Dynamic Model of a Hydraulic Servo System for a Manipulator Robot

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Abstract

In this master thesis, a mathematical model of a hydraulic servo system for a manipulator robot is completed by using several different methodologies. The models proposed are particularly tuned for the DeLaval VMS robotic arm. The parameter identification of the robotic arm is accomplished by dividing the model into several subsystems and investigating each system separately by using catalogue data, experimental data and construction drawings. Furthermore, the assumptions are proposed based on the literature review and the expertise of in-house engineers.

After completion of parameter identification several different mathematical models including linear and nonlinear methodologies are introduced. It is demonstrated that the improved-nonlinear model can successfully mimic the movement of the robotic arm with relatively small errors and it is found to be fairly reliable. Moreover, the errors incurred when chamber pressures are compared with experimental data are found to be relatively small. Furthermore, the improved linear model have successfully delivered an accurate position estimation especially for the medium valve opening, while the chamber pressures are relatively less accurately predicted.

The study further carries out sensitivity (uncertainty) analyses to investigate the crucial parameters of the model since it is sometimes very problematic to precisely estimate these parameters. It is found out that the flow coefficient and supply pressures have remarkable impact on the results of the simulations. Therefore, it is strongly advised that these parameters should be very carefully evaluated during the modeling process.

Finally the bulk modulus models are compared and the influence of the bulk modulus is revealed.

FOREWORD

First and foremost, I owe my deepest gratitude to Fredrik Nilsson for guiding me through the project with the right feedback, advice and support, and making me feel comfortable at DeLaval. This thesis project would not have been possible without you.

In a second instance, I would like to thank to my supervisor Stefan Östlund and my coordinator Nils Brandt for their guidance and feedback through the project.

I want to also express my gratitude to Ricardo Giscombe and all the colleagues in DeLaval, since they were always ready to help and to give me the right working environment through the year.

Finally, yet importantly, words are inadequate in offering my thanks to my beloved family for their love and blessing.

Yalçın Efe
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Notations

Symbol	Description
α_d	Discharge coefficient (-)
σ	Viscous friction parameter (-)
ρ	Density (kg/m^3)
τ	Time constant (s)
φ	Loss factor (-)
α	Piston area ratio (-)
A	Area (m^2)
A_p	Piston area (m^2)
c	Wave speed in oil (m/s)
C_{hA}	Hydraulic capacitance of chamber A (m^3/Pa)
C_{hB}	Hydraulic capacitance of chamber B (m^3/Pa)
c_s	Stribeck velocity parameter (-)
c_v	Flow coefficient (m^3/sec $\sqrt{Pa}$)
$c_{v.com}$	Compensated flow coefficient (m^3/sec $\sqrt{Pa}$)
d_v	Valve spool diameter (m)
E	Effective bulk modulus (Pa)
E'	Effective bulk modulus (Pa)
F	Force (N)
$f(x)$	Valve overlap function (-)
F_c	Coulomb friction (-)
F_{c0}	Coulomb friction parameter (-)
F_{ext}	External force (N)
F_f	Friction force (N)
F_L	Load force (N)
F_s	Static friction (-)
F_{s0}	Stribeck velocity parameter (-)
F_v	Viscous friction (N)

$H(x_v, d_f)$	Valve hysteresis compensation parameter (-)
$K(x_v)$	Valve input signal compensation parameter (-)
K_d	Capacitance term (m^5/Pa)
K_Q	Valve control signal term (1/sec)
$K_{Qp,A}$	Flow-pressure coefficient at chamber A (m^3/sec)
$K_{Qp,B}$	Flow-pressure coefficient at chamber B (m^3/sec)
K_{px}	Valve sensitivity coefficient
$K_{Qx,A}$	Flow gain at chamber A (m^3/sec Pa)
$K_{Qx,B}$	Flow gain at chamber B (m^3/sec Pa)
$m_{A,fl}$	Fluid mass at side A (kg)
$m_{B,fl}$	Fluid mass at side B (kg)
m_p	Piston mass (kg)
m_t	Total mass (kg)
p	Pressure (Pa)
p_A	Chamber pressure A (Pa)
p_B	Chamber pressure B (Pa)
p_N	Nominal pressure drop (Pa)
p_S	Supply pressure (Pa)
p_T	Tank pressure (Pa)
Q	Fluid flow rate (m^3/s)
Q_A	Flow to chamber A (m^3/s)
Q_B	Flow to chamber B (m^3/s)
Q_{Le}	External leakage flows (m^3/s)
Q_{Li}	Internal leakage flow (m^3/s)
Q_N	Nominal flow (m^3/s)
t	Time (sec)
T_h	Damping term (Pa/sec)
$u*$	Valve input signal (-)
V	Control volume (m^3)
V_A	Piston chamber volume (m^3)
V_B	Ring chamber volume (m^3)
$V_{pl,A}$	Pipeline volume at A side (m^3)
$V_{pl,B}$	Pipeline volume at B side (m^3)
x_p	Piston position (m)

x_v	Spool position (-)
y(t)	Hydraulic servo system output (m)
$y_{ref}(t)$	Reference signal (m)
z(t)	Disturbances (N)

Abbreviations

CCD	Charge-coupled Device
HSS	Hydraulic Servo System
LTI	Linear Time-invariant
PI	Proportional-integral
VMS	Voluntary Milking System

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1 Introduction

This chapter provides a brief introduction to the Delaval Voluntary Milking System and sets a ground for the purpose and the limitations of the study. Finally, the chapter ends with a short description of the project outline.

1.1 Background

DeLaval has around 125 years of innovation and experience in the dairy industry by supporting dairy farmers to manage their farms in an appropriate way (DeLaval, 2013). The Voluntary Milking System (VMS™) is a milking system that is developed for automatically milking cows. It enables cows to decide if they want to be milked and the system harnesses up to 3000 liters of milk per day per unit.

1.1.1 VMS multipurpose robotic arm

DeLaval VMS robotic arm is an advanced robotic arm consists of two links driven by three hydraulic actuators. The dynamic control of the arm is maintained by a hydraulic system and; hence, it is relatively fast, quiet and robust. The arm can adjust itself to various type of cows and this leads to longer lifetime milk production per cow. Moreover, the hydraulic power provision benefits lower energy consumption and lower maintenance costs (DeLaval, 2013).

The main purpose of the arm is to handle the teat preparation before milking (including optional pre-spray), to attach and reattach the teat cups, to align the milk tubes and lastly to take-off the cups and spray the teats.

When the cow enters the VMS, firstly the gripper fetches the teat cleaning cup from the teat cup magazine and sterilizes the teats. Afterwards, the teat cups are attached to the teats by a gripper. It should be noticed that the fast and accurate teat localization for a quick and high attachment rates are sustained by an optical CCD camera equipped with dual lasers given that the position of the teats differ for every cow. Once the milking is completed, the teats are disinfected by spraying a chemical shim from a mouthpiece on the arm.

Figure 1.1. Delaval VMS multipurpose robotic arm (DeLaval, 2013)

The arm is modeled according to a human arm's flexible movement range. The general benefits of the advanced robotic arm can be listed as:

- low energy consumption
- high safety for humans and cows due to low running pressures
- fast and accurate dynamics
- requirement for very little maintenance
- compact design.

1.2 Purpose

The main purpose of this thesis is to obtain a flexible and robust model that can simulate hydraulic servo dynamics of the robot arm integrated in DeLaval's foremost voluntary milking system. In this context, the main goals are

- to simulate the hydraulic dynamics of the robot arm and to validate the model by using experimental setup built by DeLaval
- to investigate the crucial parameters and reveal the influence of these parameters on the model reliability.

1.3 Limitations

The thesis is carried out in cooperation with DeLaval; therefore, the model intensively focuses on the current robot arm used in the VMS. The other limitations of the study are:

- Neither pipeline dynamics between the valve and the actuator nor between the supply and the valve are included.

- This study concentrates on the hydraulic dynamics of the robot arm rather than investigating the entire dynamics of VMS robotic system.
- The system is investigated in open loop manner and a development of a complicated controller is avoided.
- A Quasi-two-dimensional[1] experimental setup is built for the validation of the simulation model; however, the influence of the gravity is further discussed.
- The load force on the actuator and flow through valves are not directly measured from the experimental setup. These quantities are calculated indirectly.

1.4 Outline

This study begins with an introduction to hydraulic systems. A literature review is included in the introduction. Chapter 3 gives a detailed description of mathematical models for hydraulic servo systems. Then, in Chapter 4 the description of models that have been used for the simulation of Delaval's hydraulic robot arm and further assumptions made in the process of building the simulations are discussed. Chapter 5 presents the results that are obtained with the process/methods described in the previous chapters and the final chapter is devoted to discussion of the results and recommendations for future work.

[1] The experiments are intented to be implement only x and y axis. However, small deviations occur in the z axis and this phonemena requires further considerations on the gravity. Further discussions are introduced in section 5.1

2 Introduction to Hydraulic Servo Systems

This chapter is intended to deliver an introduction to hydraulic systems and present a brief overview of the relevant aspects of the area. The identification of Delaval's hydraulic system is also presented in this section. The chapter ends with a detailed literature review.

2.1 Hydraulic Servo Systems

With the beginning of 20th century the fluid power technology rapidly evolved with the help of the first generation hydraulic drives that basically compromise some flow control devices that are driving the hydraulic actuator in an open loop manner (Jelali & Kroll, 2004).

After the Second World War, the servo control industry had built up sufficient knowledge to implement accurate closed loop techniques. After that day, the closed loop servo control systems had rapidly advanced and this introduced a wide range of applications where these systems can be practiced.

In recent years, electrical drives have become more popular in the domain of high-performance motion control. However, hydraulic servo-systems are still present in wide diversity of applications that include current industrial motion systems such as machining plants, robotics, motion simulators, fatigue testing systems and so on (Walters, Eng, & Mech, 1991).

This chapter presents an overall description of hydraulic servo systems with a brief summary of several subsystems that are often included in a typical hydraulic servo system.

2.1.1 Fundamentals of Hydraulic Servo Systems

It is possible to divide a hydraulic servo system into a few basic groups of individual components, interconnected to perform a specific function in the system. In this context, a simple structure of a hydraulic system mainly comprises

- hydraulic power supply
- control elements
- actuators
- other auxiliary elements (data transmission elements etc.) (Walters, Eng, & Mech, 1991)

The hydraulic power supply generally consists of a pump and ancillary equipment such as accumulators, relief valves etc. The pump transforms the mechanical power from a prime mover (electric or diesel motor) into hydraulic power at the actuator. Furthermore, valves are used to regulate flow direction, power, and the amount of fluid and pressure delivered to the actuator. Finally, an actuator (e.g. linear or rotary) converts the hydraulic power into the desired form of mechanical power at the output.

The fluid medium lubricates the components, seals in valves and cools the system while providing direct transmission and control of the power.

Various system components are linked together with connectors to regulate the power of the fluid under pressure.

Finally, the fluid storage and conditioning components ensure necessary quality and quantity, and cooling of the fluid.

2.1.1.1 Hydraulic Power Supply

The source of a hydraulic power in servo systems can generally be regarded as pumps. Hydraulic servo-systems often practice pressure controllers in conjunction with variable-displacement pumps, but other controllers may be used to regulate flow, pressure and flow combined, or input power (Younkin, 2003).

There are different configurations in which the power supply circuits can be designed. In an *open circuit system,* the pump draws the fluid from a tank, which is utilized by the system, and the return flow exhausts back to the tank. In a *closed circuit*, there is a continuous flow between a motor and a pump. This effectively means that the pump output flow is sent directly to the hydraulic motor and then returned back in a continuous motion to a pump.

2.1.1.2 Control elements

Hydraulic control valves are devices that use mechanical motion to control a source of fluid power. Thus, it is fair to say that the valves are the most crucial mechanical (or electrical) link to the fluid interface in hydraulic systems. Based on their functional purposes, they diverge in arrangement and complexity (Merritt, 1967).

There are principally four different classes of valves in hydraulics (Jelali & Kroll, 2004):

a) **Pressure valves** are commonly used to maintain the system in a predetermined pressure level to ensure the system safety
b) **Check valves** are a special class of directional valves since they only allow fluid flow in one direction while blocking the flow in the reverse direction.
c) **Flow control valves** regulate the flow rate of a fluid. For example, they may be used to limit the maximum speed of cylinders or motors.
d) **Directional control valves** are one of the most essential devices in hydraulic systems. They permit the fluid to flow into various paths from one or more sources. They

generally contain a spool inside a cylinder that is mechanically or electrically regulated. The position of the spool restricts or permits the flow; thus, it controls the fluid flow. Therefore, the proportional variable control enables infinitely adjustable and rapid variable settings of actuators based on force, speed and stroke position.

The configuration of directional control valves can be classified according to (Merritt, 1967):

a) the number of "ways" that flow can enter and leave the valve. Since all the valves require a supply and a return; and often at least one line to the load, valves are frequently equipped either three-way or four-way.
b) the number of switching (or discrete) positions. This number varies from one in a primitive valve to the usual two or three. However, up to five positions or more can be practiced with special valves.
c) the technique of valve actuation that forces the valve mechanism to move itself to an alternative position. Either external signal commands such as electrical, manual or pilot practice or internal signal commands such as pilot pressure or spring force could be realized to move the valve mechanism.

2.1.1.2.1 Centre types of the directional control valves

Type of the valve center may directly affect the certain characteristics of a valve. In this sense, a "lap" represents the width of the lands relative to the width of the ports in the valve bore (see Figure 2.1.a).

An open center or underlap valve's land width is smaller than the port width in the valve sleeve. This effectively means that for a small amount of time all the valve ports could be connected to each other. In return, this leads to a smooth, pressure-peak-free switching during the cross-over. On the other hand, there is a possibility that undesired actuator movements may occur and; thus, they are often preferred in closed-loop applications (see Figure 2.1.b) (Jelali & Kroll, 2004).

A closed center or overlap valve's land width is larger than the port in the valve sleeve. In this context, for a small amount of time all the valve ports are disconnected from each other. This prevents system pressure on the actuator to collapse during the cross-over. However, this can also lead to undesirable pressure peaks (Totten, 2000).

*A critical center or zero valve*s' land width is equal to the port in the valve sleeve. The majority of commercial valves are listed as zero-lapped valves, basically they can obtain linear flow-signal curve (Jelali & Kroll, 2004).

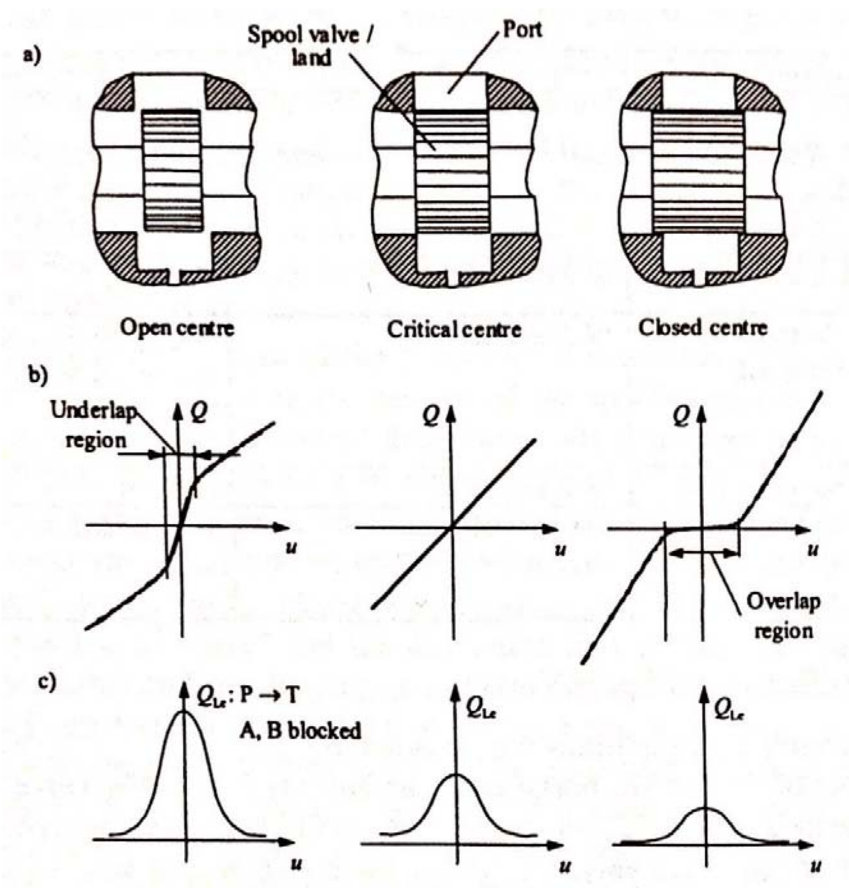

Figure 2.1. a) Definition of center types b) their corresponding flow signal graphs c) leakage flow curves (Jelali & Kroll, 2004)

2.1.1.3 Actuators

A hydraulic actuator converts hydraulic power provided by a pump and processed by control elements into useful mechanical work. The actuators can produce linear, rotary, or oscillatory output.

One of the most frequently used hydraulic actuator is a hydraulic cylinder (also called a linear hydraulic motor) that is used to generate a unidirectional force through a unidirectional stroke. Single acting actuators allow the application of hydraulic power only in one direction while double acting actuators enable users to practice hydraulic force in both direction (Ilango & Soundararajan, 2012).

Double-acting actuators with rods on both sides are called symmetric/synchronizing cylinders and can provide equal force generation in both directions. It should be also noticed that the void volume filled by the hydraulic fluid is also identical for these hydraulic cylinders; hence, the subsequent piston speeds are also the same. However, the symmetric cylinders are greater in length and consequently more expensive than double acting cylinders. Therefore, the majority of actuators that are used in the industry are as double acting cylinders (Jelali & Kroll, 2004) (see Figure 2.2).

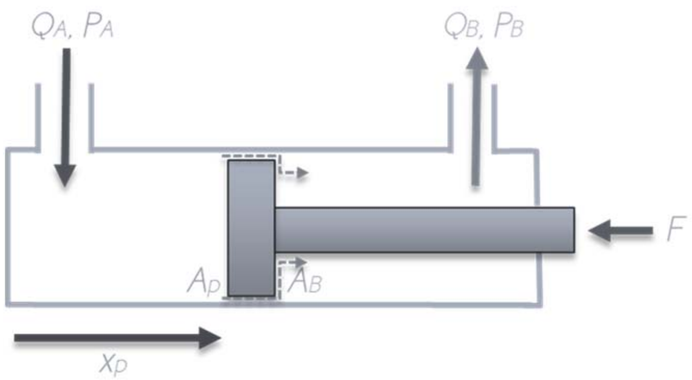

Figure 2.2. A simple representation of a double acting cylinder

2.1.1.4 Other elements

Control loops: Generally, one or more hydraulic actuator outputs are used for controlling purposes; and, this forms a closed loop hydraulic control system as shown in Figure 2.3.

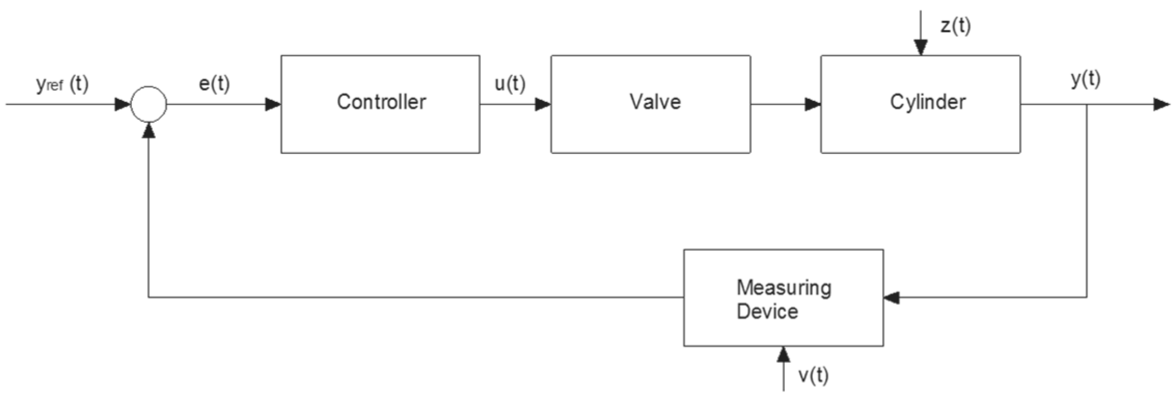

Figure 2.3. Block diagram of a hydraulic servo system in a closed control loop

The electronic measuring devices sense the system output y(t) and based on this measurement, the output is forced to follow the reference signal $y_{ref}(t)$ even though there are disturbances due to load; z(t) and due to sensor errors; v(t) (see Figure 2.3). If a deviation from the reference signal occurs (means an error at summing point), the controller undergoes a corrective action since the error is fed back to the controller until it vanishes. In this context, it is crucial that the tracking is accurate, even though the dynamics of the plant could change during operation.

On the contrary to a closed-loop system, an open loop system has no feedback from the load. Thus, correction is not possible if the load behavior deviates from the desired projection. However, it should be noted that there exist many applications where an open loop control system is adequate enough and able to avoid the extra cost of a closed loop system (Younkin, 2003).

Sensors/Transducers: Many of the current servo system applications require a high degree of accuracy in the system response and hence require precise devices to measure the controlled variables or other state variables such as (Jelali & Kroll, 2004):

- pressure
- flow
- position (angle)
- velocity (speed)
- acceleration/deceleration
- flow.

It should be also understood that the accuracy of measured parameters is crucial since the controlled variables can never be more precise than the measuring device's tolerances. This also implies that the frequency, at which the measurements are taken and transferred to the electronic controller, is essential since it can directly affect the system's response and dynamic behavior (Norvelle, 1999).

