Rightarrow m_{min} = \left\lceil \frac{n+1}{2} \right\rceil$

If we can arrange all the pieces in minimum size board, we do not need to find a better solution. For size k board we can arrange $2k-1$ pieces satisfying the given condition. For size $m_{min} \Rightarrow 2\left\lceil\frac{n+1}{2}\right\rceil - 1$



which is $\geq n$ since $2\left(\frac{n+1}{2}\right) - 1 = n$

(C) consider the case where cards $1 \dots k$ are in the bottom of the heap, then it might be the case that we complete the process in $n-k$ steps.

For this to happen, we have the following condition

→ for each numbered card (having number m) at index i in the pile (b) $m > k$, $i \leq n-k$

it must be in the hand after $(m-k-1)$ operations

[because at that point cards $1 \dots m-1$ will be at bottom of heap]

$\Rightarrow i \leq m-k-1$

If this condition is not met, we have to start fresh which means ~~there is no card in the bottom of the pile (satisfy 1...k order~~ we have to start by placing card 1, then 2... so on. Suppose we start this process right away.

For each numbered card (having number m) $m>0$ and index i $(i \leq n)$ in the pile (b)

it must be in the hand after $(m-1)$ operations

$\Rightarrow$ if $i \leq m-1 \rightarrow$ ✓ otherwise we need atleast $(i-m+1)$ operations to have this card in a valid position

$\Rightarrow$ let M be the maximum number of operation (wrt each card in pile) then we have to perform atleast M operations before we start the process

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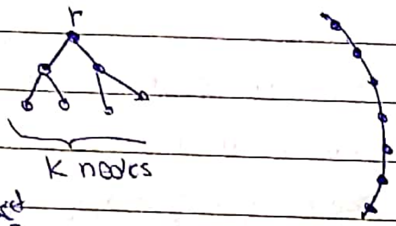
The answer is $(M+n)$. (Note: The M steps would simply be: placing blank card in bottom of pile. If there are no blank cards in hand then $M=0$)

why?

(D) consider a tree having k nodes and an arc having k points



let the root be r

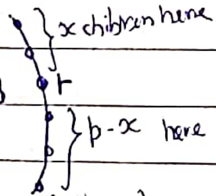
~~The~~ let the number of immediate children of r be p

Number of ways to place ~~select~~ the root $\Rightarrow (p+1)$

~~Suppose we fix~~ Now, ~~there can be~~ if there are x children on one side of the root, then there will be $p-x$ children on the other side $(0 \leq x \leq p)$

for each child c_i let it have ~~c~~ $count(c_i)$ nodes

$\Rightarrow$ we have to allocate $count(c_i)$ consecutive points for each child



Note $\Rightarrow$ $k = \sum_{1 \leq i \leq p} count(c_i) + 1 = count(r)$

What happens if they are not consecutive? → edge intersection

The number of ~~ways to~~ configurations is

→ ~~This situation can be thought of as~~ number of permutations of $p+1$ elements (root and p children) times number of configurations for each subtree.

→ It must be noted that for every permutation of these $p+1$ elements, we can make a valid tree (~~because~~ can prove by induction)

$\Rightarrow$ let $f(r)$: Number of configurations for the above described situation

$$f(r) = (p+1)! \times \prod_{1 \leq i \leq p} f(c_i)$$

* To solve the given problem, assume some node to be the root (let it be r)

Find $f(r)$. Now notice that ~~f~~ f was for an arc, not a circle

~~The answer~~ $\Rightarrow$ we can cyclic shift the nodes $n-1$ times to get ~~n~~ $(n-1) \times f(r)$ more configurations

$\Rightarrow n \times f(r)$ is the answer

→ Note: Repeating this process for different roots will lead to addition of duplicate cases since root is just a relative position