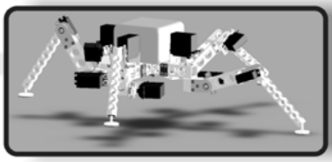
Kinematics of nonholonomic systems

Robert Grepl

2008



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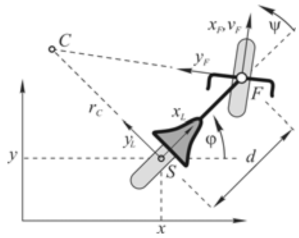
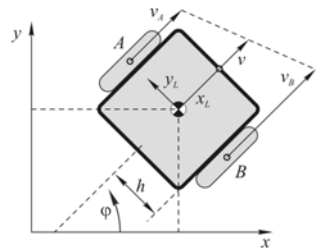
Contents

- what is nonholonomic system
- kinematics of nonholonomic systems
- dynamics of …
- a few notes about car dynamics

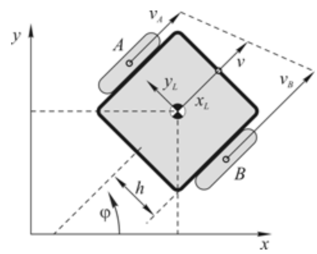
Kinematics of wheeled vehicles

- application: car, bicycle, wheeled mobile robot
- motivation: every control must start with kinematics
  (the simplest one is using kinematics only)
- two coordinate systems:
  - wheels = “joints”
  - position and orientation of car = “cartesian”
- two kinematics task:
  - forward kinematics
    known joints, compute cartesian
  - inverse kinematics
    …



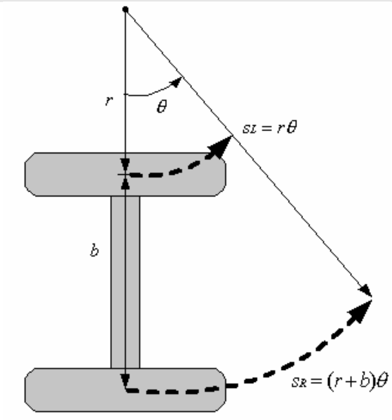


What is peculiar on “wheeled kinematics” ?



- soccer robot (car,…) can **globally** occupy arbitrary position in plane
  = has 3 dof in plane
- but **locally**, it can only
  rotate and move forward ! = 2 dof
- trajectories:
  - in normal manipulators, two different trajectories of **q** leading to particular point **q**_final means also the same final position of manipulator
  - in wheeled devices this is not true !
- consequence: modelling and control of wheeled devices is complicated (still open scientific problem)
  (parking problem)

Intuitive approach



- requirements:
  - R – the radius of trajectory
  - v – peripheral velocity
- solution: …

- note: basic robotic soccer algorithms are based on:
  - potential field planning algorithm
  - kinematics only

9	8	7	6	5	4	3	2	1	0
9	8	7	6	5	4	3	2	1	1
9	8	7	6	5	4	3	2	2	2
9	8	7					3	3	3
9	8	8	8	9	10		4	4	4
9	9	9	9	9	9		5	5	5
10	10	10	10	9	8		6	6	6
11	11	11	10	9	8	7	7	7	7
	12	11	10	9	8	8	8	8	8
		11	10	9	9	9	9	9	9

Systematic approach

- consider system with *n* dof and *k* nonholonomic constraints
- each constraint is of form

$$\mathbf{w}_i\,\dot{\mathbf{q}} = 0 \qquad i = 1,\ldots,k$$

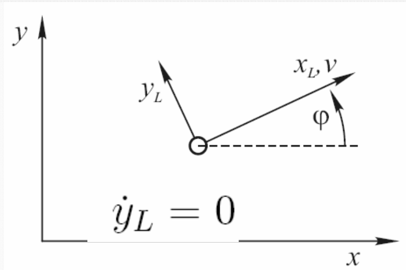
- all constraints:

$$\mathbf{A}(\mathbf{q})\,\dot{\mathbf{q}} = \mathbf{0} \qquad \mathbf{A}(\mathbf{q}) = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \cdots \\ \mathbf{w}_k \end{bmatrix}$$

- find set of *n-k* linearly independent vectors s

$$\mathbf{A}(\mathbf{q})\,\mathbf{s}_j = \mathbf{0} \qquad j = 1,\ldots,(n-k)$$

$$\mathbf{A}(\mathbf{q})\mathbf{S}(\mathbf{q}) = 0$$



$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \mathbf{R}\begin{bmatrix}\dot{x}_L\\ \dot{y}_L\end{bmatrix} = \begin{bmatrix}c_\varphi & -s_\varphi\\ s_\varphi & c_\varphi\end{bmatrix}\begin{bmatrix}v\\ 0\end{bmatrix} = \begin{bmatrix}v\ c_\varphi\\ v\ s_\varphi\end{bmatrix}$$

$$\dot{x}\tan\varphi - \dot{y} = 0$$

$$\begin{bmatrix}\tan\varphi & -1 & 0\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\varphi}\end{bmatrix} = 0$$

$$\mathbf{S} = \begin{bmatrix}\cos\varphi & 0\\ \sin\varphi & 0\\ 0 & 1\end{bmatrix}$$