2.2 Delaval's hydraulic robot arm

Among the wide variety of hydraulic actuator and valve combinations, Delaval's robotic arm is based on the valve-cylinder combination shown in Figure 2.4. The system consists of a proportional valve and a differential cylinder (sometimes also referred as single rod), identified as $A+\alpha A$ configuration and most a commonly used valve-actuator combination due to its compact size (Jelali & Kroll, 2004).

A control signal u is applied to the valve as a voltage. The main purpose of the input signal is to regulate oil flow through the valve orifices. The power supply unit maintains variable oil supply pressure p_S, while the return flow fed to a tank is at around atmospheric pressure p_T.

The piston divides actuator volume into two different chambers, given as chamber A and B. Fluid flows in or out off the chambers resulting in the movement of the piston. This fluid flow varies the pressures in the chambers, p_A and p_B respectively, and generates the required pressure difference across the chambers to mobilize the load at the actuator. In this sense, it can be noticed that the piston motion depends on the load of the actuator.

A constant oil flow is maintained by a constant valve opening and pressure drop over the valve which in turn results in a linear movement of the piston. However, it should be stressed that the oil is compressible; thus, the chambers act as two springs.

2.3 Overview of the models proposed in the literature

The literature survey is started with an investigation of general strategies on modeling and simulation. In this context, a very enlightening study is carried out by Maria (1997) to provide introductory tutorial on modeling, simulation and analysis. The main questions in the study are "*What is modeling and simulation? What is simulation modeling and analysis? What types of problems are suitable for simulation? How to select simulation software? What are the benefits and pitfalls in modeling and simulation?*". The author claims that the intended audience is those who are involved in system design and modification - system analysts, and the study delivers a road map for developing a simulation model, designing a simulation experiment, and performing a simulation analysis.

In this study, once the road map is drawn and the answers of above questions are found; more specific literature analysis on hydraulic valve/cylinder systems is carried out. It is found out that there are several different models that focus on the different aspects of the hydraulic system.

Menshawy, Moghazy, & Lotfy (2009) investigate another relevant case that explores the dynamic performance of an electrohydraulic system, containing a proportional directional valve. They build a model based on a verified theoretical model of the directional proportional valve, which was previously presented and published by Menshawy (2006). They also verify their simulation by constructing an experimental setup and cross-checking the results. It should be noted that the simulation is built in Matlab/SIMULINK environment.

Tenali (2008) studies a larger scale of electro-hydraulic servo actuator and builds also a mathematical model for applications like aircraft flight control machinery. The author uses Matlab/Simulink models of the actuator and its components and the time response of the linear actuator is also derived by using Matlab/Simulink Software.

It should be noted that even though these studies have different settings, they all are built in Matlab/SIMULINK. The authors given above agree that the available simulation blocks in SIMULINK library reduce the time required for implementation of such models. Moreover, it is also easy to implement a hydraulic system in SIMULINK since there are turn-key block models for some hydraulic components such as valves and actuators.

Dasgupta & Murrenhoff (2011) prefer to examine their closed-loop servo-valve controlled hydro-motor drive system by using Bondgraph simulation technique. They validate their findings of servo-valve model by a comparison with the established results of Gordic et al., (2004). The further focus is established on the variation of the parameters of the PI controller that regulates the servo-valve in their study. A very similar study is also carried out by Dasgupta & Watton (2005); however, they focus on the different pressure-flow characteristics across the valve ports and the orifice.

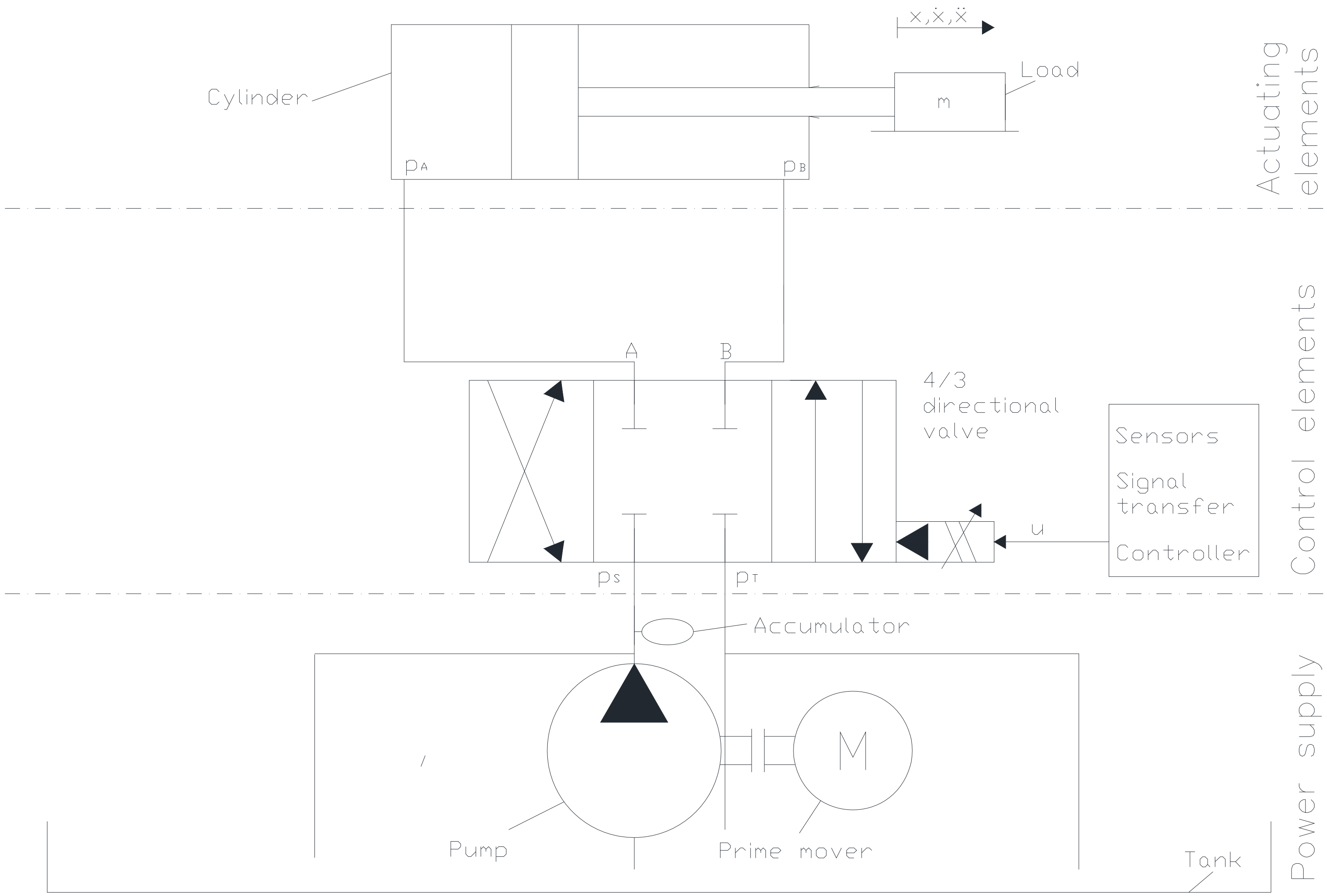

Figure 2.4. Valve-cylinder combination with power supply

One of the most interesting research is carried out by Eryilmaz & Wilson (2006). The authors develop nonlinear equations for a generic proportional valve model and these equations are used to obtain simplified flow rate expressions under commonly recognized assumptions. These equations rely on a set of geometric spool properties and physical model parameters for the flow rate through the valve ports. The study focuses on procurement of a single set of flow rate equations that are applicable regardless of valve center type. They argue that these unified model equations are useful for simulations and nonlinear controller designs.

At this point, it should be pointed out that contrary to SIMULINK models, Matlab script models are often more flexible since users can adapt any part of the script to any other script softwares without a requirement of SIMULINK simulation packages. Moreover, the script models often provide more transparency for its user. Besides, the literature study reveals that the authors are mainly using commonly agreed flow and pressure equations with minor changes. These modifications are mostly done to adapt general equations to more specific cases with reasonable assumptions.

There are also more component or parameter focused studies present in the literature. For example, Mitianiec & Bac (2011) have developed a mathematical model for the hydraulic valve timing system. It focuses on the complex physical phenomena that occur between the valve components and the hydraulic liquid. The mechanical parts of the valve are simulated by SIMULINK while the electric control parameters in order to acquire required lift and timing of valves are obtained with the help of the YARIS SI 1.3 l engine. Similarly, Wei & Xianxiang (2010) propose a second order differential equation to describe the relationship between the effective orifice area and the input signal. On the contrary, Kilic, et al., (2012) develop a model to predict accurately the pressures of a servo-valve controlled hydraulic cylinder accurately by using advanced modeling tools such as artificial neural networks.

The final remark can be made on the book of Jelali & Kroll (2004) since it presents a complete description for each part of a hydraulic-servo system with many different implementation options. However, it should definitely be noted that some of the equations in the book have typos or calculation mistakes; thus, it is advised to readers that not all the equations in this book should be taken as granted.

In this context, it can be concluded that the model and the simulation used in this study is built on using several different modules/concepts proposed in the literature. It is fair to say that the most of the simulation is built upon the suggestions by Jelali & Kroll (2004). However, Eryilmaz & Wilson (2006)'s unified equations are found to be very practical in several occasions and implemented in the current model. Moreover, the SIMULINK models are used as a guide to generate relevant scripts and many modules are translated into Matlab script in the light of previously proposed models. Finally, further improvements on specific components such as valves and hydraulic actuator are made based on the advices given by Wei & Xianxiang (2010) and Kilic, et al., (2012).

The executed model can be regarded as a specific implementation of models proposed in the literature and an experimental setup built by DeLaval. Further improvements are made on the external forces as a result of the movement of the hydraulic robot arm.

3 Mathematical modeling of hydraulic components

This chapter describes mathematical models of essential hydraulic components and comprises the most relevant dynamic and non-linear effects encountered in hydraulic servo-systems. Necessary assumptions are made to keep the model as simple and relevant as possible. It must be noted that the complete theoretical discussion of fluid motion in fluid mechanics is not the scope of this study.

3.1 The fundamental equations of fluid mechanics

Fluids are described as bodies that continuously and permanently deformed under shear stress. It is also proven that the liquids cannot return to their original state after the deformation (Bar–Meir, 2011). In this regard, the normal tension on the surface component of a fluid is given as pressure. The magnitude of the pressure at the given point is identical in all directions. The unit is given as force per unit area in a static fluid and can be calculated as:

$$p = \frac{Force}{Area} = \frac{F}{A} \ [\frac{N}{m^2}] \tag{3.1}$$

3.1.1 Bulk Modulus

It is commonly assumed that the hydraulic fluids are incompressible. However, the fact remains: the fluids present some degree of compressibility (Totten, 2000).

The assumption of an incompressible fluid can be very practical in systems where a tight control of the response is not required and the operation pressures are fairly moderate. In other cases where high pressures(e.g. larger than 100 bars) are applied to a large volume of fluid; a noteworthy amount of energy may be used to compress the fluid by squeezing the fluid's molecules closer together. This leads to a delayed response of the system meaning that a loaded actuator may not move until upstream fluid has been compressed enough. Besides, a compressed fluid stores the energy and this stored energy may cause the actuator to continue its movement even after the closure of the control valve has closed (Jelali & Kroll, 2004).

In this context, George & Barber (2007) define the bulk modulus briefly as:

"The bulk modulus is a measure of resistance to compressibility of a fluid"

In a hydraulic system that operates in the high pressure range, the bulk modulus should be carefully considered since ignoring the bulk modulus could compromise the response time of the system. Besides, applied pressure levels can affect the behavior of the system rather than only compressing the fluid. Therefore, it is essential to design systems with as little fluid as possible between the valve and the actuator (Cundiff, 2002).

3.1.1.1 Influence of entrained air

It is common experience that the hydraulic fluids are aerated in use. Therefore, the entrained air in the hydraulic fluids can lead to significant variations on bulk modules since the air is much more compressible than the oil (George & Barber, 2007).

It should be stated that there are several equations presented in the literature for the estimation of the bulk modulus such as by Jelali & Kroll (2004), Totten (2000) and Wang, Gong, & Yang (2008); however, the expressions given in these proceedings require an accurate determination of many quantities (e.g. the volume of the air entrained in liquid), and thus the experimental data are difficult to use in practice. On the other hand, the agreement made on that the influence of entrained air is generally more significant in the low-pressure region. This phenomenon occurs due to the solubility of air in fluids and increases with pressure. The designers should also be aware of that the air dissolved in a fluid at high pressure can form bubbles in case of large pressure drops — a phenomenon which is known as cavitation.

3.1.1.2 Influence of temperature

Another important parameter is the fluid temperature since fluids tend to expand with increasing temperature, which in turn leads to increase in pressure (Cundiff, 2002).

It should be pointed out that compressing a fluid leads to an increase at the fluid temperature. In a rapid compression, the generated heat cannot be dissipated quickly and this may cause even further pressure increase. However, the generated heat can be dissipated when compression occurs slowly. Thus, the system behavior must be carefully investigated to avoid such phenomenon.

3.1.1.3 Effective bulk modulus

It should be stressed that estimating all these variety of parameters could be very difficult in practice. Thus, many researchers have driven empirical formulas for the estimation of effective bulk modulus E', which includes the effects of temperature and entrained air merged from direct measurements. A frequently used estimation of effective bulk modulus for hydraulic cylinders is given by Lee & Lee (1990)

$$E'(p) = a_1 E_{max}\left(a_2 \frac{p}{p_{max}} + a_3\right) \tag{3.2}$$

with the parameters $a_1 = 0.5$, $a_2 = 90$, $a_3 = 3$, $E_{max} = 18000$ bar and $p_{max} = 280$ bar.

Eggerth (1980) expresses effective bulk modulus as

$$E^I(p) = \frac{1}{k_1 + k_2(p/p_0)^{-\lambda}} \tag{3.3}$$

with parameters k_1 and k_2 found in Table 3.1; p_0 is assumed to be 10 bar.

Table 3.1. Parameters of Eggerth's formula (Eggerth, 1980)

Temperature (°C)	k_1 (10^{-10} m²/N)	k_2 (10^{-10} m²/N)	λ
20	4.943	1.9540	1.480
50	5.469	3.2785	1.258
90	5.762	4.7750	1.100

The influence of the temperature on the effective bulk modulus, estimated by using Eggerth's formula is shown in Figure 3.1.

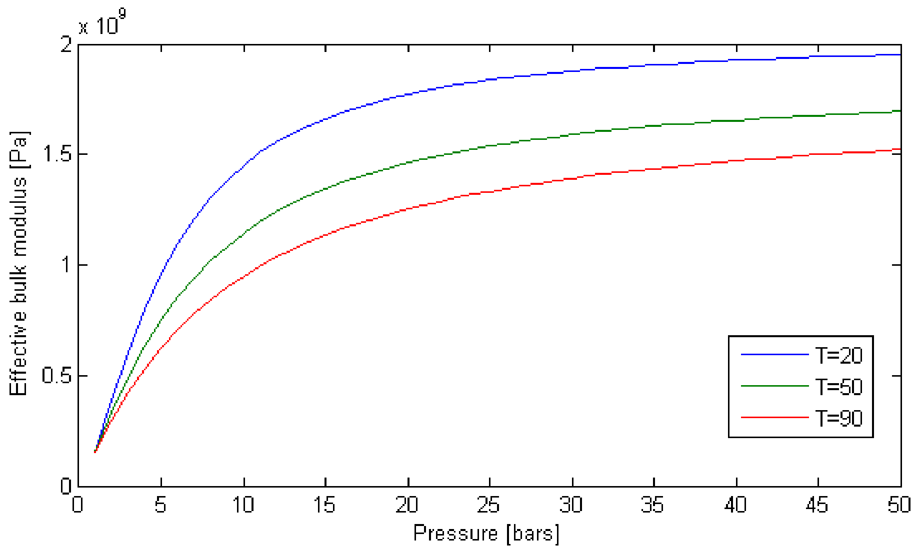

Figure 3.1. The influence of temperature on effective bulk modulus estimated by using Eggerth's formula

Jelali & Kroll (2004) argue that these approximations are sufficient for design purposes; although, experimental data are always preferable.

3.1.2 Flow through orifices

The simple description of an orifice can be given as *"an opening in a vessel, through which the liquid flows out*". Orifices are sudden sharp edged restrictions in the flow passages. The opening area of an orifice can be fixed and variable (Kundu, Cohen, & Dowling, 2011).

The purpose of orifices is to control the flow or to generate a pressure difference between the chambers. Most of the orifice flows occur in high Reynolds numbers and this region is assumed to be of major importance. The Reynolds number (Re) is a dimensionless quantity which is defined to help prediction of similar flow patterns in various fluid flow conditions. Flows with high Reynolds Number are regarded as turbulent flows (Jelali & Kroll, 2004).

The flow estimation in an orifice is governed by speed of the flow and the opening area of the orifice and

$$Q = Av = A\sqrt{\frac{2}{\rho\varphi}(p_1 - p_2)} \tag{3.4}$$

where ρ is the density of fluid and φ is assumed to be the loss factor that depends on the geometry and Reynolds number. Instead of using equation (3.4), it is more practical to use a modified orifice equation given as

$$Q = \alpha_d A\sqrt{\frac{2}{\rho}\Delta p} \tag{3.5}$$

The parameter α_d is called the discharge coefficient and can be calculated theoretically by

$$\alpha_d = \pi/(\pi + 2) = 0.611 \tag{3.6}$$

This value can be used for the sharp edged orifices as long as the flow is turbulent (Jelali & Kroll, 2004).

3.1.3 Flow equations in valves

Similar to orifices, the flow through valves are also defined by orifice equations with a linear relationship between the valve spool position x_v and the flow area given as

$$Q(x_v, \Delta p) = c_v x_v\sqrt{(p_1 - p_2)} = c_v x_v\sqrt{\Delta p} \tag{3.7}$$

with the flow coefficient

$$c_v = \pi d_v \alpha_d \sqrt{\frac{2}{\rho}} \tag{3.8}$$

where d_v is the diameter of the valve spool. In the equation above, the spool opening area is πd_v.

It should be pointed out that the flow coefficient can be best estimated by an experiment. However, such an experiment is generally difficult and requires very precise instruments (Jelali & Kroll, 2004). Therefore, the flow coefficient is generally determined by using catalogue data of the valve suppliers with the help of the formula:

$$c_v = \frac{Q_N}{\sqrt{\Delta p_N}} \frac{1}{x_{v,max}} \tag{3.9}$$

where Q_N is the nominal flow, Δp_N is the nominal pressure drop, and $x_{v,max}$ is the maximum stroke of the valve.

3.1.4 Continuity equation and pressure transients

For a defined quantity of matter, control mass or control volume, it is possible to derive the conservation laws for the mass, momentum and energy. For example, the integral form of the continuity equation (mass) for the given control area seen in Figure 3.2 can be expressed as

$$\int_1^2 \frac{\delta(\rho A)}{\delta t} ds + \rho_2 v_2 A_2 - \rho_1 v_1 A_1 = 0 \tag{3.10}$$

where the density $\rho = \rho(t, s)$ is often not constant in hydraulic servo systems (see section 3.1.1). This equation can be formulated in the differential coordinate-free form

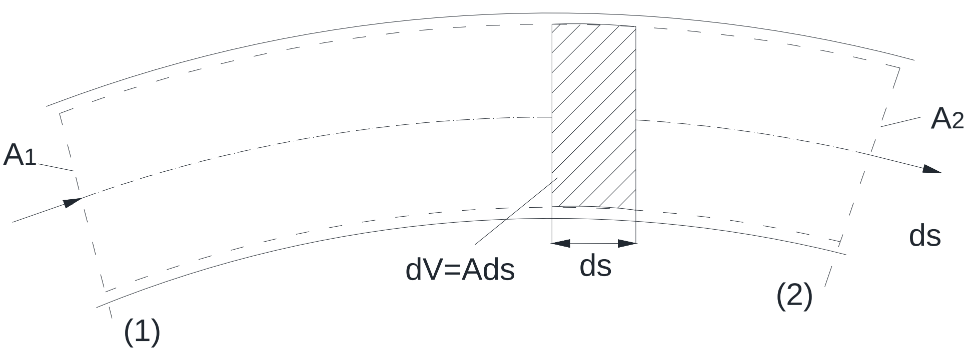

Figure 3.2. The description of a control tube

$$\frac{\delta\rho}{\delta t} + div(\rho v) = 0 \tag{3.11}$$

If the characteristics of the equation above are investigated, it would be seen that there exist two special cases which are

$$if\ \rho = constant \qquad div\ v = 0 \tag{3.12}$$

$$if\ steady\ flow \qquad div(\rho v) = 0 \tag{3.13}$$

Now it is assumed that the control volume is V and the stored fluid inside the control volume has the mass of m with a mass density of ρ. Moreover, the flow rate is steady which implies that the fluid mass stored inside the volume must be equal to the outgoing mass flow rate subtracted from the incoming mass flow rate. Hence, the equation can be rewritten as

$$\sum \dot{m}_{in} - \sum \dot{m}_{out} = \frac{d(\rho V)}{dt} = \rho\dot{V} + V\dot{\rho} \tag{3.14}$$

In a constant temperature, the density change can be written in a linearized form that is given as

$$\rho = \rho_i + \frac{\rho_i}{E}p \tag{3.15}$$

If (3.14) and (3.15) are combined

$$\sum Q_{in} - \sum Q_{out} = \dot{V} + \frac{V}{E}\dot{p} \tag{3.16}$$

Now, if the volume is fixed (V=V_0), (3.16) becomes

$$\dot{p} = \frac{E}{V_0}\left(\sum Q_{in} - \sum Q_{out}\right) \tag{3.17}$$

3.2 Elementary models of HSS components

The purpose of this section is to gather some of the most essential component models of hydraulic servo-systems, including valves, cylinders, pipelines and power supplies.

