Friction and Friction Compensation

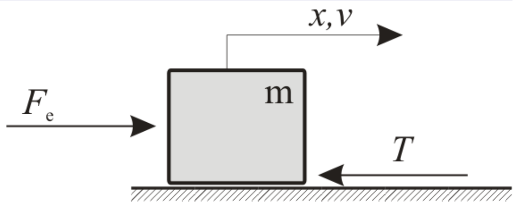
Robert Grepl
Mechatronics Laboratory
Institute of Solid Mechanics, Mechatronics and Biomechanics
Faculty of Mechanical Engineering, Brno University of Technology

2009

Dissipative forces

- viscous friction
- drag force
- dry friction

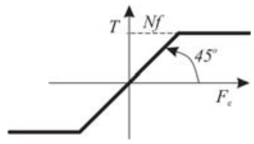
General Coulomb friction :: Intro



- equation of motion

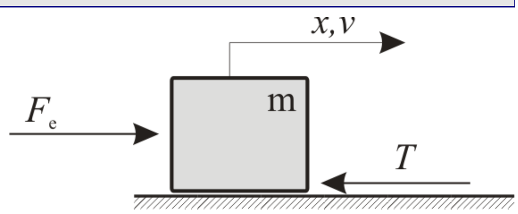
$$m\ddot{x} = F_e - T$$

- T? $T = Nf$



- ... but only for v>0
- generally:
  - v > 0: $m\ddot{x} = F_e - T$
  - v = 0: $0 = F_e - T$

General Coulomb friction :: Definition of T

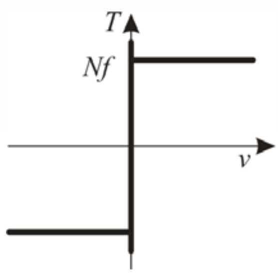


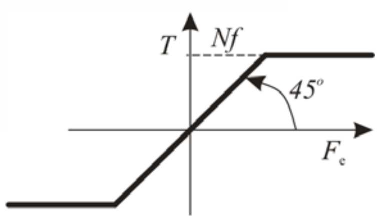
- Coulomb friction for general velocity and with stiction included:

$$T = \begin{cases} \mathrm{sgn}(v) F_C & \text{pokud } v \neq 0 \\ F_e & \text{pokud } v = 0 \wedge |F_e| < F_C \\ \mathrm{sgn}(F_e) F_S & \text{pokud } v = 0 \wedge |F_e| > F_C \end{cases}$$

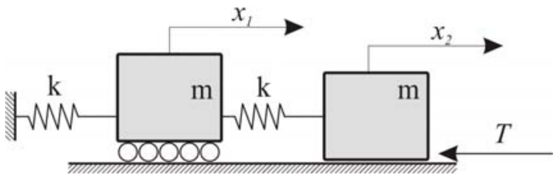
$$F_C = N f_{kin}$$

$$F_S = N f_{stiction}$$





General Coulomb friction :: Example



- equations

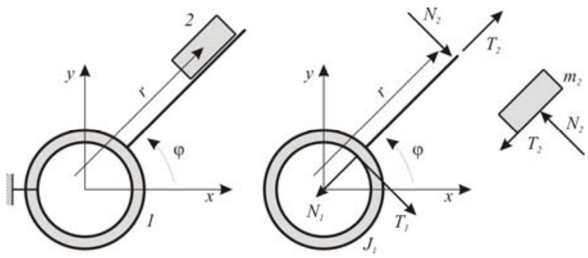
$$m\ddot{x}_1 = -2kx_1 + kx_2$$
$$m\ddot{x}_2 = kx_1 - kx_2 - T$$

$$T = \begin{cases} \mathrm{sgn}(v)F_C & \text{pokud } v \neq 0 \\ F_e & \text{pokud } v = 0 \wedge |F_e| < F_C \\ \mathrm{sgn}(F_e)F_S & \text{pokud } v = 0 \wedge |F_e| > F_C \end{cases}$$

- we need Fe, ...?

$$F_e = kx_1 - kx_2$$

Example 2



- dynamic eq.:

$$m\ddot{r} - m\dot{\varphi}^2 r = -T_2$$
$$(mr^2 + J)\ddot{\varphi} + 2mr\dot{r}\dot{\varphi} = -T_1 h$$

- constraints: $T_2 - N_1 = 0$

$$-T_1 - N_2 = 0$$

Pozor, tady je chyba.

