1 State space control (LQG) of stable aircraft

1.1 Creation and description of the model

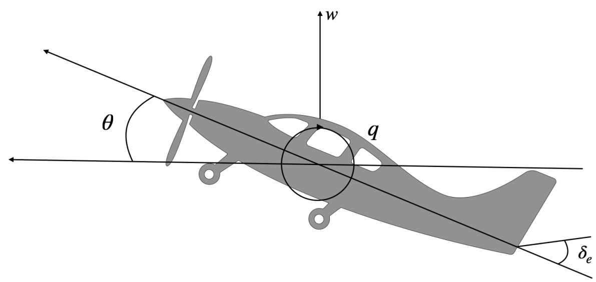

Figure 1: Aircraft longitudinal motion.

β	Sideslip angle
p	Roll rate
r	Yaw rate
Φ	Roll angle
w	Vertical velocity
q	Pitch rate
θ	Pitch angle
δ_e	Elevator deflection

TODO: More about the system model.

The dynamic model of the aircraft system 1 is taken from the article [**?**]. For longitudinal motion:

$$\beta = p = r = \Phi = 0 \tag{1}$$

The transfer function is represented in state-space form by matrices:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{2}$$

$$A = \begin{bmatrix} -0.3149 & 235.8928 & 0 \\ -0.0034 & -0.4282 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -5.5079 \\ 0.0021 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$D = 0$$

$x^T = \begin{bmatrix} w & q & \theta \end{bmatrix}$

$u = \delta_e$

The implemented Model in Simulink is presented in the following diagram 2.

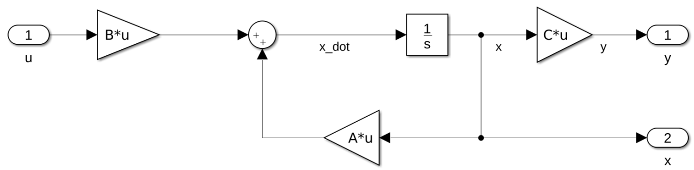

Figure 2: Plant model in Simulink.

In the Matlab system can be represented by the state space function:

```
>>sys = ss(A,B,C,D)
```

1.2 Open loop response

The system is stable only if all the real parts of the eigenvalues are negative.
The dynamics of the model is determined by the following parameters:
Pols:

$$\begin{aligned} 0.0000 + 0.0000i \\ -0.3715 + 0.8938i \\ -0.3715 - 0.8938i \end{aligned}$$

The real parts of eigenvalues are negative. The system is **stable**.
Controllability matrix rank: 3
If the rank of Controllability matrix $Co = [B \ AB \ A^2B \ \ldots \ A^{n-1}B]$ is equal to n (number of states), the system is **controllable**.
Observability matrix rank: 3
If the rank of Observability matrix $O = [C \ CA \ CA^2 \ \ldots \ CA^{n-1}]$ is equal to n, the system is **observable**.

The whole system is described by 2 equations. Open-loop impulse response is presented on figure 3. The result corresponds to our expectations.

Open-loop step response is presented on figure 4. The result corresponds to our expectations. Aircraft started to increase pitch angle.

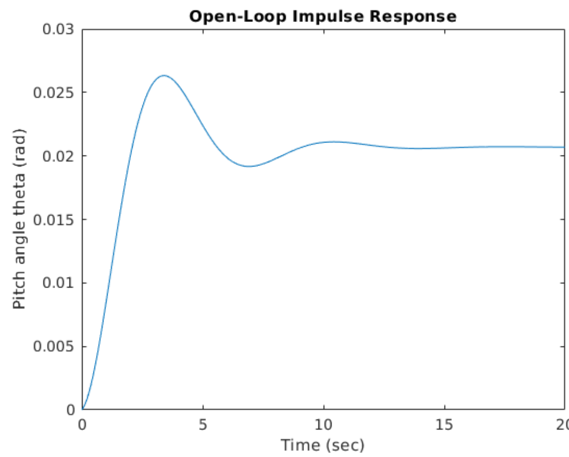

Figure 3: Open-loop impulse response.

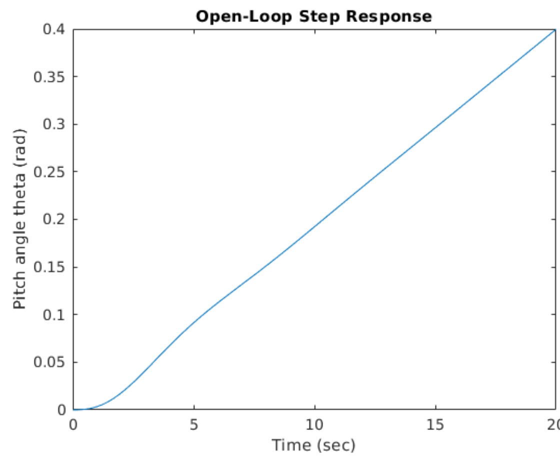

Figure 4: Open-loop step response.