3.2.1 Valves

The valves are rather complicated devices and Merritt (1967), Jelali & Kroll (2004) and Ilango & Soundararajan (2012) argue that many non-linear dynamics are present. This, in turn, results in a complex non-linear model. The non-linear effects most often encountered are

- *Dead band* of the valves with overlap
- *Square-root behavior* of the flow function
- Complicated *flow-induced forces* in addition with *friction forces*
- *Valve hysteresis* i.e., the identical change in the controller output in both directions leads to a different change in the process value.
- *Saturations* (velocity, flow, stroke etc.)
- *Response sensitivity (resolution)*, i.e., the amount of input signal change required to acquire a measurable valve flow change
- *Repeatability*, i.e., variation in output flow or pressure that may exist when the input command is repeated

3.2.1.1 Pressure-flow equations for valves

Flow through a valve orifice is described by the orifice equation (3.5), which takes the direction of the pressure drop (flow direction) into account. i.e.,

$$Q(x_v, \Delta p) = c_v x_v \sqrt{\Delta p}\, sign(\Delta p) \tag{3.18}$$

Even though (3.18) presents a simple valve model, it does not always hold in all valve applications. More specifically, the spool might not be critically centered (zero overlap), intentionally or due to manufacturing tolerances. For example, both supply and return orifices may be closed at a given time for a closed center valve. Therefore, these concerns impose a more complete valve model that can represent a wider variety of realistic operating conditions.

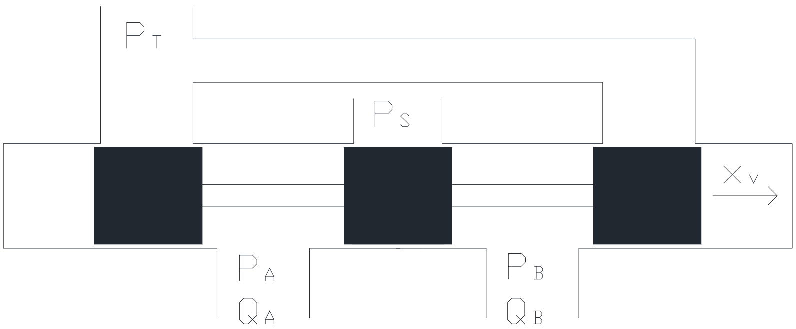

Figure 3.3. Overlapped 4/3 (four ports/three switching position) spool valve

In this context, flow relations for a proportional overlapped valve given in Figure 3.3 can be rewritten as

$$\begin{aligned} Q_A = {} & c_v f(-x_o + x_v) sign(p_S - p_A)\sqrt{|p_S - p_A|} \\ & - c_v f(-x_o - x_v) sign(p_A - p_T)\sqrt{|p_A - p_T|} \end{aligned} \tag{3.19}$$

$$Q_B = c_v f(-x_o - x_v) sign(p_S - p_B)\sqrt{|p_S - p_B|} - c_v f(-x_o + x_v) sign(p_B - p_T)\sqrt{|p_B - p_T|} \quad (3.20)$$

where x_o is the valve overlap and the function $f(x)$ is defined by

$$f(x) = \begin{cases} x & for \quad x \geq 0 \\ 0 & for \quad x < 0 \end{cases} \quad (3.21)$$

It should be noticed that the flow direction is determined by sign of $x_o$and x_v. The model assumes that all the orifices are identical. In case $x_o = 0$, the equations hold valid for critically centered valve.

The equations above also take into account a special case where the pressure of any chamber exceeds the system pressure, e.g., in case of a sudden and rapid reversing of an actuator with a large load. This effectively means that the cylinder would act as a pump in such a case.

Radial clearance and Leakage Flows: Equations (3.18-3.21) take into consideration the pressure-flow behavior of valves. The valves often have radial clearance due to manufacturing tolerances. Therefore, there always exist some extra leakage flows in valves which influences performance, behavior and associated pressure-flow curves for *small openings*. Some other complex models also consider the fact that rounded edges and radial clearances cause laminar flow for small port openings (Merritt, 1967), (Jelali & Kroll, 2004) and (Mitianiec & Bac, 2011).

However, it should be stressed that outside these regions, the equations given above fit very well. Besides, these non-linear dynamics can be added to the model by changing c_v to

$$c_{v.com} = K(x_v)c_v \quad (3.22)$$

The compensation parameter $K(x_v)$ varies between 0 and 1 based on the valve input signal. The value of $K(x_v)$ can be determined by using catalogue data; however, a more accurate description can also be made by using experimental data in case it is possible.

Valve hysteresis: Hysteresis is the point of widest separation between valve flow/input signal characteristic curve with increasing input relative to decreasing input. This phonemena can be seen in Figure 3.4 for a proportional valve with an overlap. This effect can be compensated by an additional parameter $H(x_v, d_f)$ that depends on the valve input signal and flow direction

$$c_{v.com} = K(x_v)H(x_v, d_f)c_v \quad (3.23)$$

for the valve hysteresis and varies motsly between 0.80 and 1.

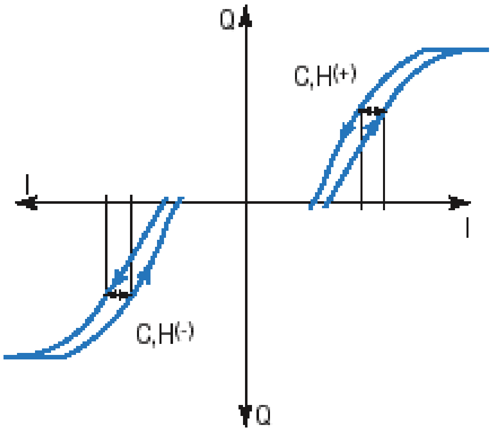

Figure 3.4. The valve hysterisis (Johnson, 2013)

Valve dynamics: The valve dynamics involves a large set of parameters that are seldom known within some range, and mostly completely unknown. Information on these parameters is generally acquired from diverse sources such as manufacturer's catalogues and drawings, literature and heuristic/manual optimization. On the other hand, an accurate analytical description would be time consuming and extremely difficult to verify (Dasgupta & Watton, 2005), (Gordic, Babic, & Jovicic, 2004), (Jelali & Kroll, 2004) and (Merritt, 1967).

Nowadays manufacturer catalogues provide step responses and time constants and this information can be used to model simplified valve dynamics such as a first order system (Fisher, 2013).

The response time of the valve is defined by a parameter called T_{63}, which is the duration measured from the beginning of the input signal change until the output reaches 63% of the corresponding transformation (Fisher, 2013). Here, the valve dynamics approximate a first order system with a time constant of τ. First order linear time-invariant (LTI) systems are characterized by the differential equation

$$\frac{dV}{dt} + \frac{1}{\tau}V = f(t) \tag{3.24}$$

where τ describes the exponential decay constant and V is a function of time t

$$V = V(t) \tag{3.25}$$

The right-hand side of (3.25) is defined as the forcing function f(t). It describes an external driving function of time, which can be given as the system input, to which V(t) is the response, or system output. A very classical example of f(t) is the Heaviside step function, denoted by u(t)

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \qquad (3.26)$$

In this context, if the forcing function is replaced with a step input, the equation becomes

$$\frac{dV}{dt} + \frac{1}{\tau}V = Au(t) \qquad (3.27)$$

The general solution to (3.27) for time t > 0, given that $V_{(t=0)} = V_0$ can be formulated as

$$V(t) = V_0 e^{-t/\tau} + A\tau(1 - e^{-t/\tau}) \qquad (3.28)$$

It should be noted that the long-time solution would be independent of time and the initial conditions.

$$V_\infty = A\tau \qquad (3.29)$$

The step response of a system for initial value $V_0=0$, $A=1$ and $\tau = 30\ ms$ can be seen in Figure 3.5. It can be easily observed that the output is estimated as 63% of the input after $= 30\ ms$ is reached.

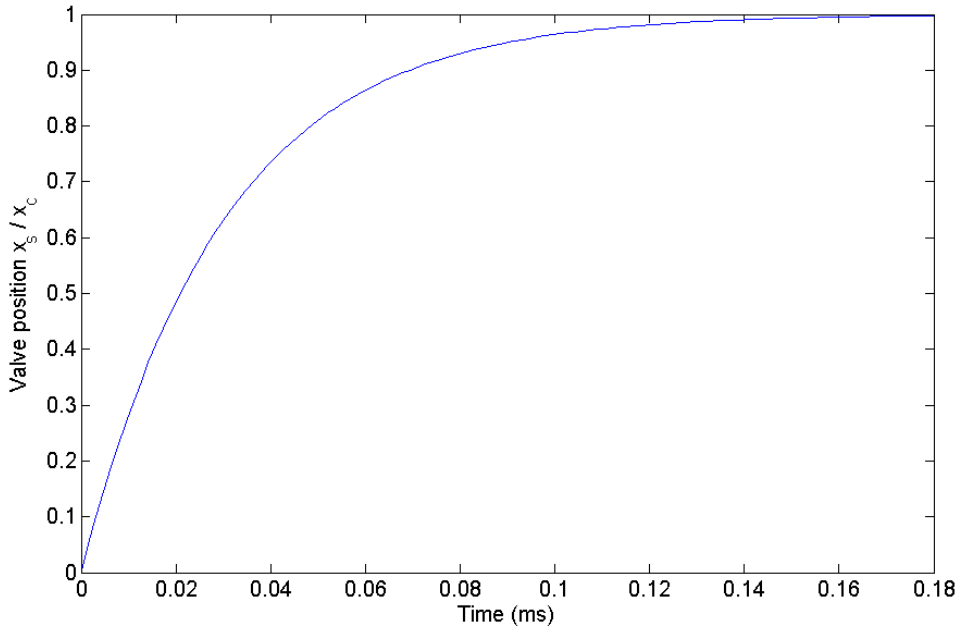

Figure 3.5. The step response of the valve for $\tau = 30\ ms$

3.2.2 Pipelines

The connection of hydraulic components is carried out by pipelines. Jelali & Kroll (2004) argue that the whole pipeline volume can be included in the corresponding cylinder chamber as an inefficient volume if the length of the pipeline is smaller than a certain limit. This limit is given by

$$l < c/10f_{max} \tag{3.30}$$

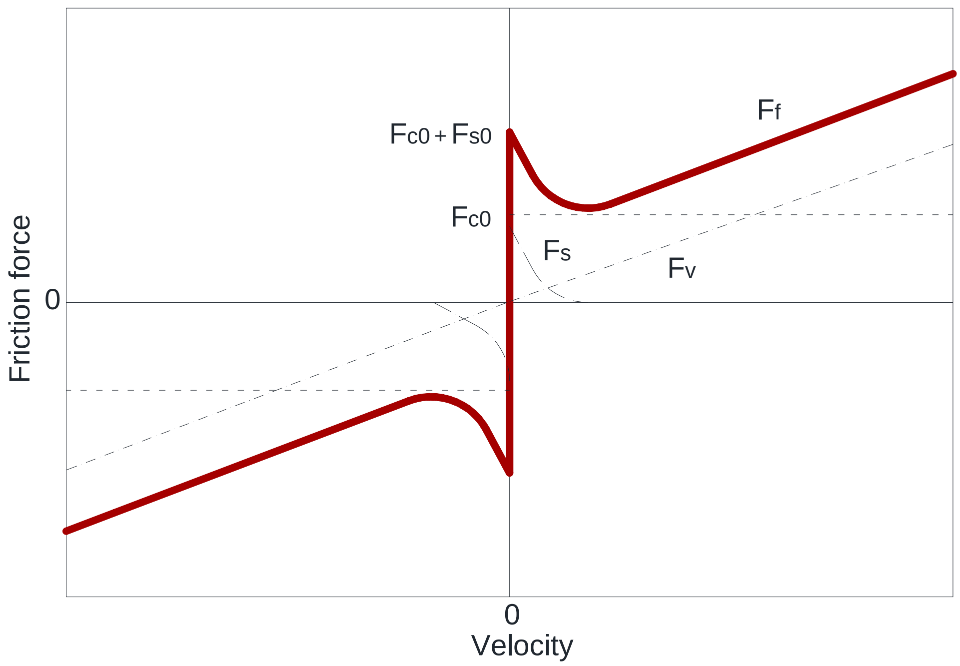

Figure 3.6. Velocity-dependent friction force (Stribeck curve)

where c is the wave speed in oil and f_{max} is the maximum value of the interesting frequency. If the estimated length exceeds the length of pipeline, an additional simulation model for the pipelines must be created.

Goodson & Leonard (1972) suggest using (3.31) for estimation of wave speeds in rigid lines

$$c_s = \sqrt{\frac{E}{\rho}} \tag{3.31}$$

The bulk modulus in the (3.31) can be replaced by the effective bulk modulus *E'* and in this way the influence of entrained air and mechanical compliance can be taken into account (see section 3.1.1).

Applications when pipelines have to be modeled are listed below:

1. In the application where the oil supply unit is detached from the hydraulic servo system with long connecting lines (Jelali & Kroll, 2004). This type of configuration can lead to undesired oscillations: it should be appointed that Viersma (1980) has investigated these systems and provided a lot of insight about the dynamics of such systems.
2. In some applications the servo valves cannot be located on the actuator chambers due to space restrictions. Such a setup results in a relatively long pipeline between the valve and actuator chambers, hence the pipeline modeling is required in these cases (Jelali & Kroll, 2004).
3. In some very specific cases, the modeling of supply and return lines stands in the heart of the modeling process (e.g., aircraft brake hydraulic systems). However, these pipelines include quite different parts, such as rubber hoses etc. This, in turn, may have significant impact on the generation of considerable pressure peakings and oscillations; hence, an additional hydraulic notice.

It should be stated that further investigation of hydraulic pipelines and some specific models are given in the literature by Jelali & Kroll (2004) Novak, Guinot, Jeffrey, & Reeve (2010) and Younkin (2003) in case the reader desires to know more about pipeline modeling. However, these more sophisticated models are out of the scope of this research simply because of the given conditions above: they do not apply to the model that is investigated in next chapter.

3.2.3 Hydraulic cylinder

The theoretical modeling of hydraulic cylinders is well-known and the basic principles are given earlier by various authors such as Viersma (1980) and Merritt (1967). However, an experimental setup can improve the accuracy of the parameters used in the cylinder model. Therefore, the theoretical model is slightly simplified to develop a compact model that can be easily identified by experiments.

The most important non-linear effects that are essential for the cylinder model are

- *Geometrical asymmetry* due to the difference on the area of each side of the piston
- Pressure dependence of *bulk modulus* in combination with the fluid elasticity as well as the elasticity of mechanical compliance
- "*Actuator stiffness*" dependent on the *position* i.e., the damping ratio of the transient dynamics varies with cylinder position
- Friction forces involve with a highly non-linear Coulomb friction term.

It should be stressed that the compressibility of the oil influences the dynamics of hydraulic servo systems remarkably. The hydraulic fluid shows a spring like properties and this introduces a second order mass-spring system whose natural frequency restricts the bandwidth of any hydraulic servo-system abruptly. The system can be damped by leakage flow and viscous friction but their capabilities of damping are relatively small (Jelali & Kroll, 2004).

3.2.3.1 Pressure dynamics in cylinder chambers

To begin with, (3.17) can be applied to each of the cylinder chambers, it yields

$$Q_A - Q_{Li} = \dot{V}_A + \frac{V_A}{E'(p_A)}\dot{p}_A \tag{3.32}$$

$$Q_B + Q_{Li} - Q_{Le} = \dot{V}_B + \frac{V_B}{E'(p_B)}\dot{p}_B \tag{3.33}$$

where V_A is the piston chamber volume and V_B is the ring volume chamber. The given volumes include both the valve connecting line and the chamber volumes. Moreover, Q_{Li} and Q_{Le} describe the internal and the external leakage flows respectively. The volumes of the chambers can be given as

$$V_A = V_{pl,A} + \left(\frac{S}{2} + x_p\right)A_p = V_{B0} + x_pA_p \tag{3.34}$$

$$V_B = V_{pl,B} + \left(\frac{S}{2} + x_p\right)A_p = V_{B0} - \alpha x_pA_p \tag{3.35}$$

where $V_{pl,A}$ and $V_{pl,B}$ are the volumes of the pipelines at each side respectively. Therefore each volume has an *efficient part* (i.e.,the volume of the chambers) and *inefficient part* (i.e., the volumes of the pipelines between the valve and the actuator). The time derivatives of the volumes can be formulated as

$$V_A = \dot{A}_px_p \tag{3.36}$$

$$V_A = -\alpha\dot{A}_px_p \tag{3.37}$$

(3.32) and (3.33) can be reformulated to provide pressure dynamics in cylinders

$$\dot{p}_A = \frac{1}{C_{hA}}\left(Q_A - x_pA_p - Q_{LiA} - Q_{LeA}\right) \tag{3.38}$$

$$\dot{p}_B = \frac{1}{C_{hB}}\left(Q_B + \alpha x_pA_p + Q_{LiB} - Q_{LeB}\right) \tag{3.39}$$

The hydraulic capacitance of each chamber can be written as

$$C_{hA} = C_h(p_A, x_p) = \frac{V_A(x_p)}{E'_A(p_A)} = \frac{V_{pl,A} + (x_{p0} + x_p)A_p}{E'_A(p_A)} \tag{3.40}$$

$$C_{hB} = C_h(p_B, x_p) = \frac{V_B(x_p)}{E'_A(p_A)} = \frac{V_{pl,B} + (x_{p0} + x_p)A_p}{E'_B(p_B)} \tag{3.41}$$

Internal leakage from one chamber to another can be given as

$$Q_{Li} = C_{Li}(p_A - p_B) \tag{3.42}$$

where C_{Li} is the internal leakage flow coefficient. The external leakage from chambers to case drain can often be neglected.

3.2.3.2 The piston motion equations

The governing equation for the piston motion can be found by using Newton's second law on the piston forces. Thus, it becomes

$$m_t\ddot{x}_p + F_t(\dot{x}_p) = (p_A - \propto p_B)A_p - F_{ext} \tag{3.43}$$

It should be pointed out that the total mass m_t is sum of piston m_p and the mass of hydraulic fluid in the cylinder chamber and in the pipelines; $m_{A,fl}$ and $m_{B,fl}$ respectively:

$$m_t = m_p + m_{A,fl} + m_{B,fl} \tag{3.44}$$

Hence, the mass of the fluid can be calculated by

$$m_{A,fl} = \rho[V_{pl,A} + (x_{p0} + x_p)A_p] \tag{3.45}$$

$$m_{B,fl} = \rho[V_{pl,B} + (x_{p0} + x_p)\alpha A_p] \tag{3.46}$$

The one of the important aspect of the motion equation can be regarded as friction forces on the piston (Ilango & Soundararajan, 2012). The common friction model is given by the Stribeck friction curve (after Stribeck, 1902) and a typical curve can be seen in Figure 3.6. This curve models the friction forces as a function of the speed and the formulation is given as

$$\begin{aligned} F_f(\dot{x}_p) &= F_v(\dot{x}_p) + F_c(\dot{x}_p) + F_s(\dot{x}_p) \\ &= \sigma(\dot{x}_p) + sign(\dot{x}_p)\left[F_{c0} + F_{s0}exp\left(-\frac{|\dot{x}_p|}{c_s}\right)\right] \end{aligned} \tag{3.47}$$

The characteristics of the curve are determined by three parameters: viscous friction F_v, static friction F_s, and Coulomb friction F_c. The parameter σ is the viscous friction, F_{c0} is the parameter for Coulomb friction, F_{s0} and c_s (known as Stribeck velocity) are the parameters for static friction.

4 Simulation models

This chapter firstly characterizes and interconnects the subsystems of Delaval's robot arm and then presents a nonlinear model built upon the relevant mathematical modeling knowledge given in the previous chapters. Further model simplifications are also proposed and several linear models are derived. Additionally, determination of the specific model parameters is represented for a complete understanding of the process.

4.1 Characterization and interconnection of subsystems

The complexity of the modeling of system can be reduced by breaking it into a number of subsystems (see Figure 4.1).

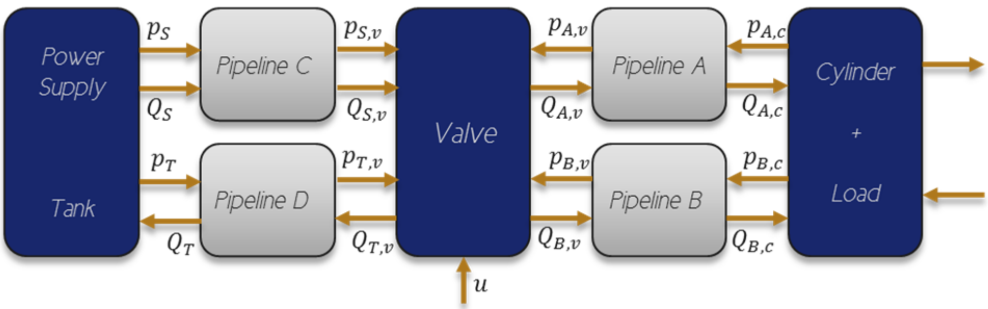

Figure 4.1. Subsystems of hydraulic servo-systems with interconnections'

I. Depending on the actuator pressures, the valve converts the control signal into oil flow. The valves are developed to be fast and assumed to show linear input-output behavior; however, these devices' actual behavior is usually not ideal, which is also the case with the valve used in Delaval's robot arm. This non-ideal behavior of the valve propagates through the entire robot arm since the valve flow drives the actuator. It must be pointed out that this is the main reason why the valve, in the modeling process, is explicitly considered as a separate system.