- -> we have 6 unknowns and 4 equations -> friction eq.

$$T = \begin{cases} \mathrm{sgn}(v)F_C & \text{pokud } v \neq 0 \\ F_e & \text{pokud } v = 0 \wedge |F_e| < F_C \\ \mathrm{sgn}(F_e)F_S & \text{pokud } v = 0 \wedge |F_e| > F_C \end{cases}$$

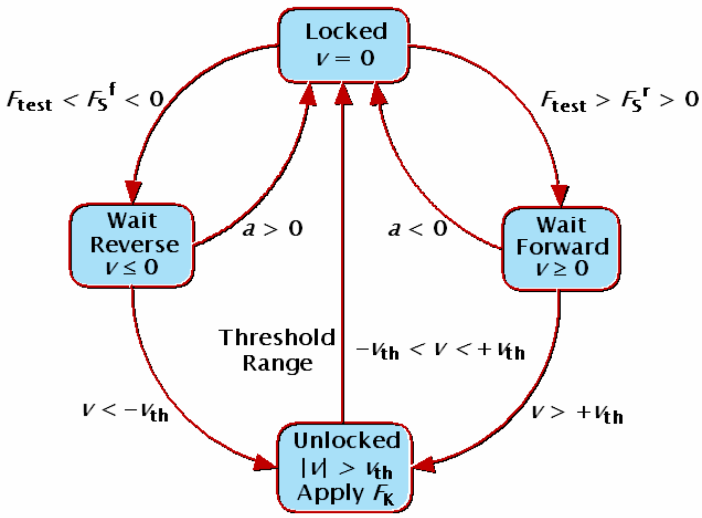
$$T_1 = f(\dot{\varphi}, N_1) \qquad T_2 = f(\dot{r}, N_2)$$

$\Rightarrow$

$$T_1 = f(\dot{\varphi}, T_2) \qquad T_2 = f(\dot{r}, T_1)$$

General Coulomb friction :: Discussion

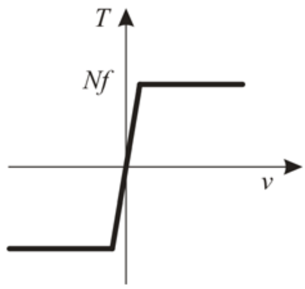
- what if we have 2 frictional joints? ... 4 combinations...
- requires computation of Fe
- similar approach is used in SimMechanics



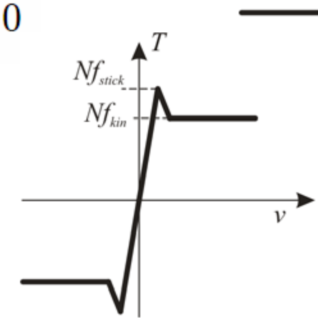
Static friction models

- Coulomb computationally efficient

$$T = \begin{cases} \min(kv, Nf), & \text{pro } v > 0 \\ \max(kv, -Nf), & \text{pro } v \le 0 \end{cases}$$



- Coulomb + stiction



Dynamic friction models

- Reset Integrator

$$\dot{p} = \begin{cases} 0 & \text{if } (v > 0 \wedge p \geq p_0) \vee (v < 0 \wedge p \leq -p_0) \\ v & \text{otherwise} \end{cases}$$

$$F_{\text{fric}} = \frac{(1+a(p))F_{\text{kin}}p}{p_0} + \beta\dot{p}$$
$$a(p) = \begin{cases} a & \text{if } |p| < p_0 \\ 0 & \text{otherwise} \end{cases}$$

- p... state – bending of virtual bristle
- parameters:
  - Fkin - kinetic friction
  - a - stiction (increase relatively to Fkin)
  - beta - damping
  - p0 – range of stiction

Dynamic friction models

- LuGre

$$\dot{z} = v - \sigma_0 \frac{|v|}{g(v)} z$$
$$g(v) = \alpha_0 + \alpha_1 \mathrm{e}^{-(\frac{v}{v_0})^2}$$
$$F_{\text{fric}} = \sigma_0 z + \sigma_1 \dot{z} + (\alpha_2 v)$$

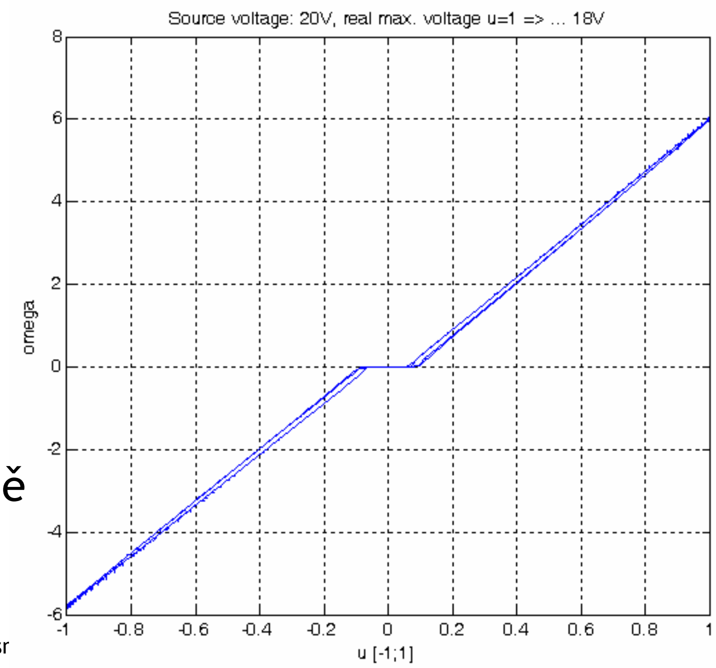
- z – state
- parameters:
  - alpha0 – kinetic friction
  - alpha1 - stiction
  - v0 – stribeck velocity
  - sigma0 – stiffness of bristles
  - sigma1 – damping of bristles
  - (alpha2 – viscous friction)

