1 Models

1.1 General 1DOF model dynamic

$$m\ddot{x} + b\dot{x} + kx = u \tag{1}$$

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = \frac{1}{m}u \tag{2}$$

$$\ddot{x} + 2\delta\dot{x} + \Omega^2 x = \frac{1}{m}u \tag{3}$$

where $\delta = \frac{b}{2m}$ is damping and $\Omega = \sqrt{\frac{k}{m}}$ is natural frequency of system.

$$\ddot{x} + 2\zeta\Omega\dot{x} + \Omega^2 x = \frac{1}{m}u \tag{4}$$

where $\zeta = \frac{b}{2\sqrt{km}}$ is damping ratio.

1.2 Model overview

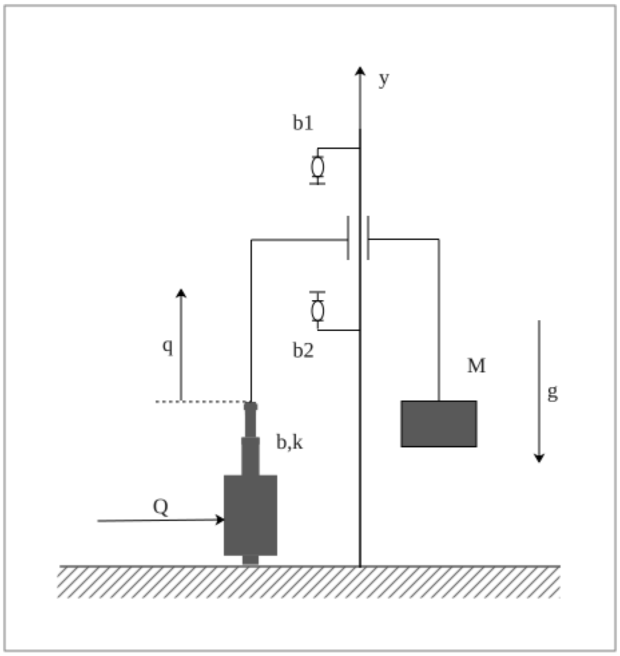

Figure 1: Schematic model

1.3 Hard stop

Hard stop can be represented as spring and dumps:

$$F_{HS} = \begin{cases} K_p(x - g_p) + D_p v & \text{for } x \geq g_p \\ 0 & \text{for } g_n < x < g_p \\ K_n(x - g_n) + D_n v & \text{for } x \leq g_n \end{cases} \tag{5}$$

Possible parameters:

K_p	10^6	$[kg/s^2]$
K_n	10^6	$[kg/s^2]$
D_p	350	$[kg/s]$
D_n	350	$[kg/s]$
g_n	0	$[m]$
g_p	0.2	$[m]$

1.4 General physical principles

1.4.1 Thermodynamics

p	Pa	pressure
V	m^3	volume
m	kg	mass
n	mol	amount of substance
R	$Jkg^{-1}K^{-1}$	ideal gas constant
r	$Jkg^{-1}K^{-1}$	mass-specific gas constant
T	K	temperature
S	m	area
z	m	hight
w	ms^{-1}	flow speed
H	J	enthalpy
ν	m^3kg^{-1}	specific volume
Q	J	heat shared with environment
W_T	J	work
c_p	$Jkg^{-1}K^{-1}$	is the specific heat at constant pressure
c_v	$Jkg^{-1}K^{-1}$	is the specific heat at constant volume
$g = 9.81$	ms^{-2}	gravity acceleration
$\gamma = 1.4$(air)	$-$	heat capacity ratio (isentropic expansion factor)

1.4.2 Equation of state

Generally $pV = nRT$ but for air purpose were $r = \frac{pv}{T} = R = 287.1[Jkg^{-1}K^{-1}]$ following equation can be used 6:

$$pV = mrT \tag{6}$$

1.4.3 Isothermal process

Used in some papers 7:

$$p_1V_1 = p_2V_2 = const \tag{7}$$

1.4.4 Adiabatic process

Adiabatic process 8:

$$p_1V_1^\gamma = p_2V_2^\gamma = const \tag{8}$$

Heat capacity ratio:

$$\gamma = \frac{c_p}{c_v} \tag{9}$$

Mayer's relation:

$$c_p = c_v + R \tag{10}$$

1.4.5 Bernoulli's principle

Bernoulli's principle 11:

$$H_1 + \frac{mw_1^2}{2} + mgz_1 + Q = H_2 + \frac{mw_2^2}{2} + mgz_w + W_T \tag{11}$$

$$H_1 - H_2 = -\int_1^2 V dp = c_p(T_1 - T_2) = c_p T_1 (1 - \frac{T_2}{T_1}) \tag{12}$$