II. The hydraulic actuator draws oil flows Q_A and Q_B and provides mechanical displacement to the robot arm to carry out the assignment presented in section 1.1.1. The outputs of the actuator are given as actuator pressures p_A and p_B, and position x_p or velocity $\dot{x}_p$ while the only input is the external force F_{ext}. F_{ext} could be any unexpected event that may lead additional force on the arm.

The inertia (impedance) of the actuator base is developed large enough to avoid parasitic energy exchange between the base and the actuator. This effectively means that no parasitic motion of the base can occur.

III. If there is a long actuator stroke and relatively long pipelines, pipelines between the valve and the actuator should be considered as a third subsystem. This is simply because of the compressibility and the inertia of the oil; pressure waves travel with a velocity inside the pipelines and are almost ideally reflected at the end of the line. Because of these dynamics, pressures and flows at the cylinder side must be separated from those at the valve side (can be observed in Figure 4.1 since these parameters named with different indices at each side). However, it is not the case in VMS system and this phenomena is neglected.

IV. Similar to the pipelines between the cylinder and valve, the pipelines between the power supply system and the valve can be additionally considered as a fourth subsystem in case the distance between main line and accumulator/valve is large. However, in Delaval's hydraulic system, the valves are placed relatively closer to the cylinder.

V. The power supply cannot maintain the constant pressure flow.

It should be also stressed that the pipeline dynamics can be neglected in case of low-frequency behavior since they do not play a significant role in the input/out behavior. Therefore, the model of the hydraulic servo-system is simplified into two subsystems (valve + cylinder) with a non-ideal power supply.

4.2 Structured nonlinear model

In this section, a reasonably simple model that includes most of the dynamic and non-linear effects that are relevant to a hydraulic servo system will be presented.

4.2.1 Assumptions

The assumptions are made to simplify the derivation of the non-linear model. These assumptions are

- Turbulent flow occurs through the flow of the valve.
- The inefficient volumes such as pipelines between the valve and the actuator are included in relevant cylinder chambers as inefficient volumes.
- The friction model includes coulomb, static and viscous friction (e.g. Stribeck model)
- The surroundings of the actuator and the load are rigid.

- Any possible dynamic behavior of pressure in the pipelines between valve and actuator is assumed to be negligible (i.e., valve is located on the actuator).

4.2.2 The model description

A clear overview of the proposed non-linear model is shown in Figure 4.2. It can be seen that the model consists of three different modules given as valve dynamics, pressure dynamics and the mechanical part. It should be noted that F_L is the load force, F_f is the friction force and F_{ext} is any external force acting on the cylinder.

For positive control input signal $u \geq 0$, the pressure dynamics model takes the form of

$$\begin{bmatrix} \dot{p}_A \\ \dot{p}_B \end{bmatrix} = \begin{bmatrix} \frac{E'_A(p_A)}{V_A(x_p)}\left[-A_p \dot{x_p} - K_{Li}(p_A - p_B)\right] \\ \frac{E'_B(p_A)}{V_B(x_p)}\left[+\propto A_p \dot{x_p} + K_{Li}(p_A - p_B)\right] \end{bmatrix} + \begin{bmatrix} \frac{E'_A(p_A)}{V_A(x_p)} c_{v,com}\sqrt{p_S - p_A} \\ -\frac{E'_B(p_B)}{V_B(x_p)} c_{v,com}\sqrt{p_B - p_T} \end{bmatrix} \quad (4.1)$$

and for negative control input signal $u < 0$

$$\begin{bmatrix} \dot{p}_A \\ \dot{p}_B \end{bmatrix} = \begin{bmatrix} \frac{E'_A(p_A)}{V_A(x_p)}\left[-A_p \dot{x_p} - K_{Li}(p_A - p_B)\right] \\ \frac{E'_B(p_A)}{V_B(x_p)}\left[+\propto A_p \dot{x_p} + K_{Li}(p_A - p_B)\right] \end{bmatrix} + \begin{bmatrix} \frac{E'_A(p_A)}{V_A(x_p)} c_{v,com}\sqrt{p_A - p_T} \\ -\frac{E'_B(p_B)}{V_B(x_p)} c_{v,com}\sqrt{p_S - p_B} \end{bmatrix} \quad (4.2)$$

These equations can be further simplified by assuming constant bulk modulus and fixed chamber volumes; however, this leads to additional errors. Therefore, further simplifications are avoided and the pressure dynamics model will be kept as it is given.

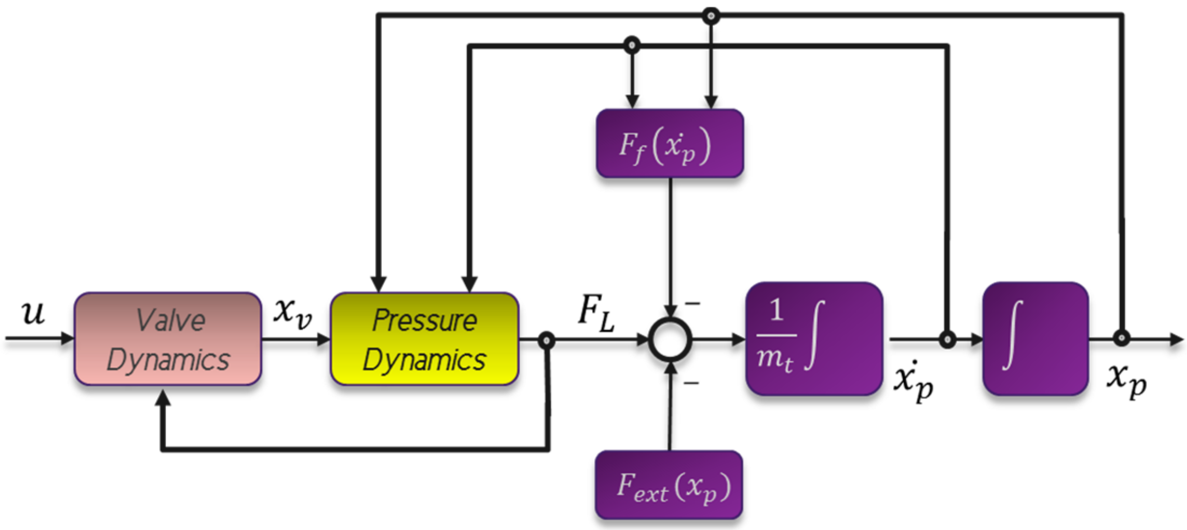

Figure 4.2. Structured model of the proposed hydraulic servo-system

Firstly, the valve dynamics module can be examined. The valve dynamics uses a first order model to convert the reference signal to actual valve opening. This is given by

$$\frac{dV}{dt} + \frac{1}{\tau} V = u \tag{4.3}$$

where u is given as the reference input of the valve.

However, it should be remembered from section 3.2.1 that the valve coefficient is dependent of input signal, flow direction and the pressures $x_v(u^*, P, x_{flow})$; hence, these non-linear effects are also included in the valve module.

Finally the mechanical module is formulated

$$\begin{bmatrix} \dot{x}_P \\ \ddot{x}_P \end{bmatrix} = \begin{bmatrix} \int \ddot{x}_P \\ \frac{1}{m_t(x_p)} \left[(p_A - \propto p_B) A_p - F_f(\dot{x_p}) - F_{ext}(x_p) \right] \end{bmatrix} \tag{4.4}$$

It should be stressed that this model maintains a reasonably well described non-linear hydraulic servo model with practical assumptions (see section 4.3.1).

It is also possible to further develop linearized and even more simplified models from the given non-linear model. These will be investigated further in this chapter.

4.3 Structured linearized model

The linearization of the model starts with linearization of the pressure–flow equations of the valves [2]. Reformulation of (3.18) with help of Taylor series at a particular operating point $P_0 = (x_{v0}, p_{A0}, p_{B0})$[3]

$$\Delta Q = \frac{\partial Q}{\partial x_v} |_{P_0} \Delta x_v + \frac{\partial Q}{\partial p} |_{P_0} \Delta p + \cdots \tag{4.5}$$

results in

$$\Delta Q_A = K_{Qx,A} \Delta x_v + K_{Qp,A} \Delta p_A \tag{4.6}$$

$$\Delta Q_B = K_{Qx,B} \Delta x_v + K_{Qp,B} \Delta p_B \tag{4.7}$$

By using (4.6) and (4.7); the flow gains can be defined as

[2] More detailed information on approximation of non-linear systems can be found in Appendix A.
[3] The deviation from the operation point at which the model is linearized is symbolized by the operator "Δ".

$$K_{Qx,A} = \frac{\partial Q_A}{\partial x_v}|_{P_0} = \begin{cases} c_v\sqrt{(p_S - p_{A0})} & for\ x_v > x_{v,olap} \\ c_v\sqrt{(p_{A0} - p_T)} & for\ x_v < -x_{v,olap} \end{cases} \quad (4.8)$$

$$K_{Qx,B} = \frac{\partial Q_B}{\partial x_v}|_{P_0} = \begin{cases} -c_v\sqrt{(p_{B0} - p_T)} & for\ x_v > x_{v,olap} \\ -c_v\sqrt{(p_S - p_{B0})} & for\ x_v < -x_{v,olap} \end{cases} \quad (4.9)$$

In the same way, the flow-pressure coefficient can be defined as

$$K_{Qp,A} = \frac{\partial Q_A}{\partial p_A}|_{P_0} = \begin{cases} \dfrac{-c_v\, x_{v0}}{2\sqrt{(p_s - p_{A0})}} & for\ x_v > x_{v,olap} \\ \dfrac{c_v\, x_{v0}}{2\sqrt{(p_{A0} - p_T)}} & for\ x_v < -x_{v,olap} \end{cases} \quad (4.10)$$

$$K_{Qp,B} = \frac{\partial Q_B}{\partial p_B}|_{P_0} = \begin{cases} \dfrac{-c_v\, x_{v0}}{2\sqrt{(p_{B0} - p_T)}} & for\ x_v > x_{v,olap} \\ \dfrac{c_v\, x_{v0}}{2\sqrt{(p_s - p_{B0})}} & for\ x_v < -x_{v,olap} \end{cases} \quad (4.11)$$

Finally, the pressure sensitivity coefficient can be given as

$$K_{px} = \frac{K_{Qx}}{K_{Qp}} \quad (4.12)$$

The coefficients K_{Qp} , K_{Qx} and K_{px} are regarded as the valve sensitivity coefficients and are specifically important in determining stability, frequency response, and other dynamic characteristics (Merritt, 1967).

In particular, the flow gain influences the open loop gain constant in the system; hence, plays an important role on the system stability. The flow-pressure coefficients influence the damping ratio of the valve/cylinder system. The valves often have quite large pressure sensitivity coefficient which presents ability of valves to breakaway large friction load with a relatively small error (Jelali & Kroll, 2004).

4.3.1 Assumptions

The linearized model has the same assumptions as given in section 4.2.1. Besides, the linearized model neglects the influence of the change of piston position Δx_v on volumes of the chambers A and B (e.g., $V_A(x_p)$ and $V_B(x_p)$ are constant), as well as change of pressures in effective bulk modulus (i.e., $E'_A(p_A)$and $E'_A(p_A)$ are constant).

4.3.2 Model description

Most parts of the non-linear model are kept except some necessary simplifications of non-linear effects.

For positive control input signal $u \geq 0$, the pressure dynamics model takes the form of

$$\begin{bmatrix} \Delta\dot{p}_A \\ \Delta\dot{p}_B \end{bmatrix} = \begin{bmatrix} \frac{E'_{A0}}{V_{A0}}\left[-A_p\Delta\dot{x}_p - K_{Li}(\Delta p_A - \Delta p_B)\right] \\ \frac{E'_{B0}}{V_{B0}}\left[+\propto A_p\Delta\dot{x}_p + K_{Li}(\Delta p_A - \Delta p_B)\right] \end{bmatrix} + \begin{bmatrix} \frac{E'_{A0}}{V_{A0}}\left[K_{Qx,A}\Delta x_v + K_{Qp,A}\Delta p_A\right] \\ -\frac{E'_{B0}}{V_{B0}}\left[K_{Qx,B}\Delta x_v + K_{Qp,B}\Delta p_B\right] \end{bmatrix} \qquad (4.13)$$

The valve dynamics module uses a first order model to convert the reference signal to actual valve opening and ignores all other nonlinearities.

Finally the mechanical module can be formulated as

$$\begin{bmatrix} \Delta\dot{x}_P \\ \Delta\ddot{x}_P \end{bmatrix} = \begin{bmatrix} \int \Delta\ddot{x}_P \\ \frac{1}{m_t}\left[(\Delta p_A - \propto \Delta p_B)A_p - F_f(\Delta\dot{x}_p) - F_{ext}(\Delta x_p)\right] \end{bmatrix} \qquad (4.14)$$

It can be seen from the descriptions that the linear model is very similar to the given non-linear model except for the influence of nonlinearities. The influence of these effects will be further investigated in Chapter 5.2.

4.3.3 Parameter feedback

The regular linear model ignores the nonlinearities in the model by applying constant parameters. This can be applicable for most of the parameters; however, the abnormalities cannot be ignored.

In this context, it is possible to further improve the linear model by implementing a parameter feedback. It should be noted that the given models up to now are solved with discrete solvers. This brings the opportunity that each constant parameter actually can be updated based on a feedback of the latest state of the system. This approach can be better understood from Figure 4.3.

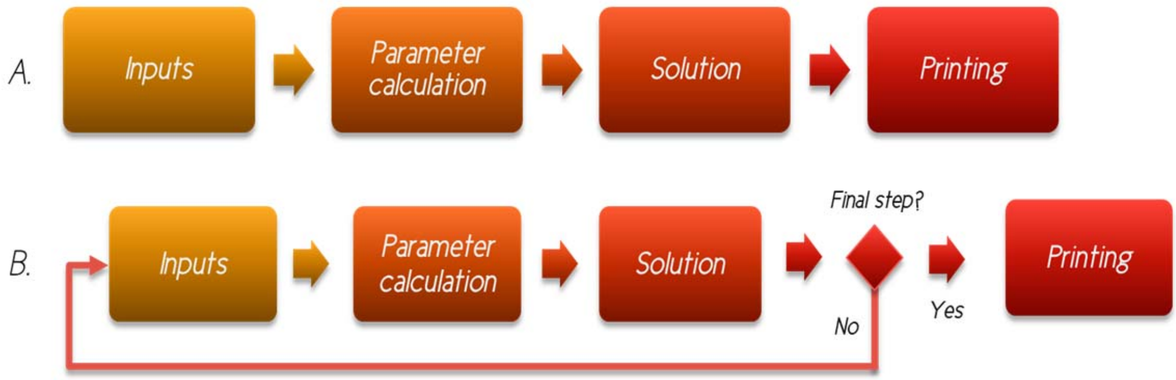

Figure 4.3. a) The solver algorithm of the linearized model b) The solver algorithm of the linearized model with a parameter feedback

It is clearly seen that the linearized model neglects the influence of the change of piston position Δx_v on the volumes of the chambers A and B as well as the change of pressures in effective bulk modulus. However, the linear model with a parameter feedback updates these parameter every time step based on the state of the system. Therefore, a relatively more accurate model with further considerations on non-linearities can be implemented by using this approach.

4.4 Simplified linearized model

The purpose of the simplified model is to further decrease the dependencies of the pressures on each chamber. In this sense, a linear relationship between the chamber pressures and the load pressure are given as[4]

$$\Delta p_A = \frac{1}{1+\alpha^3}\Delta p_L \tag{4.15}$$

$$\Delta p_B = -\frac{\propto^2}{1+\alpha^3}\Delta p_L \tag{4.16}$$

At this point, if the load pressure is rewritten and differentiated;

$$p_L = p_A - \propto p_B \tag{4.17}$$

$$\dot{p_L} = \dot{p}_A - \propto \dot{p}_B \tag{4.18}$$

Now, combining equation (4.15-4.18) and (3.38-3.39) the pressure dynamics equation becomes

[4] For further information on linearization process, see Appendix B.

$$\dot{p_L} = K_Q \Delta x_v - \frac{1}{T_h} \Delta p_L - K_d \Delta \dot{x}_p \tag{4.19}$$

The parameters K_Q, T_h, K_d in (4.19) can be expressed as

$$K_Q = \frac{E'_A}{V_A} K_{Qx,A} - \frac{E'_B}{V_B} K_{Qx,B} \tag{4.20}$$

$$K_d = A_p \left(\frac{E'_A}{V_A} + \alpha^2 \frac{E'_B}{V_B} \right) \tag{4.21}$$

and

$$T_h = \frac{1}{\frac{E'_{A0}}{V_{A0}} \left[\frac{K_{Qp,A} \alpha^2 - C_{Li}(1+\alpha^2)}{1+\alpha^3} \right] - \propto \frac{E'_{B0}}{V_{B0}} \left[\frac{K_{Qp,B} \alpha^2 + C_{Li}(1+\alpha^2)}{1+\alpha^3} \right]} \tag{4.22}$$

Finally equations (4.17-4.18) and (3.43) are combined to give the motion equation (mechanical module)

$$\begin{bmatrix} \Delta \dot{x}_P \\ \Delta \ddot{x}_P \end{bmatrix} = \begin{bmatrix} \int \Delta \ddot{x}_P \\ \frac{1}{m_t} \left[(\Delta p_L) A_p - F_f(\Delta \dot{x}_p) - F_{ext}(\Delta x_p) \right] \end{bmatrix} \tag{4.23}$$

It should be stressed that this approach reduces the state of the system by one with the help of developing a relationship between each chamber. The model neglects the influence of the change of piston position Δx_v on volumes of the chambers A and B as well as change of pressures in effective bulk modulus.

4.4.1 Parameter feedback

It is also possible to further improve the simplified linear model by parameter feedback implementation as it is done in the regular linear model. Similarly, each state parameter will be updated based on a feedback that delivers the latest state of the system. For further information, see section 4.3.3.

4.5 Further information on essential model parameters

Some of the crucial parameters can be estimated by using manufacturer's catalogue information and drawings. If the parameters cannot be found on those sheets, they can be calculated either by using data from an experiment or by an expression provided in the literature.

4.5.1 The valve flow-signal function

If the supply pressure is constant, it can be seen that a valve is partially characterized by the relationship between valve input u^* and valve flow Q, when the flow is in steady state. This relationship can be used to determine flow-signal function $Q(u^*)$. Proportional valves are characterized by straight proportional lines in ideal cases. In practice, these curves deviate from ideal cases due to saturation for larger valve inputs, valve overlap/underlap, valve hysteresis and some non-smooth behavior around zero input. A typical flow-signal curve can be seen in Figure 4.4. It should also be noted that this curve is used for the calculation of the reference flow coefficient.

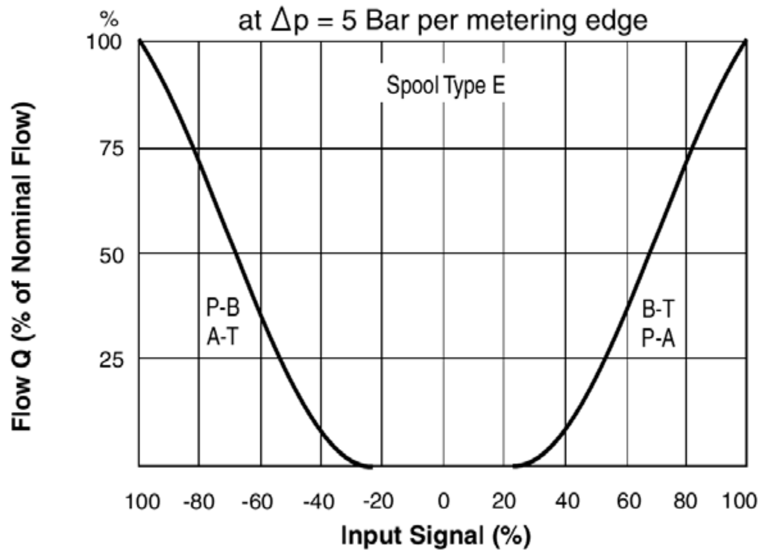

Figure 4.4. The flow-signal curve for the valve implemented in the developed model (Parker, 2013)

4.5.2 Valve overlap and underlap

This parameter is generally provided clearly in the manufacturer's catalogue information. Moreover, the valve overlap can easily be determined by using Figure 4.4, i.e., the beginning of the fluid flow for the smallest valve input signal.

4.5.3 Actuator dimensions and mass

The manufacturer's construction drawings of the actuator often precisely provide required piston areas and stroke thus, the actuator dimensions and the mass can easily be determined by using these sheets.

It is relatively more difficult to determine the inefficient volumes of the actuator such as pipelines between valves and actuator, use of safety manifolds etc. These parameters can be determined by relatively less accurately calculations with the help of using construction drawings.

The most straight forward provision of the masses of the components can be done by weighting the objects. However, less precise estimations can also be made by using construction drawings.

4.5.4 Friction forces

As it is introduced in Section 3.2.3.2, non-linear friction forces in an actuator is one of the main source of the nonlinearities in the cylinder model.

As proposed in the given section, it is generally effective enough to use the Stribeck friction model. In this model, viscous friction is dependent on the velocity of the piston and plays an effective role on the stability and damping of hydraulic system. Thus, viscous friction should never be omitted in the model.

Viersma (1980) argues that Coulomb friction is the magical factor that is responsible for the most of the non-linearity in the actuator and is always suspected when something unexpected happens. Jelali & Kroll (2004) also conclude that Coulomb friction may cause instability issues in practice, i.e., stick-slip phenomena at low speeds.