Friction in DC motor :: Experiment

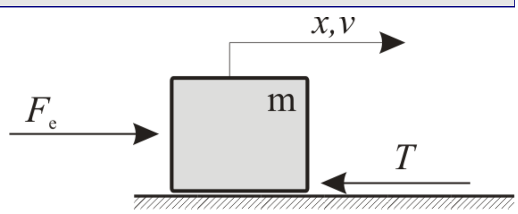
- praktická ukázka nelinearity soustavy motor + řídicí elektronika
- vstup: napětí u (norm.)
- výstup: úhlová rychl.

$$\omega = k_N u$$

- reálně: **nelinearita**
- experimentálně ověřeno, že je způsobena částečně elektronikou, ale převážně motorem (suché tření)



Friction compensation :: Why PID fails?



- equation of motion:

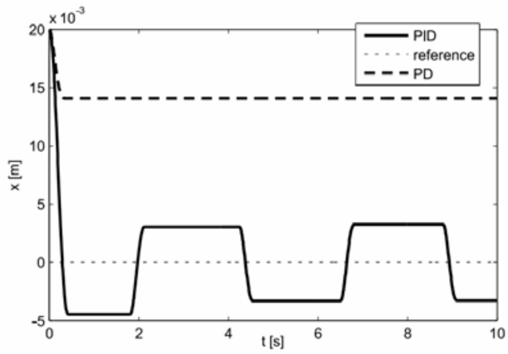
$$m\ddot{x} = F_e - T$$

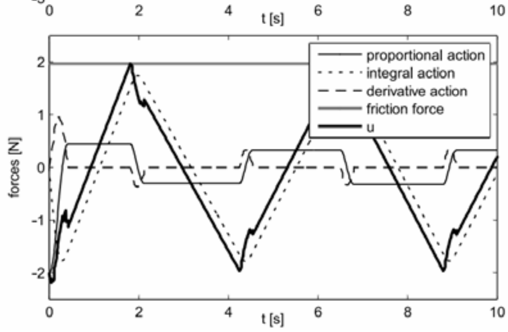
- PD regulator

$$u = K_p e + K_d \frac{\mathrm{de}}{\mathrm{dt}} \qquad e_{ust} = \frac{F_T}{K_p}$$

- PID

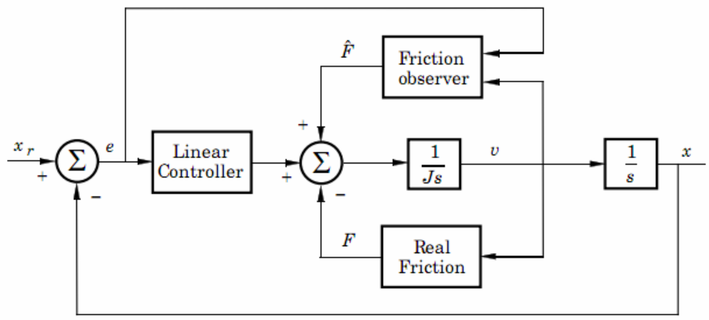
$$u = K_p e + K_d \frac{\mathrm{de}}{\mathrm{dt}} + K_i \int \mathrm{edt}$$





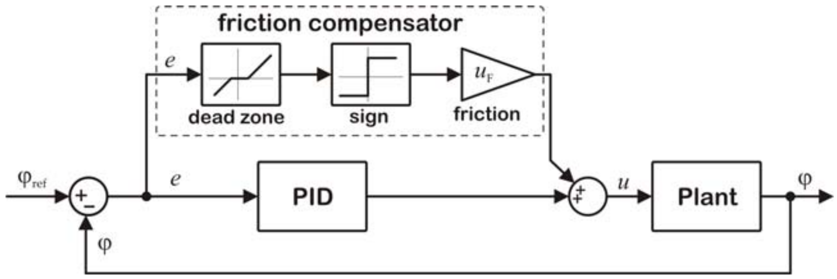
Friction compensation I.

- based on velocity measurement
- principle of feedback linearization
- problem: velocity measurement (noise)



	$e_{rms} \cdot 10^3$	$e_{max} \cdot 10^3$
No friction	3.12	9.06
With friction	13.0	63.7
Friction compensation (Coulomb)	7.85	32.7
Friction compensation (LuGre)	2.65	8.57
Overcompensation	6.72	28.5
Undercompensation	6.22	25.8

Friction compensation II.



- use position error instead of velocity

Issues

- overcompensation – oscilations
- undercompensation – steady state error
- -> online friction coeff. estimation

References

- Olsson, H.; Astrom, K. J.; de Wit, C. C.; Gafvert, M. & Lischinsky, P.
  Friction models and friction compensation
  Eur. J. Control, **1998**, *4*, 176-195