1.3 LQR design - zero control

In modeling zero control response, the input of the system will be $u = -Kx$, $(r = 0)$. Substituting into equation 2, the result of Close-loop system will correspond to equation 3.

$$\dot{x} = (A - BK)x \tag{3}$$

For calculation K gain were used LQR method. This method based on quadratic cost function J. Where Q matrix represent how "fast" controller will approximate the following state. R matrix represent how much we depend on energy that we add to system to control.

$$J = \int (x^T Qx + u^T Ru)dt \tag{4}$$

In Matlab we define Q and R matrix and use a **lqr** command.

```
Q = diag([0, 0, 500]);
R = .1;
K = lqr(A,B,Q,R);
```

Simulink model of Close-loop system is presented in the following diagram 5.

1.4 LQR design - control to the point - rscale

N gain is used to scalar our input for a full-state feedback system to eliminate the steady-state error.

$$u = -Kx + rN \tag{5}$$

where r is final point. Substituting equation 5 into equation 2, the result of Close-loop system controlled to r will correspond to equation 6.

$$\dot{x} = (A - BK)x + BNr \tag{6}$$

For calculation **N** gain in matlab were used rscale function from [**?**].

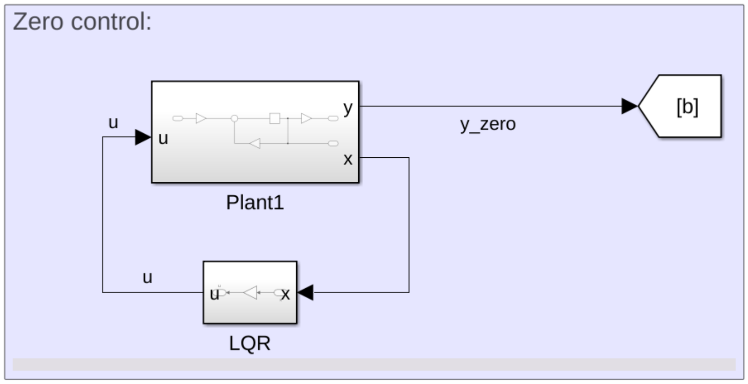

Figure 5: Zero control model in Simulink.

```
N = rscale(A,B,C,D,K);
```

Simulink model of Close-loop system controled to point is presented in the following diagram 6.

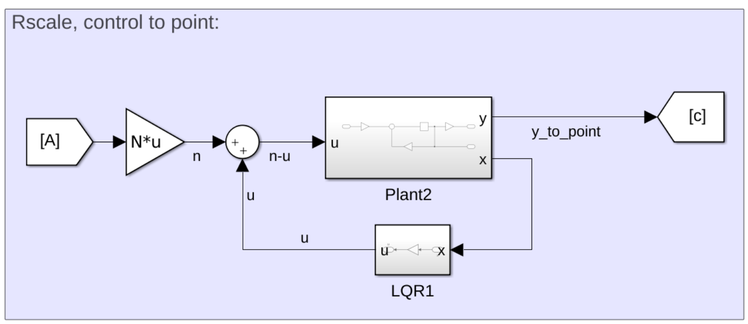

Figure 6: Rscale control to point model in Simulink.

In the Matlab system can be represented by the state space function:

```
>>sys = ss(A-B*K,B*N,C,D)
```

The Close-loop impulse and step response are presented in the following figure 7, 8.

1.5 Study of system behavior

In case Longitudinal dynamic it's possible to measure θ (pitch angle). There is no specific conditions for deflector angle actuator. The $R = 0.1$ matrix we will keep equal to 0.1. In the following figure 9, there are impulse responses for different Q matrices. Where we change x parameter from 1 to 500 with 50 as step.

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & x \end{bmatrix}$$

The following Q and R matrices will suffice for our purposes. These values will be used in the following simulations.