In this context, the friction forces can be experimentally calculated by measuring the pressures of each cylinder chamber and using

$$F_f = (p_A - \propto p_B)A_p - m\ddot{x} \tag{4.24}$$

where p_A and p_B are pressures in chambers A and B respectively. It should be noticed that the load and any external forces are neglected in this formula. Moreover, if the data would be taken in a steady state condition, the acceleration term disappears and (4.24) becomes:

$$F_f = (p_A - \propto p_B)A_p \tag{4.25}$$

It also possible to obtain the data for the friction forces from the manufacturer's data sheets whereas some of the parameters of the Stribeck friction model can be found in the literature and be adapted to the model used.

4.5.5 Leakage coefficients

The external leakage flow from actuator to casing is neglected; therefore, only the internal leakage flow needs to be determined.

The internal leakage coefficient C_{Li} can be obtained either by manufacturer's data sheets or by an experimental setup. The latter is done by fixing the piston position rigidly and taking pressure measurements from each chambers of cylinder. This experimental setup is based on the fact that a small valve opening would lead to supply pressure on the one side of the chamber. However, in case of leakage flow, there will be oil flow and this would cause a pressure drop in the relevant chamber. In this sense, the pressure in the given chamber would be less than the supply pressure while the pressure in the other chamber would equal to the tank pressure since the valve enables free flow of small amounts of oil (see flow equation in chapter 3.2.3.1).

4.6 Simulation package

The simulation package Matlab R2011b has been selected even though literature mainly proposes Matlab/SIMULINK. In house Delaval experts conclude that Matlab script models are often more flexible since the user can adapt any part of the script with any other Matlab or other software in DeLaval. Moreover, the script models often provide more transparency to its user.

5 Results

The chapter starts with a description of the experiment carried out to acquire measurement data for the validation. The model results are obtained with the methods described in the previous chapters. They are compiled, analyzed and compared with the experimental data. To acquire complete understanding of the model dynamics, further sensitivity analysis is also carried out.

5.1 The introduction of measured data

The validation of the simulation studies often requires experimental data; hence, an experimental setup is built at DeLaval.

In the experiment, pressure in both chambers, the pressure at the pump outlet, the displacement of the rod, the valve input signal and the elapsed time are measured. The measured data and measurement frequency for each variable are shown in Table 5.1.

Table 5.1. The measured data and measurement frequency for each variable

Measured quantity	Nomenclature	Measurement Frequency (Hz)
Chamber Pressure A	p_A	40
Chamber Pressure B	p_B	40
Pump Outlet Pressure	P_S	40
Displacement	x_p	200
Valve input signal	u	200
Elapsed time	t	200

It is assumed that the robot arm only moves in the X and Y direction and maintains the motion always parallel to the ground. In other words, there are no gravity forces acting on the cylinder in the model. However, it is unavoidable that small deviation in Z axis occurs. Even though, the gravity is not included in the models, the gravity leads to deviations on the chamber pressures on the experimental data which will be further discussed in section 6.1. Besides, the displacement is estimated with the help of the measured angle between the robot arm and the actuator rod; thus, deviation in Z axis direction leads to additional errors in the calculation of the piston position. However, the errors in the displacement can be compensated since the starting position and final position of the arm is known.

5.1.1 Friction forces

One of the crucial aspects of the model is the simulation of the friction forces. However, the verification of the proposed model (3.47) should be done by a comparison with experimental data. For this purpose, the friction forces are calculated by using the experiment proposed in section 4.5.4 and the experimental friction forces are estimated for different velocities with the help of (4.25). The comparison of values acquired by the experiment and the friction model is given in Table 5.2. It can be easily seen that the friction model estimates the related friction forces with accuracy that has a relative error smaller than ±3.6%.

Table 5.2. Verification of friction model by comparison of the simulated data with the experimental data

Experimental vs simulated friction forces			
Experimental Friction Force (N)	**Velocity (m/s)**	**Simulated Friction Force (N)**	**Relative Error**
120.53	0.165	124.20	**-3.04%**
113.7	0.123	112.44	**1.11%**
103.52	0.078	99.84	**3.55%**
94.75	0.048	91.48	**3.45%**
85.3	0.022	84.71	**0.69%**
79.86	0.01	82.64	**-3.48%**
Friction model parameters			
c_S	F_v	F_{c0}	F_{s0}
0.01	280	78	5

5.1.2 The flow coefficient

It is discussed in section 3.2.1 that the flow coefficient often deviates from the theoretical value for different valve openings. This effect is investigated by building an experimental setup and calculating the flow coefficient by using data obtained from the experiment with the help of (3.18). In the experiment, the supply pressure (p_S), chamber pressures (p_A, p_B) and position of the rod (x_p) are measured. By using this data, experimental flow coefficient value is calculated. It should be pointed out that, due to valve hysteresis, this calculation is made only for the positive displacement (see section 3.2.1.1). The estimated compensation parameter $K(x_v)$ is shown in Figure 5.1.

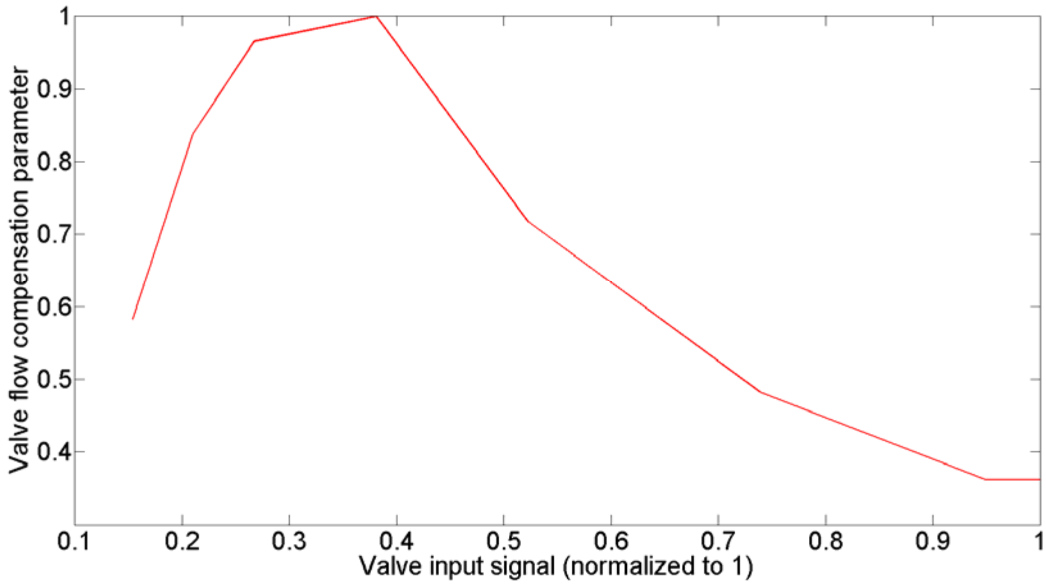

Figure 5.1. Deviation of compensation parameter $K(x_v)$ for measured data

The valve acts relatively proportional between the valve input signals 0.2 to 0.5; while, its behavior is dramatically altered in other regions. The lower flow coefficient means that less flow through the valve would occur than the theoretical estimation.

5.1.3 The valve hysteresis

It is given in section 3.2.1, that the valves often have hysteresis that should not be omitted in the mathematical models. This phenomenon is investigated by comparing the flow coefficients for both directions for the same input signal. The estimated valve hysteresis compensation parameter $H(x_v, d_f)$ can be seen in Figure 5.2.

Figure 5.2 clearly shows that the valve hysteresis is around 5% most of the time. However, the hysteresis varies between 0-12.5 % for the linear region and shows nonlinear behavior.

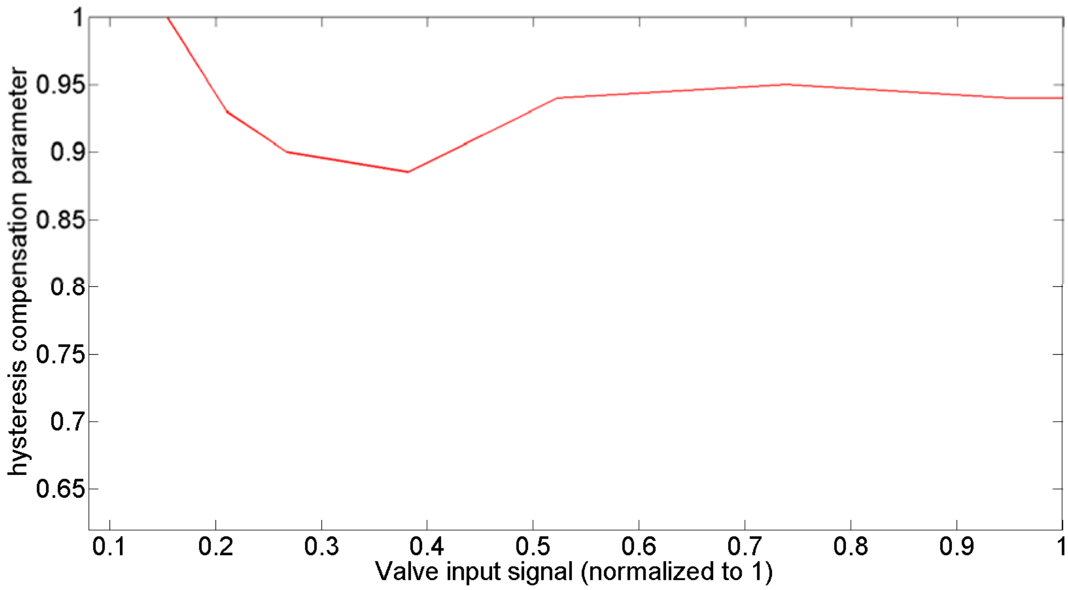

Figure 5.2. . Deviation of valve hysteresis compensation parameter $H(x_v, d_f)$

5.2 Comparison of models

In this section, a comparison of the nonlinear, linear and simplified linear models is carried out. The measured data for the given valve input signal is also included. Furthermore, it should be stressed that the compensation parameters ($K(x_v)$ and $H(x_v, d_f)$) are omitted in all models for a fair comparison, although, it can be easily implemented in nonlinear model and with the help of a feedback loop for linear models.

The comparison is made for various step input signals because of the nonlinear behavior of hydraulic system and can be seen in Figure 5.3 to Figure 5.7.

It can be seen from Figure 5.3 that none of the models can predict the displacement accurately for small openings. However, the nonlinear model shows better pressure prediction than linear model. Between the valve opening region of 25.8% to 50.5% (see Figure 5.5 and Figure 5.6), the linear model shows the highest accuracy on the displacement and pressures. For the large valve opening (see Figure 5.7), the models completely diverge from the experimental data for the displacement and the chamber pressures.