Systematic approach

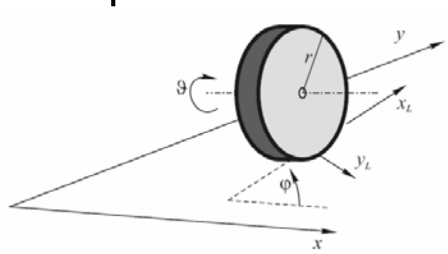
- finally, we have kinematic model
  u1 = v
  u2 = dphi
  (vector **dq** = [dx,dy,dphi] in cartesian system)

$$\dot{\mathbf{q}} = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2$$

Systematic approach

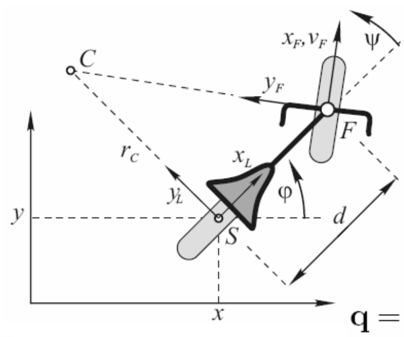
Similarly, we can obtain models for:

- disc in plane



$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \\ \dot{\vartheta} \end{bmatrix} = \begin{bmatrix} rc_\varphi \\ rs_\varphi \\ 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u_2$$

- bicycle

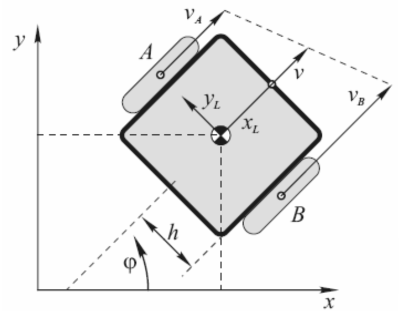


$$\dot{\mathbf{q}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} rc_\varphi \\ rs_\varphi \\ \frac{r}{d}\tan\psi \\ 0 \\ 1 \end{bmatrix} u_2$$

$$\mathbf{q} = \begin{bmatrix} x & y & \varphi & \psi & \vartheta \end{bmatrix}^T$$

Systematic approach

- differentially driven soccer robot



$$\mathbf{q} = \begin{bmatrix} x & y & \varphi & \vartheta_A & \vartheta_B \end{bmatrix}^T$$

$$\dot{\mathbf{q}} = \begin{bmatrix} \frac{rc_\varphi}{2} \\ \frac{rs_\varphi}{2} \\ -\frac{r}{2h} \\ 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} \frac{rc_\varphi}{2} \\ \frac{rs_\varphi}{2} \\ \frac{r}{2h} \\ 1 \\ 0 \end{bmatrix} u_2$$

Dynamics of nonholonomic systems

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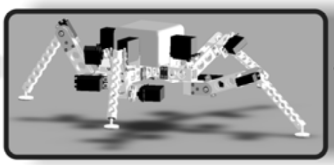

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Problem definition and two solutions

- forward dynamics =
  - known forces in joints (= wheels)
  - and unknown behaviour in cartesian space
- solution:
  - 1) normal dynamic model with unknown lateral forces computed as

$$F_R = -k\dot{y}_L$$

  - 2) reduction of model
(presentation of soccer robot feedback linearization)

Notes: car dynamics

- longitudinal dynamics
- lateral dynamics
  - kinematic approach
  - lateral slip + magic formula...
- ¼ car model
- ½ car model

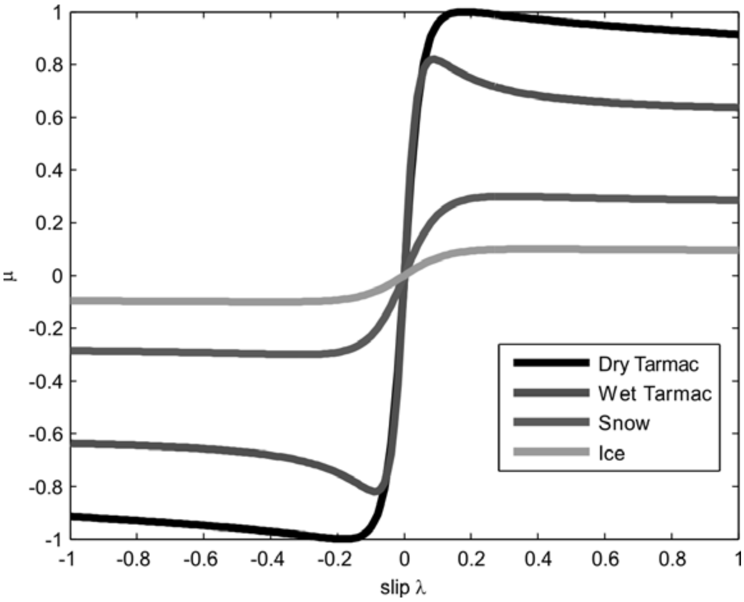
Longitudinal dynamics: Pacejka Magic formula for force in tire

- how to compute longitudinal force of the tire ?

$$F_x = \mu N$$

$$\mu = \mathrm{f}(\lambda)$$

$$\lambda = \frac{\omega R - v}{\max(v, \omega R)}$$



Conclusion

- wheeled vehicles have strange kinematics
  – "parking problem"
- kinematic model is formulated for velocities !
- positions are obtained using integration
- dynamics: two main approaches
  - using force – can include lateral slip
  - using reduction of model – good e.g. for control
- in car dynamics, tire modelling is critical
  (and to have realistic model is very difficult)