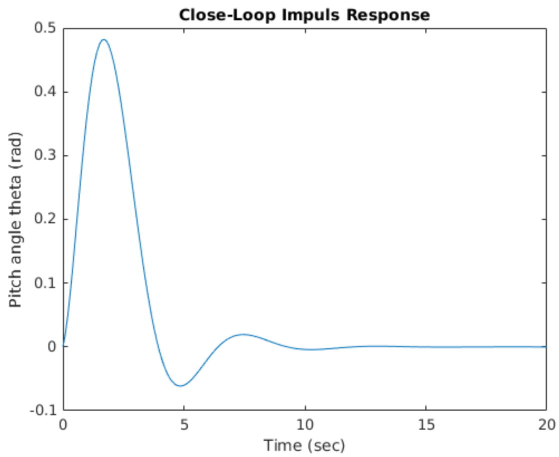

Figure 7: Close-loop impulse response.

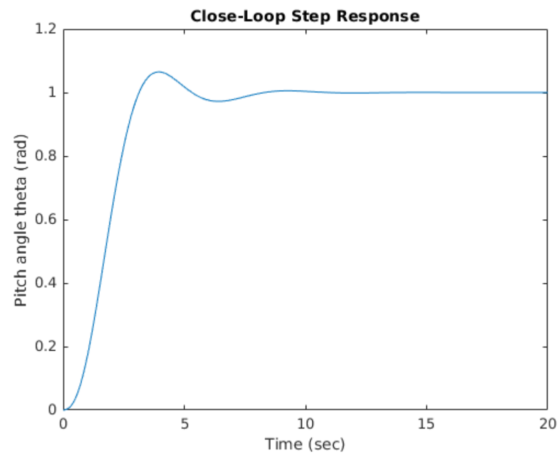

Figure 8: Close-loop step response.

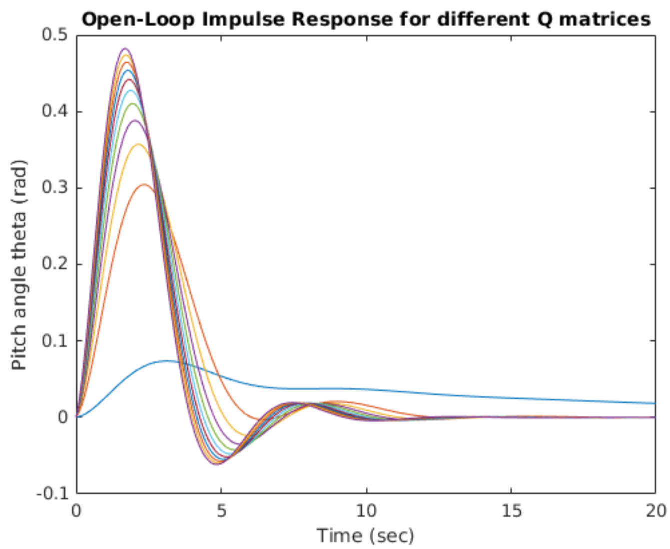

Figure 9: Impulse response for different Q matrices.

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 500 \end{bmatrix} \quad R = 0.1$$

1.6 Observer's proposal

The LQR controller can be used if we have information about the whole state x. A Kalman filter can be used to reconstruct the state from the y measurement. The following system of equations 7 representing Kalman filter.

$$\begin{aligned} \frac{d}{dt}\hat{x} &= A\hat{x} + Bu + Kf(y - \hat{y}) \\ \hat{y} &= C\hat{x} \end{aligned} \tag{7}$$

Simulink model of Kalman filter is presented in the following diagram 10.

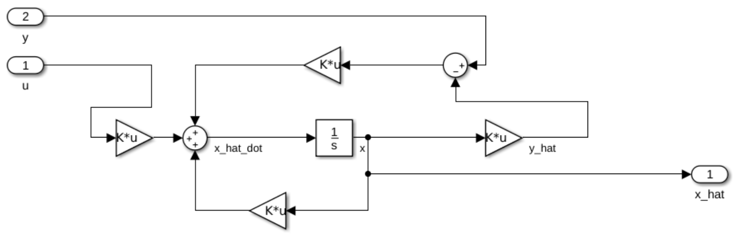

Figure 10: Kalman filter implementation in Simulink

Kalman filter use knowledge about disturbance of state and measurement noise magnitudes. Vd matrix representing covariance of state disturbance and Vn matrix is covariance of measurement noise. There are more options how to calculate Kf gain. Define Vd and Vn matrices we can use the same **lqr** function as follow:

```
Vd = .01*eye(3);
Vn = 1;
Kf = (lqr(A',C',Vd,Vn))';
```

Simulink model of system is presented in the following diagram 11.