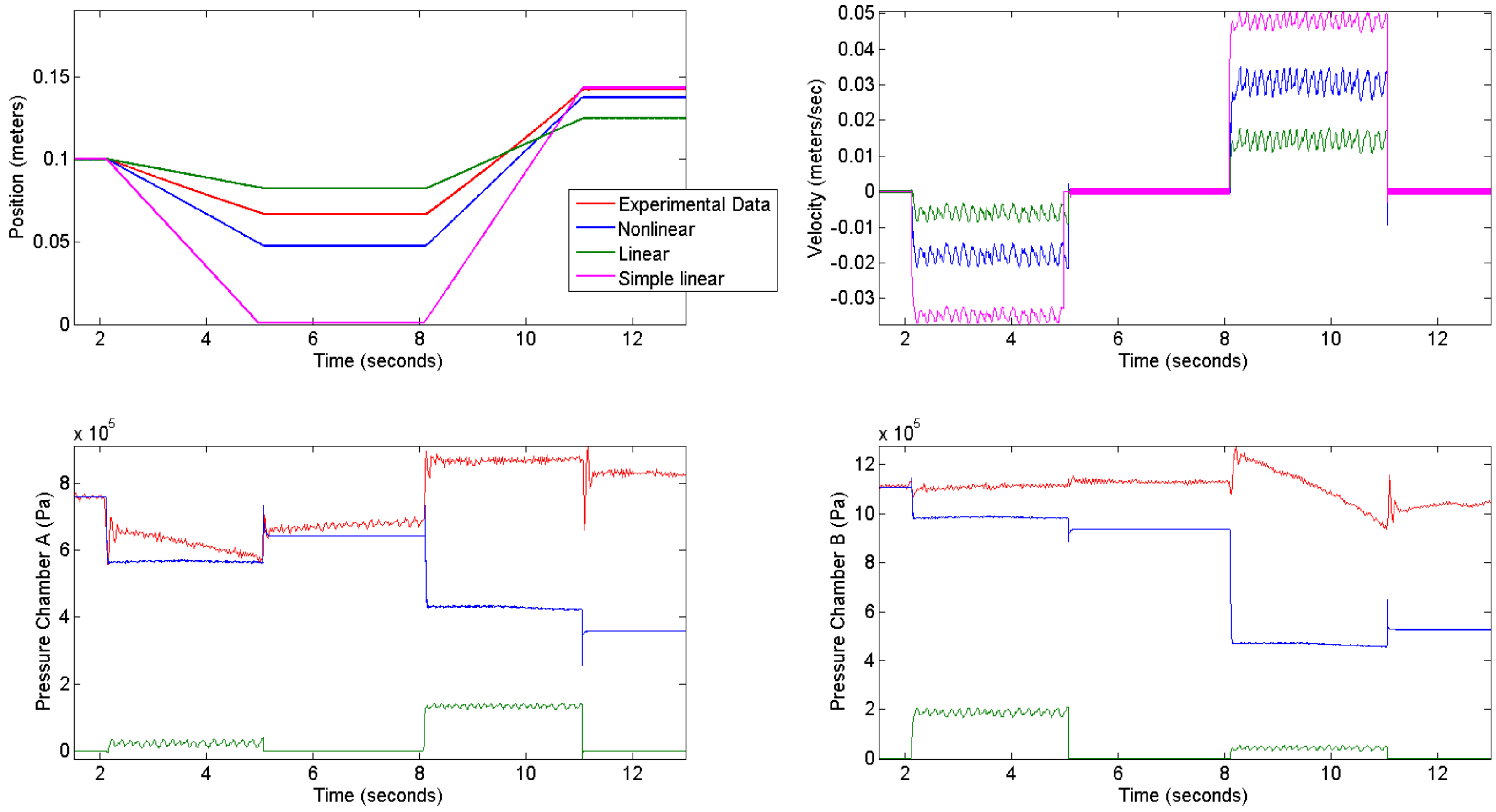

Figure 5.3. The comparison of models for the input signal of 20.3%

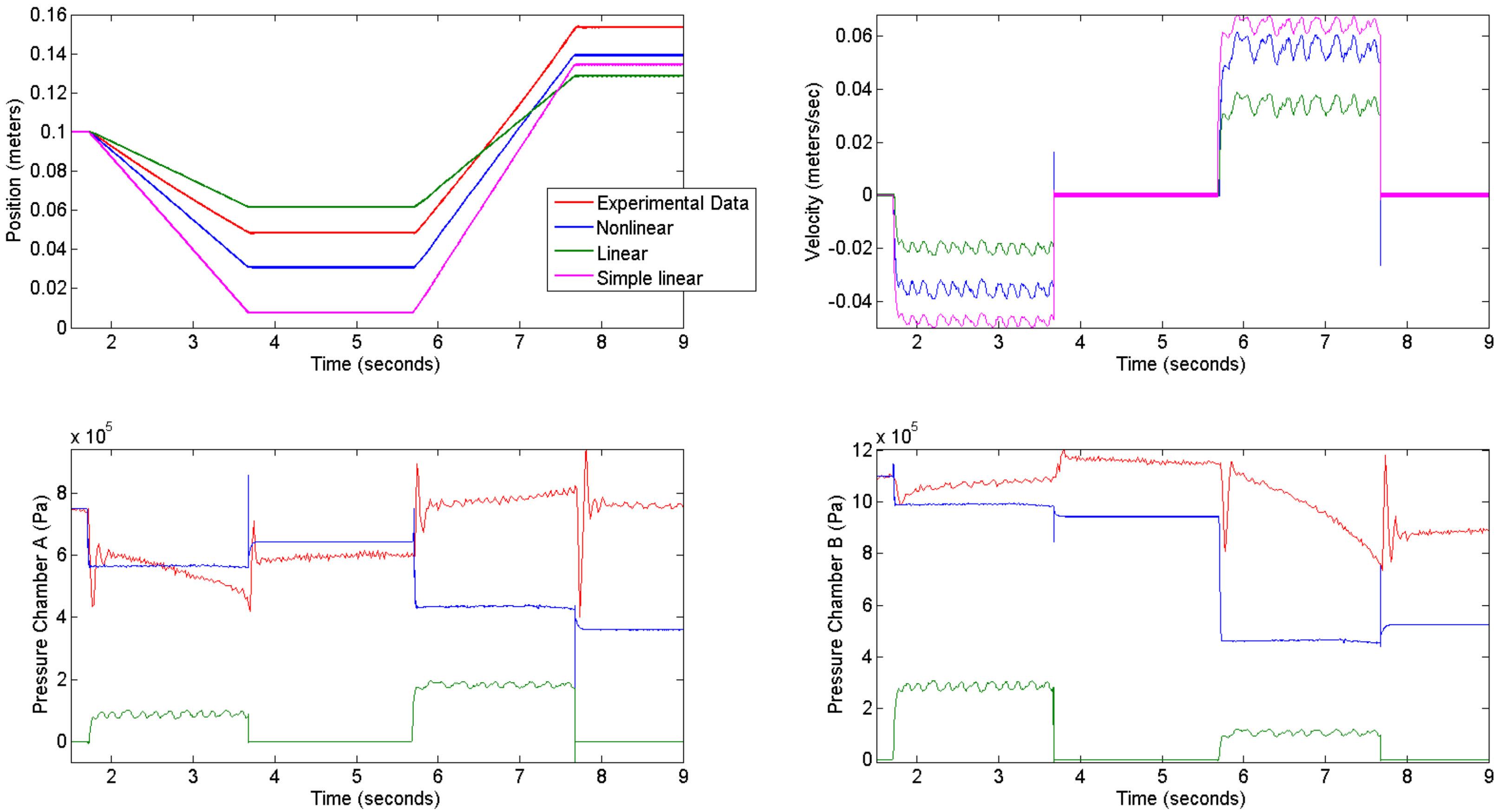

Figure 5.4. The comparison of models for the input signal of 25.8%

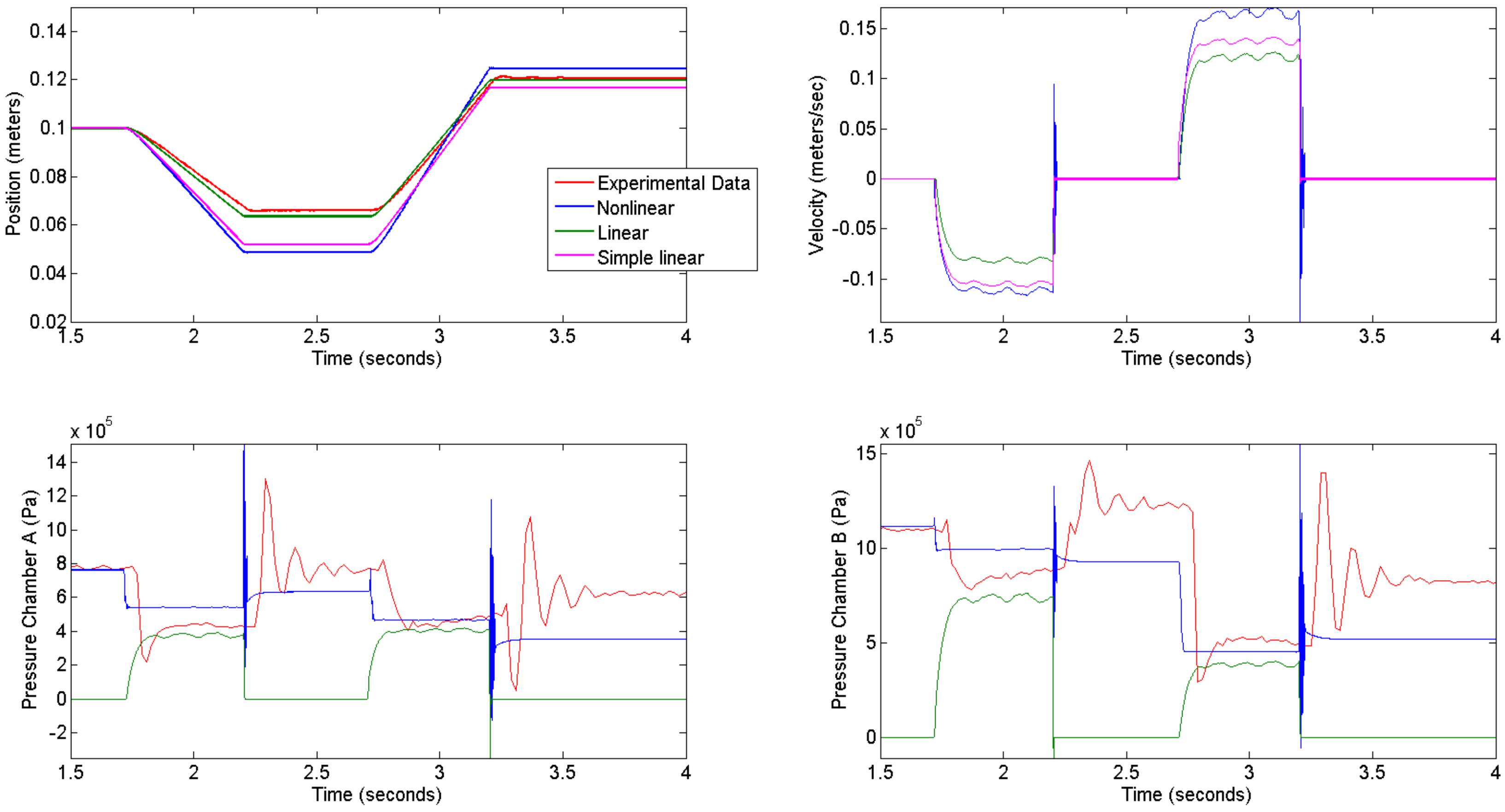

Figure 5.5. The comparison of models for the input signal of 36.8%

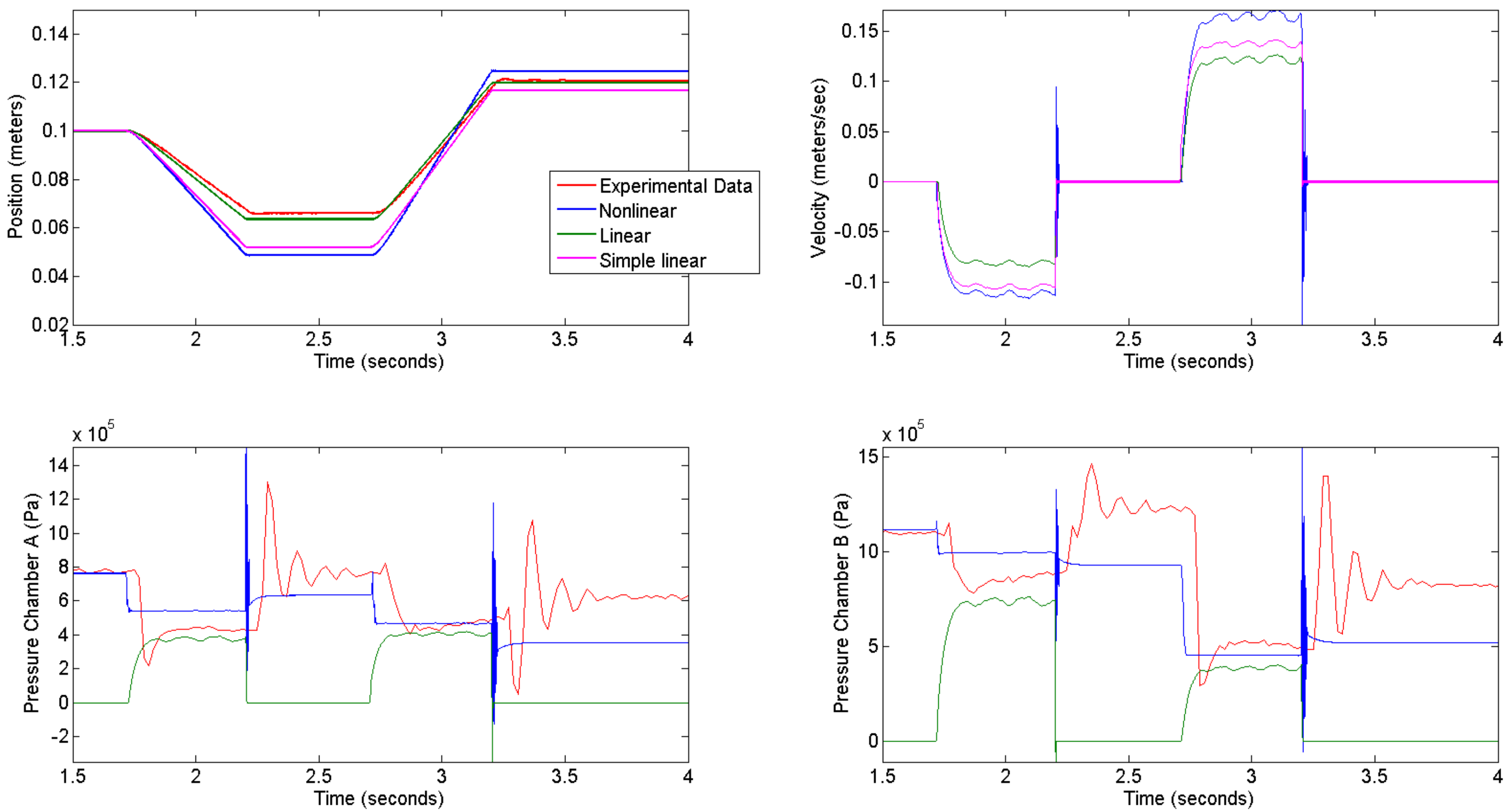

Figure 5.6. The comparison of models for the input signal of 50.5%

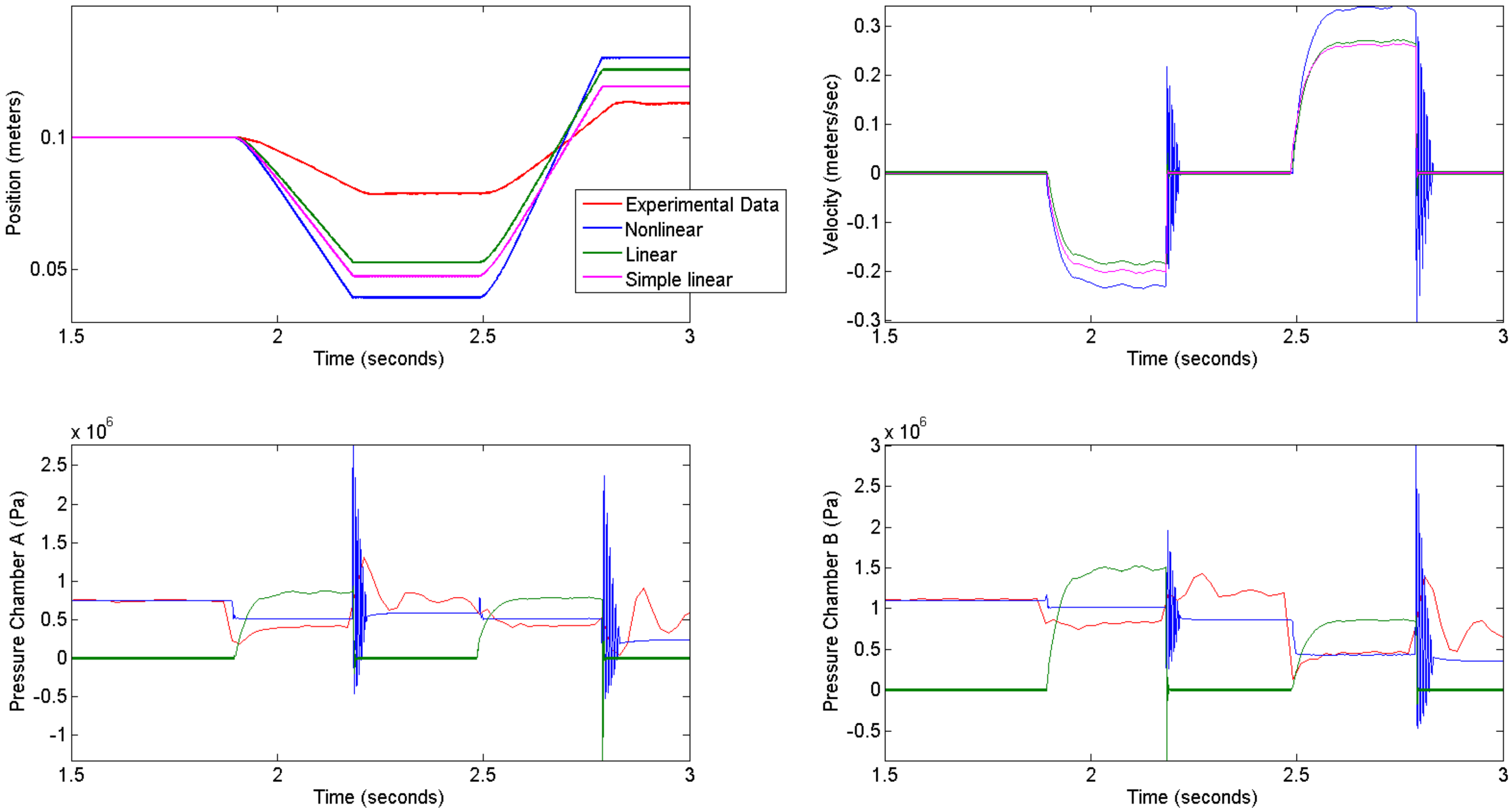

Figure 5.7. The comparison of models for the input signal of 91.6%

5.3 Comparison of improved models

In this section, a comparison of the improved nonlinear, linear and simplified linear models is carried out. In other words, the improved models include compensation parameters ($K(x_v)$ and $H(x_v, d_f)$) for valve hysteresis and valve flow nonlinearities. Moreover, the parameter feedback also updates the state variables at each time step (see section 4.3.3).

The measured data for the given valve input signal is also included. The comparison is made for various step input signals because of the nonlinear behavior of the hydraulic system and shown in Figure 5.8 to Figure 5.13.

It can be seen from Figure 5.38 that the nonlinear model can predict the displacement with a better accuracy for small openings. However, it should be noted that the accuracy of improved models are better than regular models. Between the valve opening region of 25.8% to 50.5% (see Figure 5.9 and Figure 5.12), the nonlinear model show the highest accuracy on the displacement and pressures. Similarly, the accuracy of improved models is better than regular models in this region, too. For the large valve opening (see Figure 5.13) nonlinear model is still able to forecast displacement and pressures, while other models are not. Although, they are more accurate than regular models.

5.4 Comparison of all proposed models

In this chapter, the steady state displacement errors of all models are compared in Table 5.3 to provide a complete overview to the reader. It should be given that relative errors below than 7.5% is regarded as highly accurate, between 7.5 % an 20 % is given as moderately accurate while relative errors larger than 20 % are assumed to be inaccurate.

Table 5.3. Relative displacement errors of each model

Valve Signal	Nonlinear	Improved Nonlinear	Linear	Improved Linear	Simple Linear	Simple Imp. Linear
20.3%	Inaccurate	Accurate	Inaccurate	Inaccurate	Inaccurate	Inaccurate
25.8%	Inaccurate	Accurate	Moderate	Inaccurate	Inaccurate	Moderate
36.8%	Inaccurate	Accurate	Accurate	Accurate	Inaccurate	Accurate
50.5%	Inaccurate	Accurate	Accurate	Accurate	Inaccurate	Moderate
71.3%	N/A	Accurate	N/A	Accurate	N/A	Moderate
91.6%	Inaccurate	Accurate	Inaccurate	Moderate	Inaccurate	Moderate