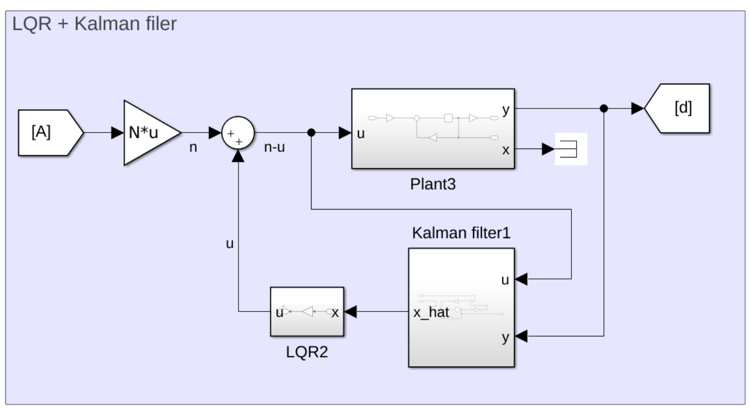

Figure 11: Rscale control to point model in Simulink.

Implementing Kalman filter in Matlab:

```
Akf = A-Kf*C;
Bkf = [B Kf];
Ckf = eye(3);
Dkf = 0*[B Kf];
sys_kf = ss(Akf, Bkf, Ckf, Dkf);
```

1.7 Study of system behavior with an observer (simulation)

The real system control contain measurement noise and disturbance. In model 12 were used w_d and w_n disturbance and noise inputs as Gaussian white noise.

The whole system with LQG implementation, disturbances and noise can be describe as following system of equations **??**.

$$\epsilon = x - \hat{x} \tag{8}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\epsilon} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - KfC \end{bmatrix} \cdot \begin{bmatrix} x \\ \epsilon \end{bmatrix} + \begin{bmatrix} I & 0 \\ I & -Kf \end{bmatrix} \cdot \begin{bmatrix} w_d \\ w_n \end{bmatrix} \tag{9}$$

Simulink model is presented in the following diagram 12.

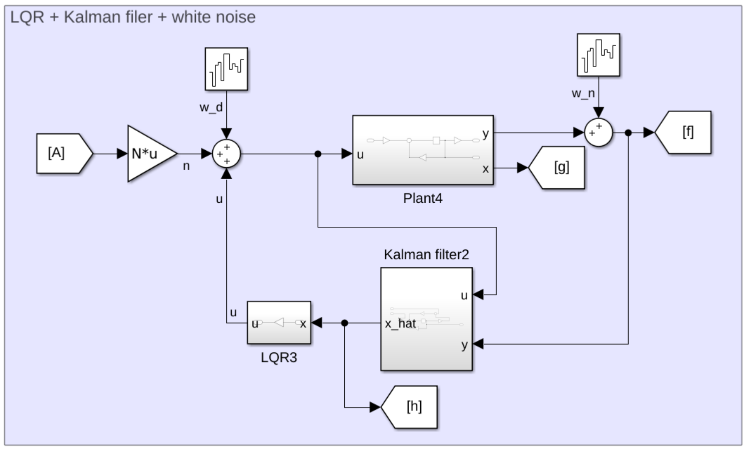

Figure 12: LQG simulink model

The following graph 13 shows the correct operation of the LQG controller. Kalman filter correctly estimates the state with permissible noise level.

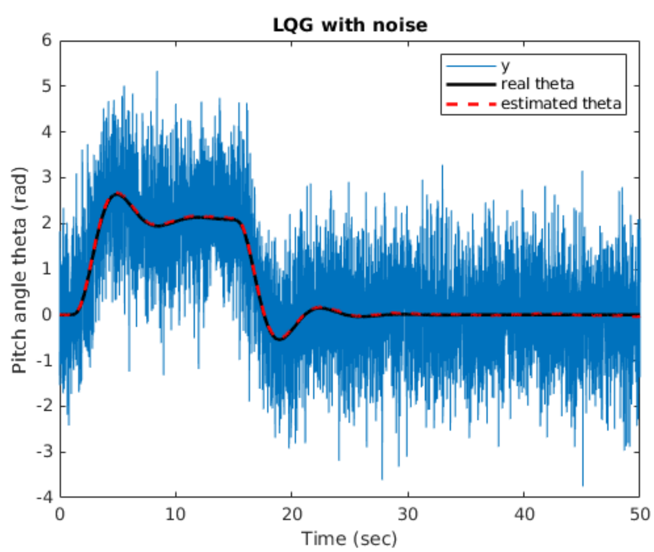

Figure 13: LQG regulator output

With increasing noise amplitude, the model is able to remain functional. This can be verified by changing the disturbance and noise magnitude in the simulink model.

1.8 Evaluation of the whole task and conclusion

All Simulink models are available in appendix.