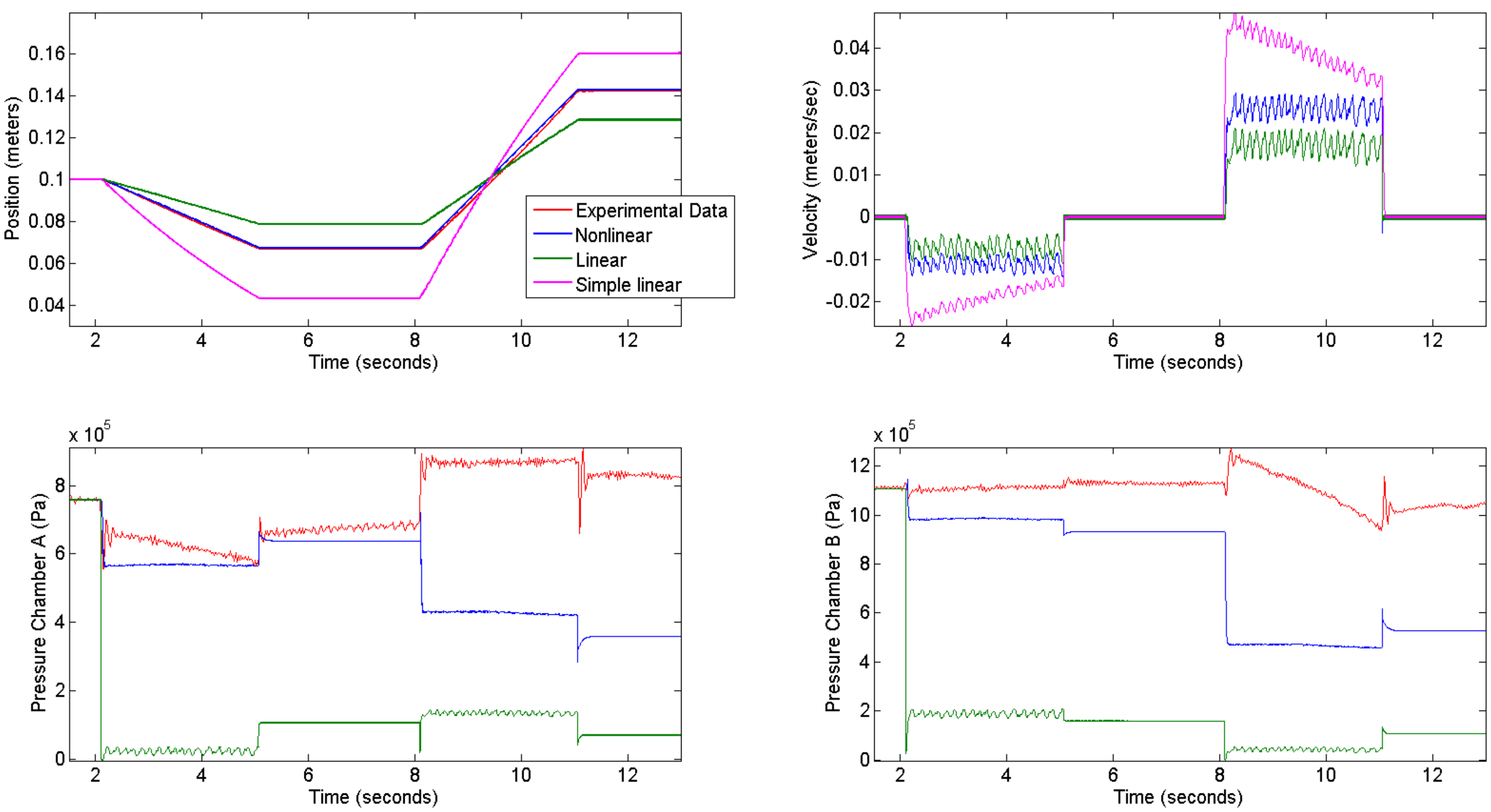

Figure 5.8. The comparison of models for the input signal of 20.3%

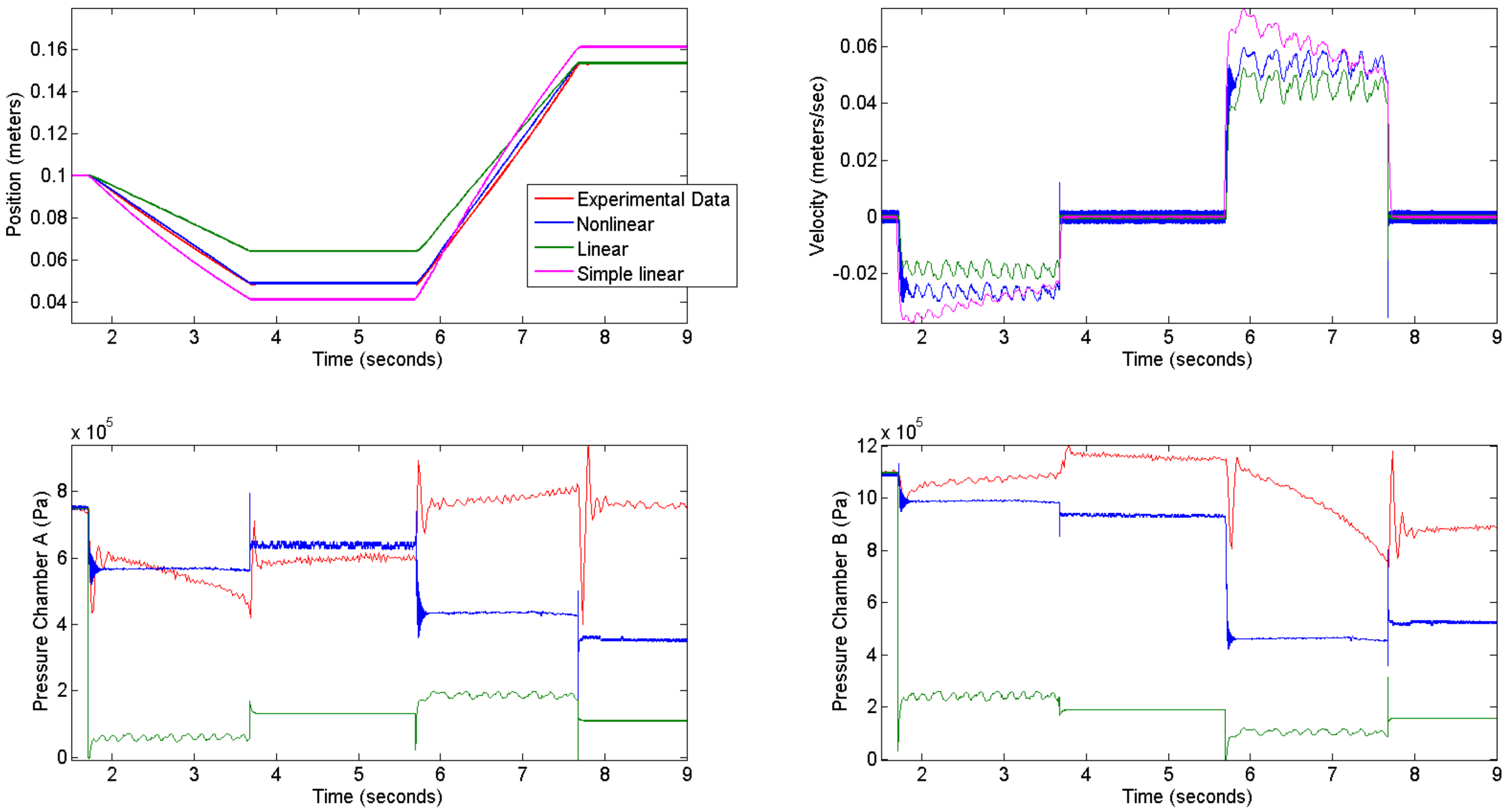

Figure 5.9. The comparison of models for the input signal of 25.8%

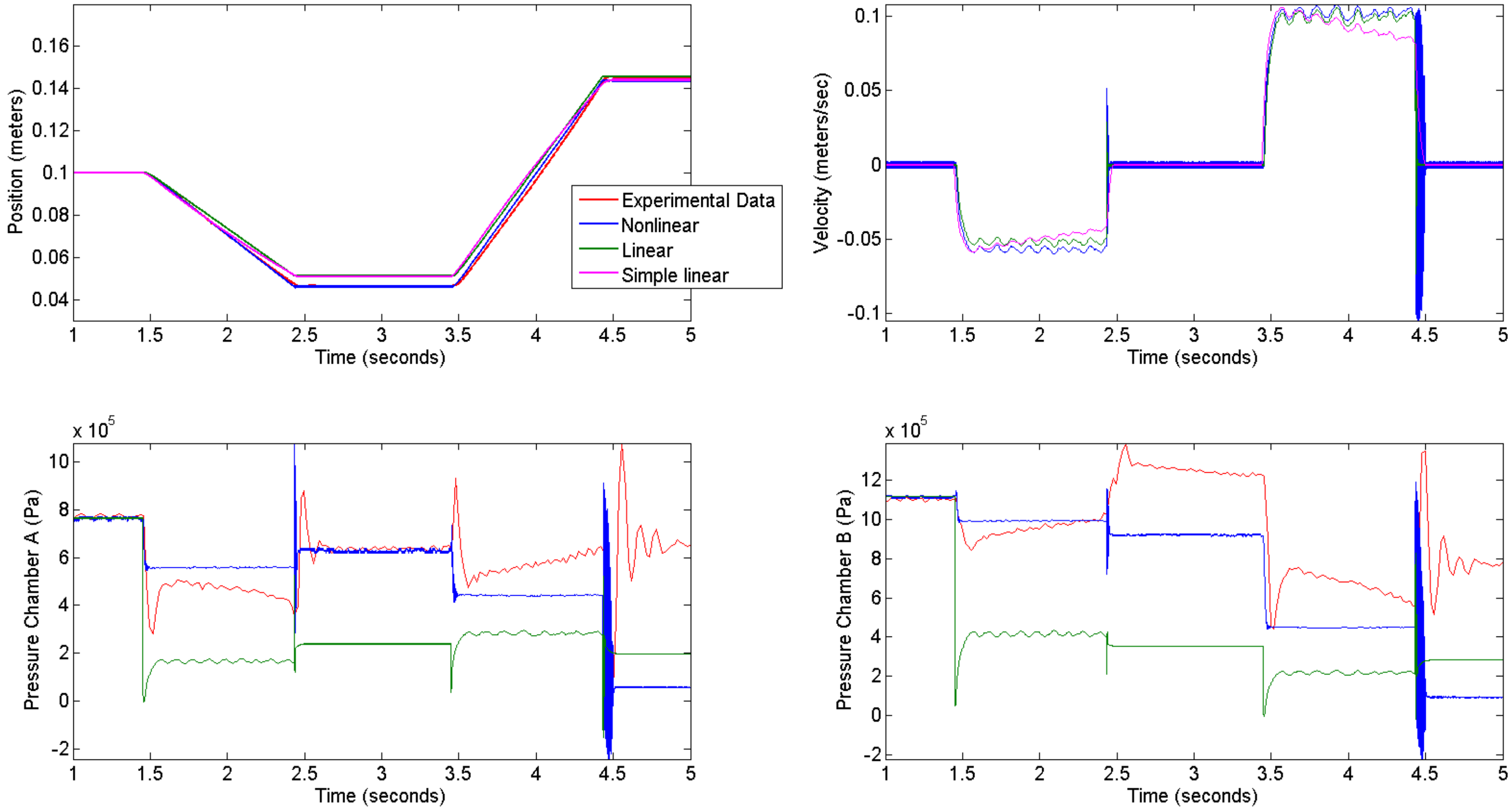

Figure 5.10. The comparison of models for the input signal of 36.8%

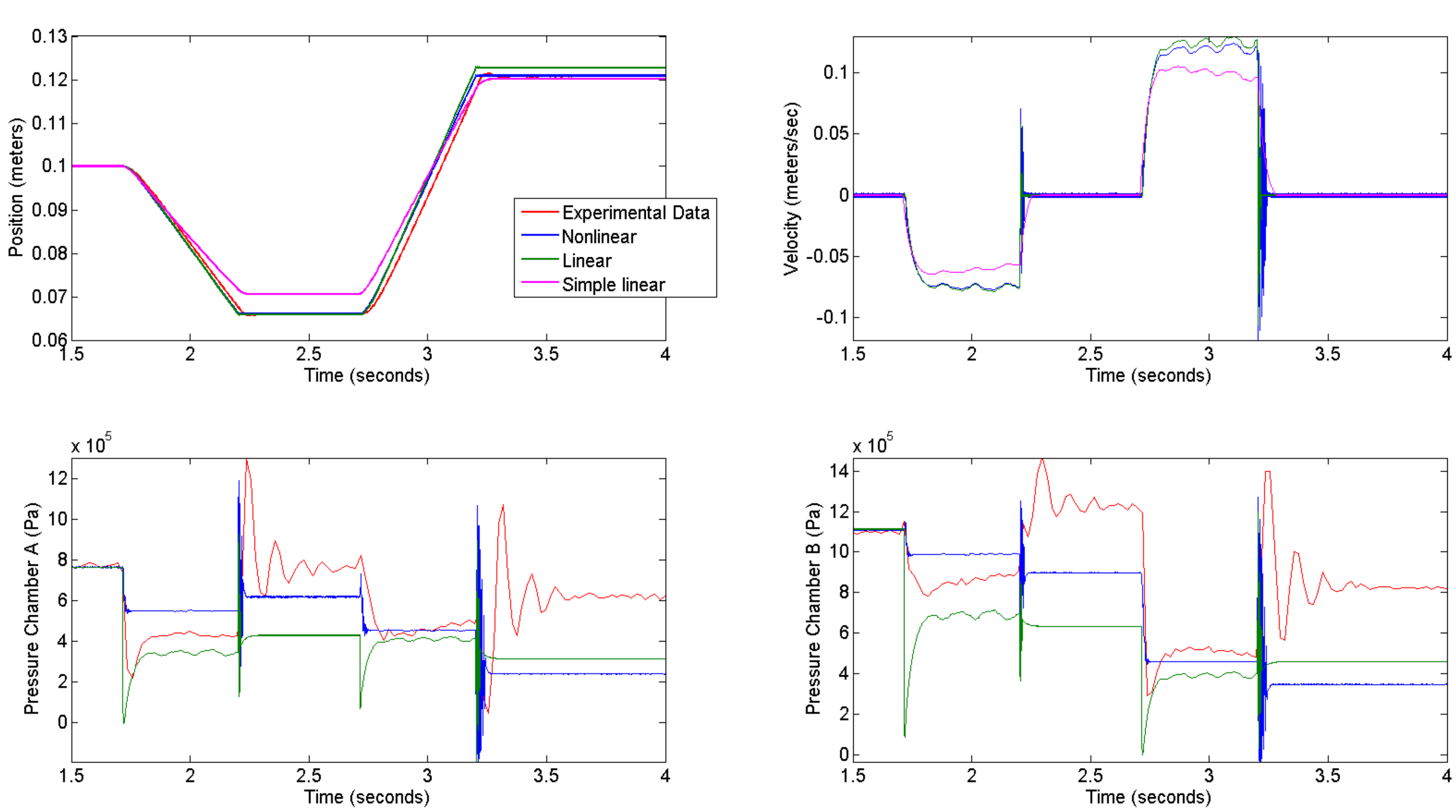

Figure 5.11. The comparison of models for the input signal of 50.5%

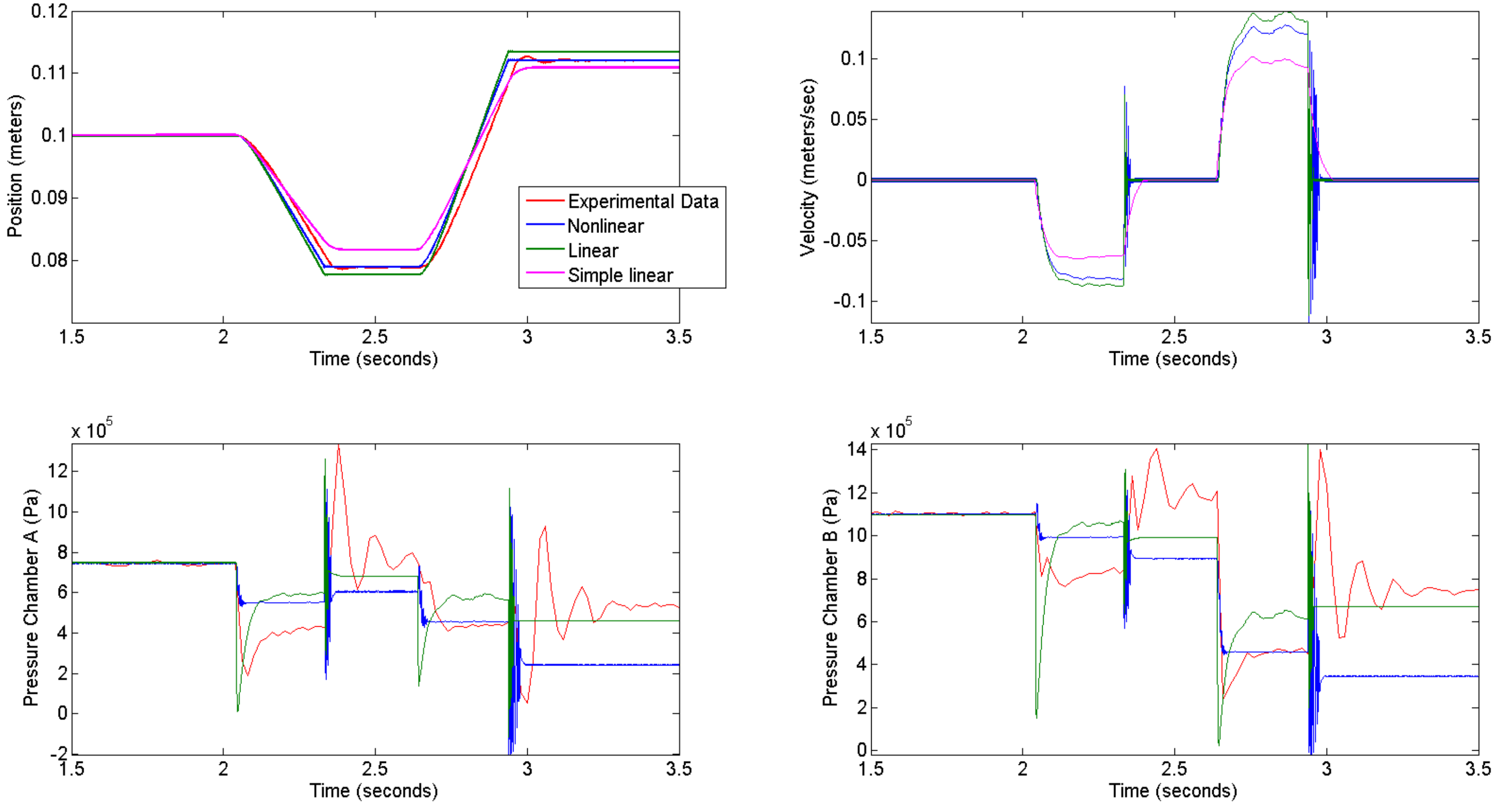

Figure 5.12. The comparison of models for the input signal of 71.3%

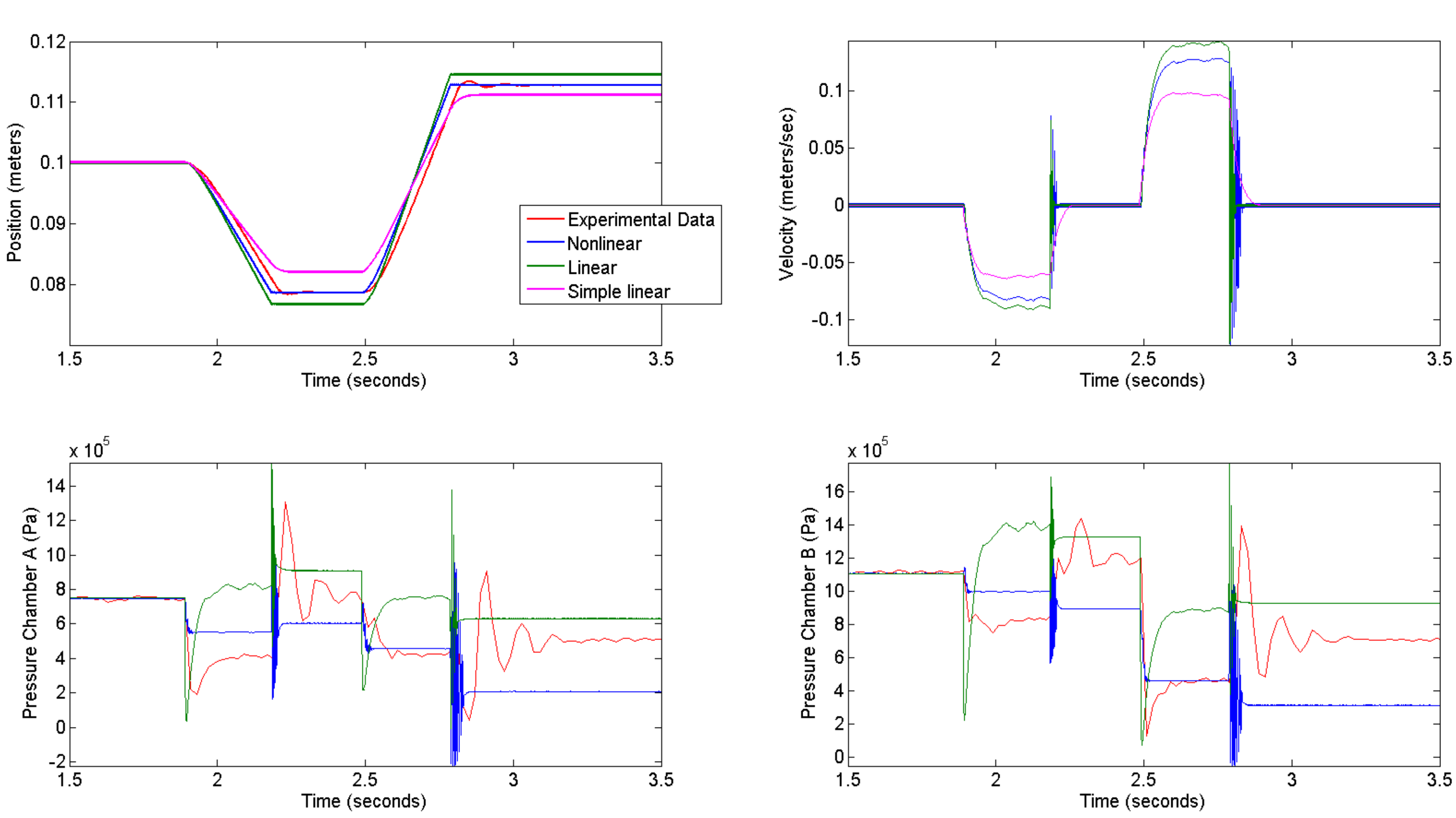

Figure 5.13. The comparison of models for the input signal of 91.6%

5.5 Sensitivity Analysis

In this section sensitivity analysis is carried out to investigate how the uncertainty in the output of the model can be apportioned to different sources of uncertainty in its inputs. It should be given that nonlinear model is used for sensitivity analyses. And while the other parameters of the simulation could be determined easily, there were uncertainties on the flow coefficient, the valve response time, supply pressures, the mass variation and external forces, therefore they will be investigated further.

The main purposes of the analysis are:

- Testing the robustness of the results since the presence of uncertainty in given parameters is unavoidable in the model.
- Reducing the uncertainty by detecting which of these parameters cause substantial influence in the output and should therefore be the focus of attention if the robustness were to be improved
- Proposing further model simplification by avoiding parameters that have no effect on the output (e.g. assuming constant supply pressure).

5.5.1 Influence of flow coefficient

Flow coefficient (c_V) is one of the most crucial parameter in the simulations and derived from the valve catalogue provided by suppliers and modified by using experimental data if it is needed in the simulations. It mainly acts as an open loop gain in the flow models and has a crucial role on the stability of the system. Thus, it becomes the most crucial variable in the flow equation even though it is sometimes extremely hard to estimate it precisely. Hence, it is very obvious that carrying out a sensitivity analysis on the flow coefficient and revealing possible outcomes are essential for the sake of the simulations. The sensitivity analysis for the flow coefficient is shown in Figure 5.14.

5.5.2 Influence of valve response time

It is fair to say that valve response time is relatively less crucial but it is still interesting to see if it affects the simulation in any unpredicted way. It must be stated that valve response time can be effected by many unknown conditions and sometimes becomes more stochastic than other deterministic parameters (see section 3.2.1) and carried out sensitivity analysis is presented in Figure 5.15.

5.5.3 Influence of supply pressures

In house DeLaval experts presented their opinion on the supply levels and the opinions suggested that the pressures can vary between 25-35 bars in the VMS. Therefore, it is essential to include this parameter in the sensitivity analyses, as shown in Figure 5.16, to increase reliability and practicality.

5.5.4 Influence of mass variation of the system

Another suggestion by in house experts of Delaval proposes that mass can vary based on various external sources and/or design variations. Therefore, it is also believed that a sensitivity analysis should be carried out on this parameter and can be seen in Figure 5.17.

5.5.5 Influence of possible external forces

The main purpose of this is to reveal the feasibility of simulation in the occurrence of any external forces (F_{ext}) and can be observed in Figure 5.18.

5.6 Further concerns on effective empirical bulk modulus

There are various effective empirical bulk modulus models proposed in the literature. Moreover, each proposal points out its own advantages; hence, it is interesting to examine the effect of different bulk modulus models for the sake of the simulation. Moreover, the theoretical cases where the bulk modulus is relatively large or small will be also investigated here as can be seen in Figure 5.19.

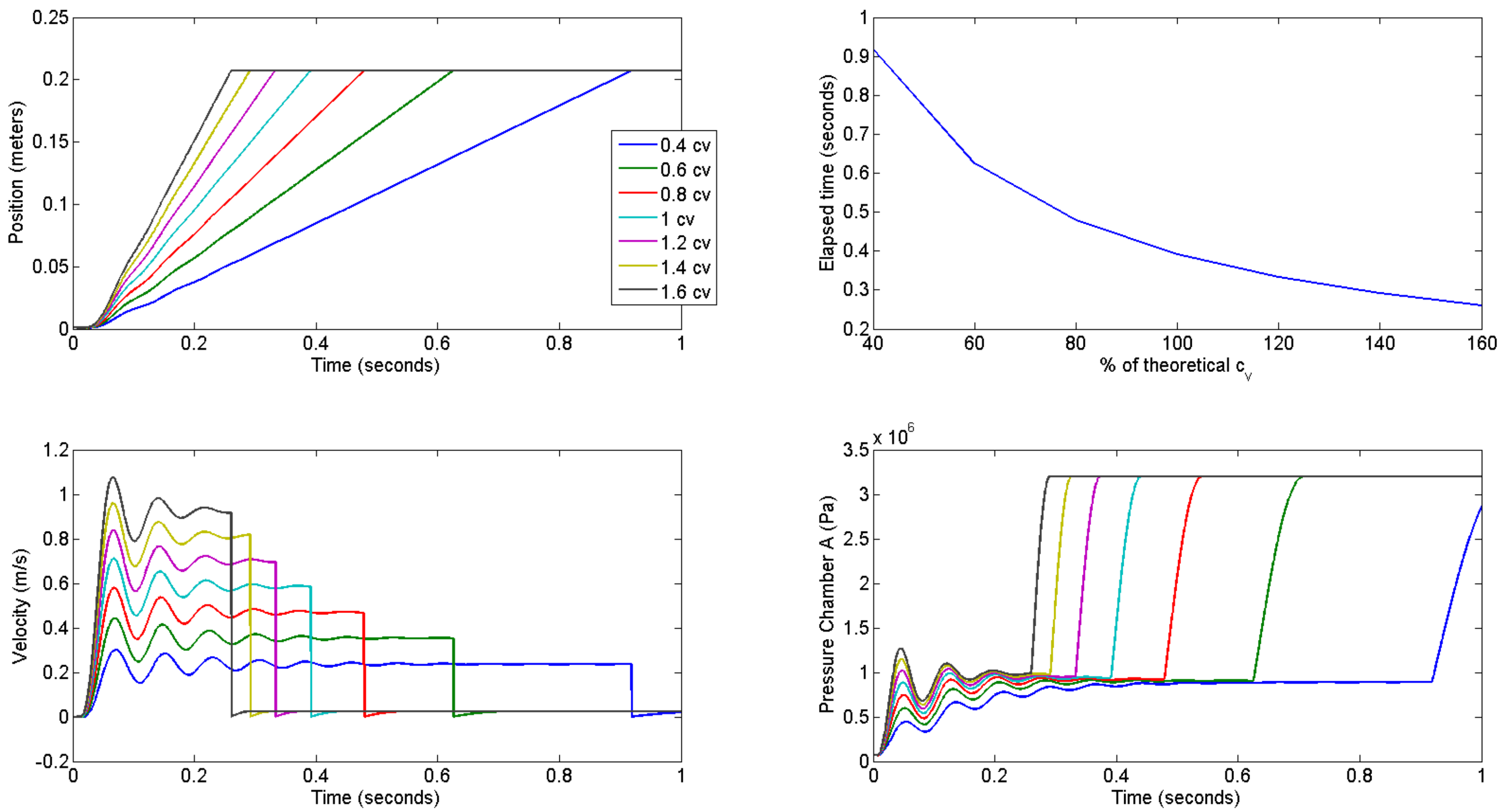

Figure 5.14. Influence of the flow coefficient on the outputs

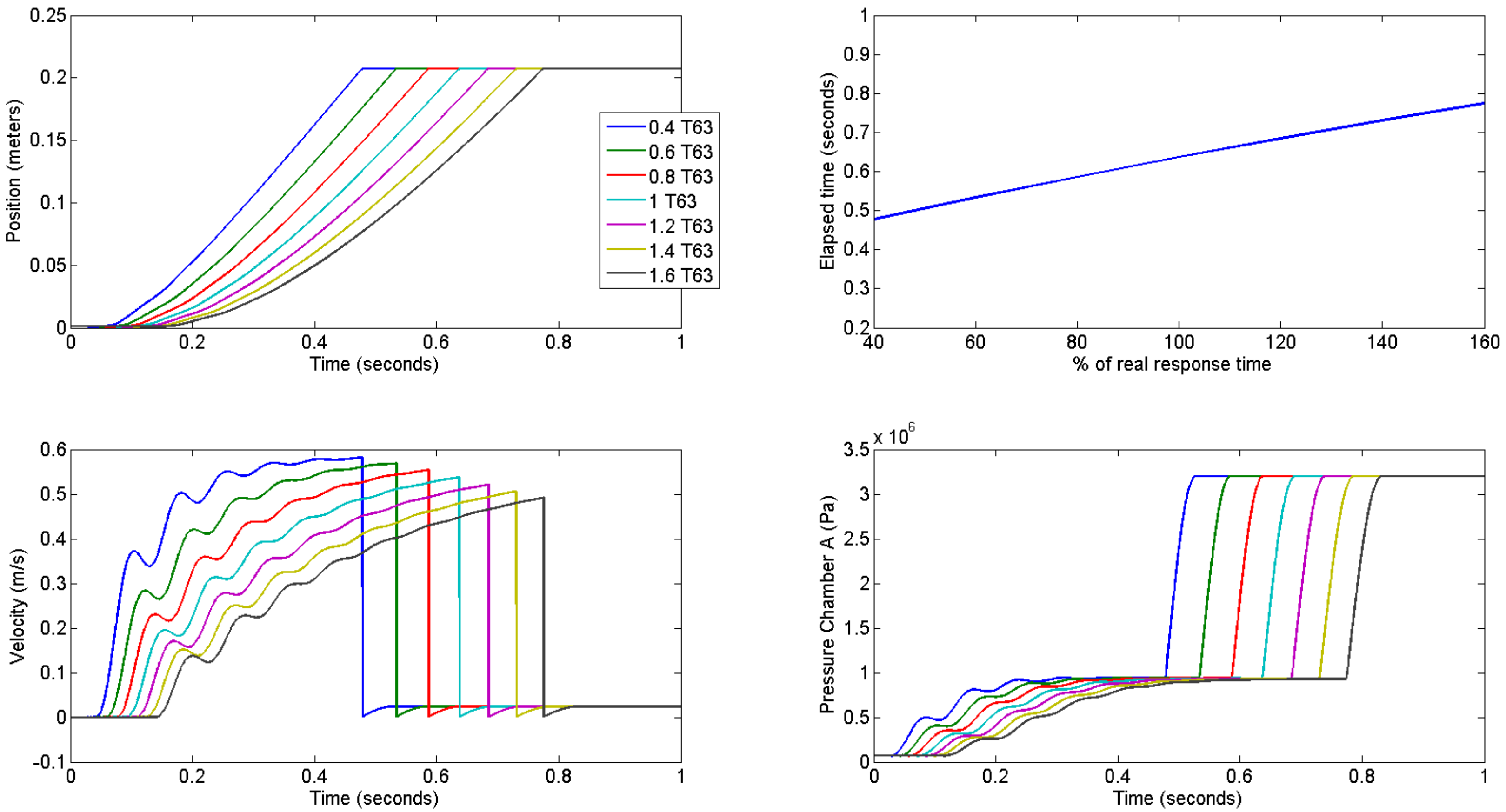

Figure 5.15. Influence of the valve response time on the outputs

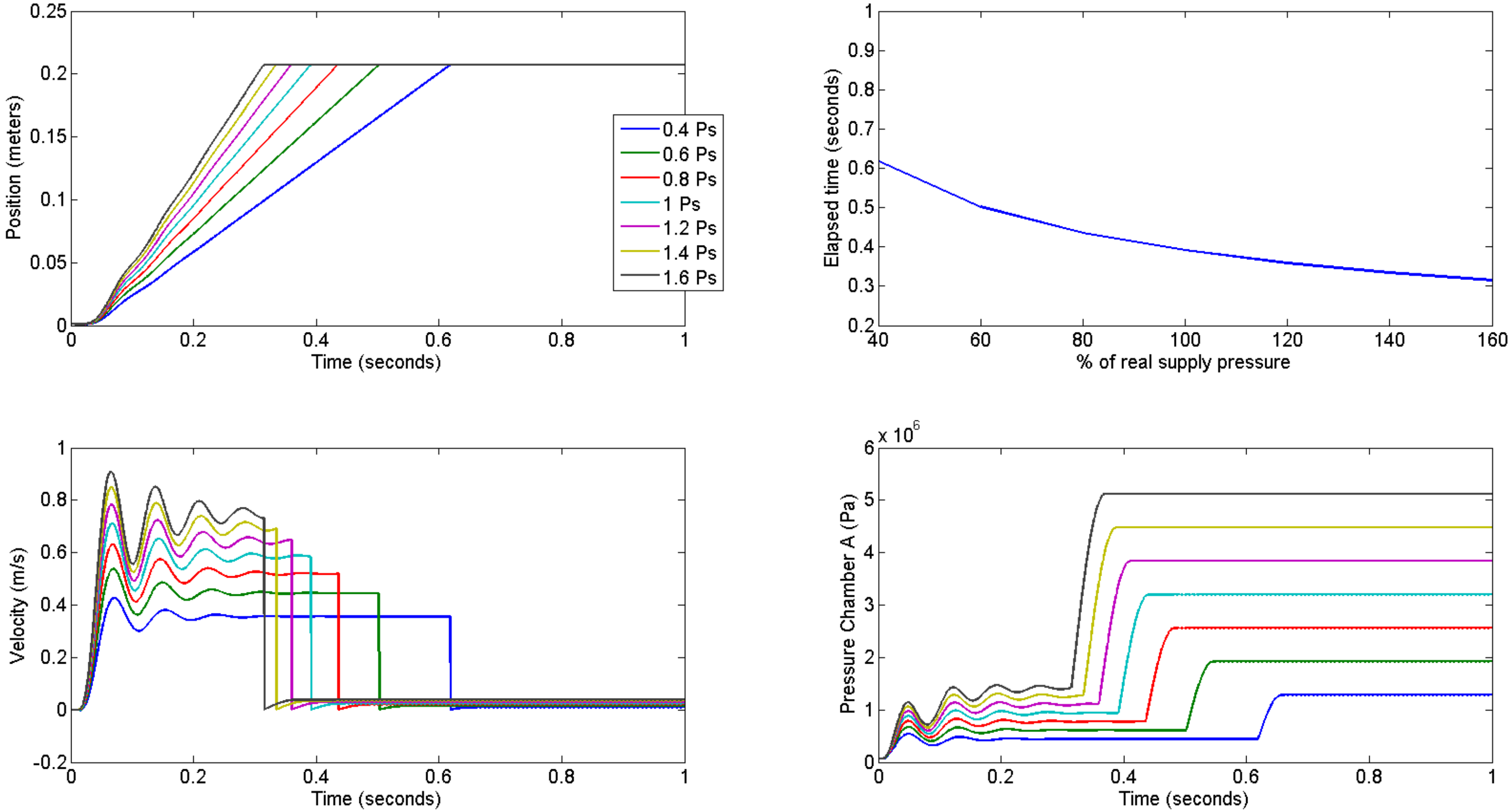

Figure 5.16. Influence of the supply pressure on the outputs

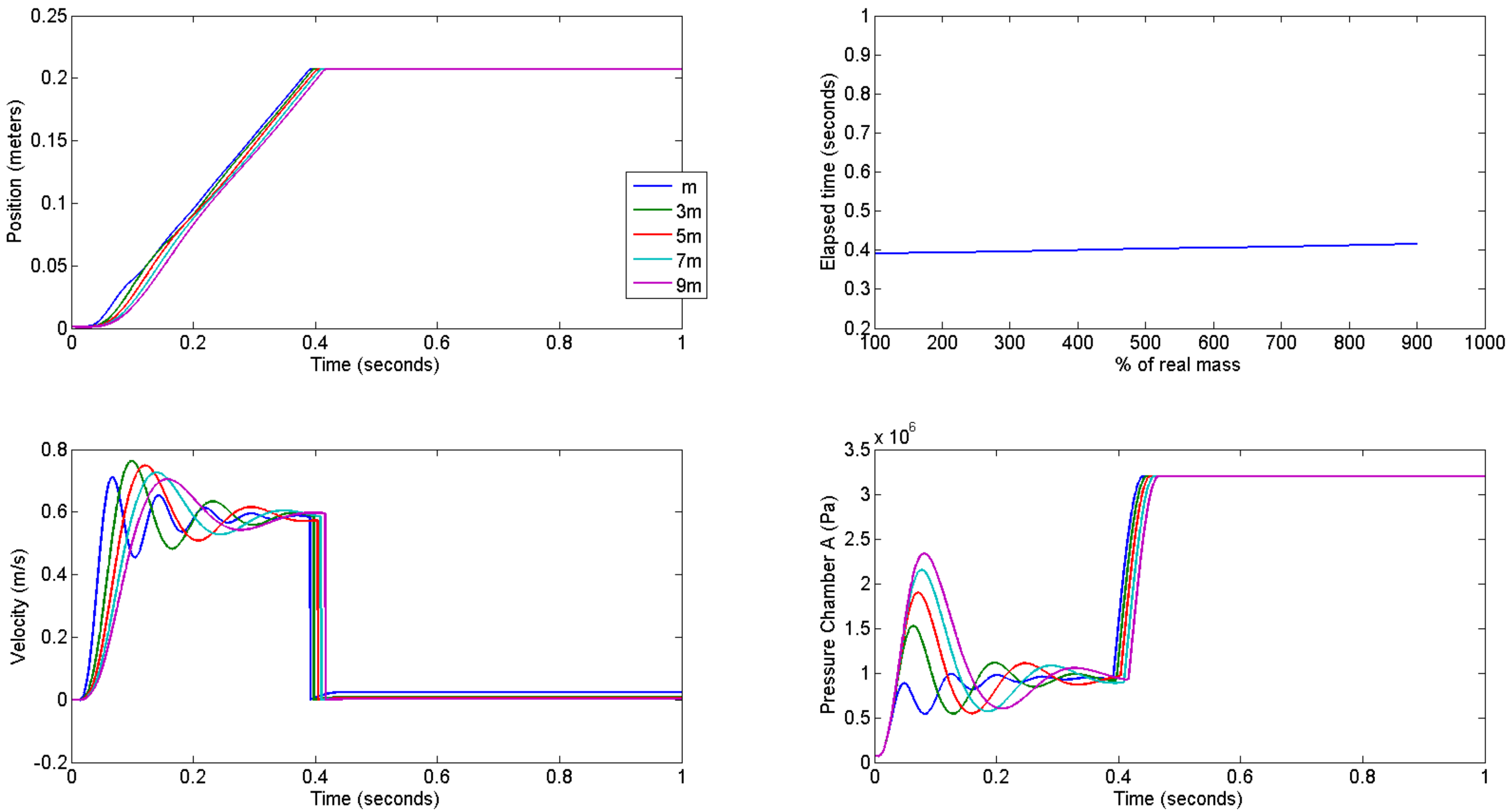

Figure 5.17. Influence of the mass variation on the outputs

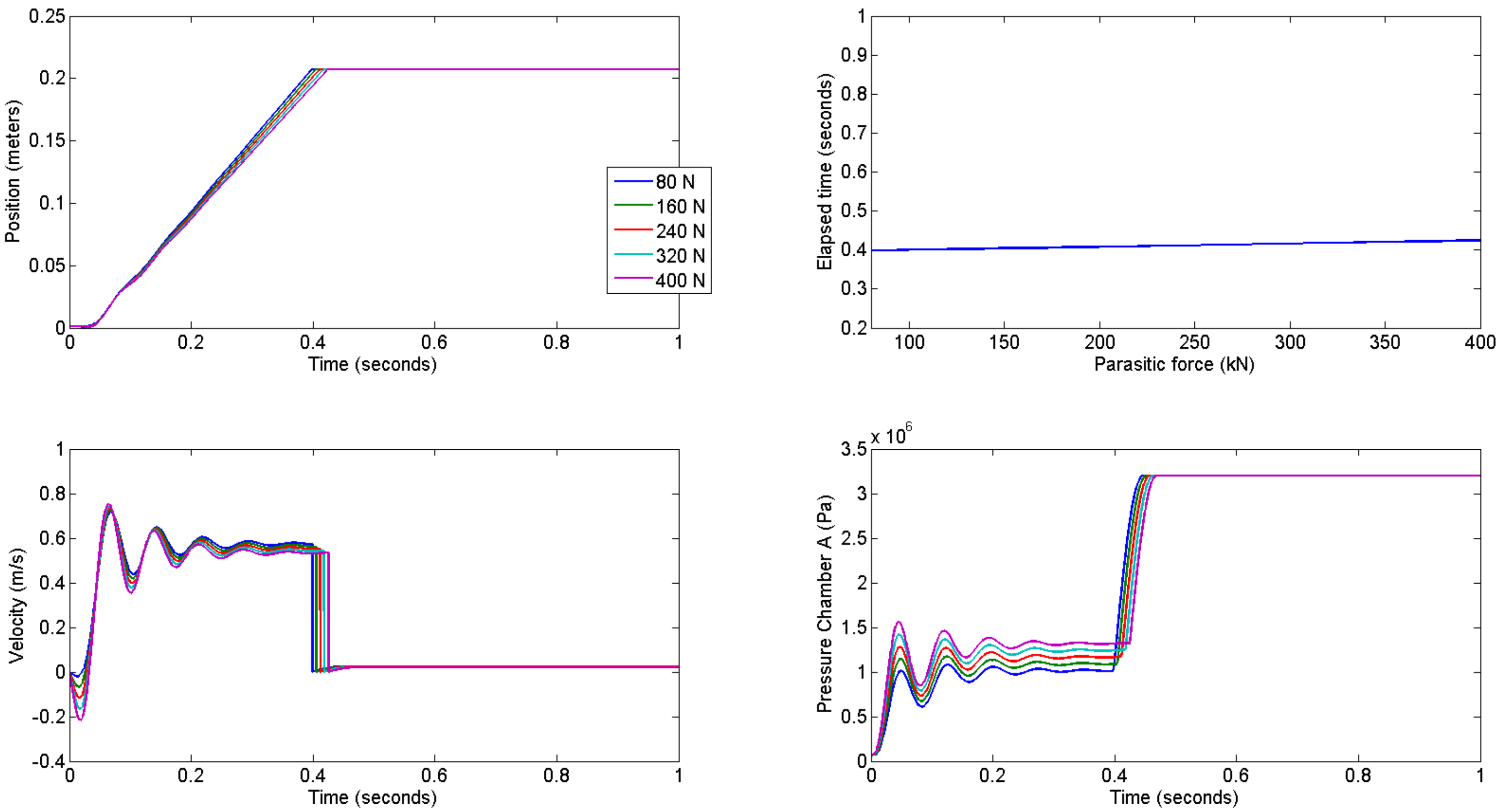

Figure 5.18. Influence of the parasitic forces on the outputs

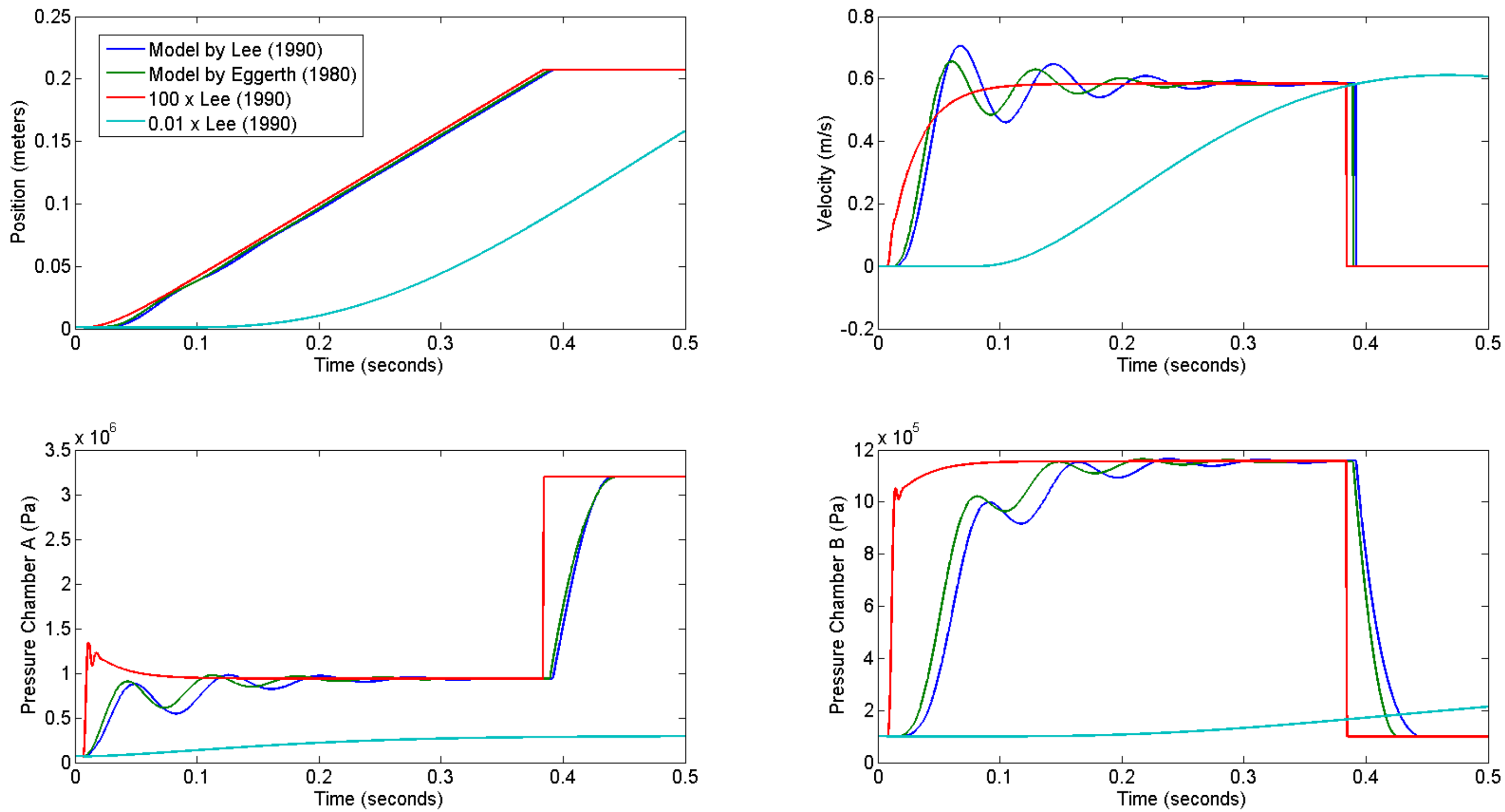

Figure 5.19. The influence of the bulk modulus

6 Discussion and Conclusions

In this chapter, the obtained results are further investigated and the crucial points are discussed. Based on the discussion, the most critical conclusions are stressed. Any possible future work is also explained in this chapter.

6.1 Discussion

Firstly, the implemented friction model predicts the friction forces with relatively small errors. This may occur due to the piston rod moves with relatively small velocities and the cylinder has been produced with small tolerances.

Even though, proportional valves often act proportionally to the input signal, it can be seen that that is not entirely correct with the current proportional valve. However, it must be stated that the VMS robot arm operates under $0.5u*$ region[5] and these nonlinearities are not encountered in the application. Moreover, valve hysteresis is another aspect of the model as it shows also nonlinear behavior and an additional attention must be paid to the determination of this parameter to accurately predict the displacement for both directions.

For the small valve input signal (u*=20.3%), none of the models accurately predicts the displacement although the linear and the nonlinear models provide more accurate simulations than the simple linear model. It can be seen that the nonlinear model can more accurately predict the chamber pressures while the linear model fails to do so (see Table 5.3). One remark should be made on the chamber pressures on the experimental data. The gravity forces affect the arm once the arm deviates from the starting position. Since the gravity forces are changing based on the displacement, this can clearly be observed on the pressures. In other words, there is an increasing force applied on the piston due to the gravity. The trend occurs simply because larger angle means that larger gravity force on the arm. When the arm stops, a steady state pressure difference between chamber A and B occurs to counteract the gravity. It is also the reason of pressure difference between the nonlinear model and the experimental data since the nonlinear model does not include the influence of the gravity.

[5] It must be noticed that the input valve signal is normalized to 1.

The valve behavior can be assumed to be linear in case valve input signal varies between 26% and 51%. Generally, the accuracy of the models is increased in this area, especially with the linear model since there are less nonlinear effects. The nonlinear model generally predicts the pressure levels accurately while the linear model predicts both values accurately enough. However, for models in large openings region (u*=91.6%), the displacement cannot be predicted by none of the models even though the chamber pressures are accurate enough.

The improved nonlinear model can predict both displacement and the chamber pressures accurately for all regions. Moreover, all of the improved models can predict both displacement and the chamber pressures more accurately except for the small openings. Especially, the simple linear model also predicts the displacement relatively accurate enough while it fails to do so when a regular simple linear model is used.

It is clear that the flow coefficient has a significant impact on the displacement of the arm, while it has no impact on the steady state pressure chambers. On the other hand, the supply pressure has a remarkable influence on both steady state chamber pressures and the displacement. The valve response time alters the time the system reaches the steady state. Both the mass variation and external forces do not have major impact on the displacement. However, the mass variation affects the dynamics of the system significantly. Larger mass leads to higher pressure level on the chamber A which is needed to overcome inertial forces. Moreover, the external forces affect both the dynamic and steady state chamber pressures.

While two different empirical models suggest similar behavior, increased bulk modulus values shows that the system would have reached steady state faster or vice versa.

6.2 Conclusions

It is clear from the discussion and Table 5.3 that none of the regular models are accurate enough to simulate the robot arm for various valve input signals. However, the linear model is accurate enough to simulate both chamber pressures and the displacement for the linear region and can be used to simulate only this region. Besides, since the nonlinear model is able to predict pressure levels more accurately for all regions; it must be used for prediction of pressures if it is necessary.

It is fair to say that the improved nonlinear model is safe to use for all valve openings and inferior to any other models. Clearly, the parameter feedback makes both the linear and the simplified linear models more reliable. However, it should be understood that the linear model contains more equations and require more calculating power than the nonlinear model if the parameter feedback is implemented. Therefore, even though the linear model is accurate enough, using the nonlinear model is still more logical for practical purposes (e.g. need for less calculation power). On the other hand, the simplified linear model requires less calculation power and can be considered for specific applications. However, the simplified linear model does not provide the chamber pressures and this can be a drawback if those variables need to be estimated.

Sensitivity analysis points out that the flow coefficient is the most crucial parameter for the estimation of the displacement of the valve and must be thoroughly investigated. Even though, most of the literature claims that implementing a constant supply pressure is a valid assumption, sensitivity analysis on this parameter stresses that such an assumption may cause major errors in both chamber pressures and estimated displacement in the simulations. In this context, it must be given that the measured supply pressures are varying in a significant range in the DeLaval VMS system.

The dynamics of the system is affected by the mass variation and the external forces; though, these parameters must be estimated carefully if dynamics of the system has the highest priority. Moreover, the valve response time can be crucial for fast dynamics and large valve response time value must be avoided for such cases. Furthermore, the bulk modulus is another important parameter that alters the system dynamics and the most appropriate bulk modulus model must be carefully selected by investigating the system properties. Then the closest empirical model can be picked with help of the literature.

6.3 Future work

It is stated that the models do not implement the effect of the gravity; although it was unavoidable in the experiment. Therefore, a gravity model can be further introduced for increased accuracy.

For the inaccurate models, the system state can be updated by using additional considerations (e.g. Kalman Filter)

More smooth description of the flow coefficient can be made by operating the robot arm for more points. This may also increase the robustness of the models.

The flow though the valves can be measured with flow sensors and this may help cross-checking some of the parameters used in the simulation. It should be noted that the flow coefficient is calculated indirectly since there is no measurement of flow through the valve available.

The DeLaval VMS robot arm contains three hydraulic actuator that drives two links. Therefore, these actuators can be connected with a more detailed mathematical model to provide complete description of the robot arm.

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