Differential form:

$$\nu dp + wdw + gdz + dw_T = 0 \tag{13}$$

1.4.6 Fluid mechanics

$\dot{m}$	kgs^{-1}	mass flow
c	ms^{-2}	speed of sound
w_k	ms^{-2}	critical flow velocity
ψ	$-$	flow coefficient
ψ_{max}	$-$	critical flow coefficient
β	$-$	ration of pressure differential
β_k	$-$	critical ratio of pressure differential

Continuity equation 14:

$$\dot{m} = S_1 w_1 \rho_1 = S_2 w_2 \rho_2 = const \tag{14}$$

1.4.7 Air expansion from tank

Assuming $W_T = 0, z_1 = z_2, Q = 0$ conditions and combine with 11 we will get 15 equation:

$$w_2 = \sqrt{2(H_1 - H_2)} \tag{15}$$

$$w_2 = \sqrt{2RT_1(\frac{\gamma}{\gamma - 1})(1 - (\frac{p_2}{p_1})^{\frac{\gamma-1}{\gamma}})} \tag{16}$$

$$\rho_2 = \frac{p_1}{RT_1}(\frac{p_2}{p_1})^{\frac{1}{\gamma}} \tag{17}$$

Together 14 16 17:

$$\dot{m} = Sp_1\sqrt{\frac{2}{RT_1}} \cdot \sqrt{\frac{\gamma}{\gamma - 1}\left(\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right)} \tag{18}$$

where:

$$\psi\left(\frac{p_2}{p_1}\right) = \sqrt{\frac{\gamma}{\gamma - 1}\left(\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right)} \tag{19}$$

Finally 20:

$$\dot{m} = Sp_1\sqrt{\frac{2}{RT_1}}\psi\left(\frac{p_2}{p_1}\right) \tag{20}$$

1.4.8 Critical flow velocity

Speed of sound:

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT} \tag{21}$$

Assume $c = w_2$ (16, 21) we will get the critical flow velocity:

$$c_2 = w_k = \sqrt{\gamma RT} = \sqrt{2RT_1\frac{\gamma}{\gamma-1} - 2w_k^2\frac{1}{\gamma-1}} \tag{22}$$

$$w_k^2 = 2RT_1\frac{\gamma}{\gamma-1} - 2w_k^2\frac{1}{\gamma-1} \tag{23}$$

$$w_k = \sqrt{2RT_1\frac{\gamma}{\gamma-1}} = \sqrt{2p_1\nu_1\frac{\gamma}{\gamma+1}} \tag{24}$$

For calculating critical pressure ratio assume $w_k = w_2$ 24 16:

$$\sqrt{2RT_1\frac{\gamma}{\gamma-1}} = \sqrt{2RT_1\frac{\gamma}{\gamma-1}\left(1-\left(\frac{p_2}{p1}\right)^{\frac{\gamma+1}{\gamma}}\right)} \tag{25}$$

$$\left(\frac{p_2}{p1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{2}{\gamma+1} \tag{26}$$

$$\tag{27}$$

$$\left(\frac{p_2}{p1}\right)_k = \left(\frac{p_k}{p1}\right) = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \beta_k \tag{28}$$

Critical pressure condition is $p_k = p_1\beta_k$.
Applying 28 to 19:

$$\psi_{max}(\beta_k) = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\sqrt{\frac{\gamma}{\gamma+1}} \tag{29}$$

For air $\beta_k = 0.528, \psi_{max} = 0.484$
Final equation for ψ:

$$\psi\left(\frac{p_2}{p_1}\right) = \begin{cases} \sqrt{\frac{\gamma}{\gamma-1}\left(\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma+1}{\gamma}}\right)} & 0.528 < \frac{p_2}{p_1} \leq 1 \\ \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma+1}}\sqrt{\frac{\gamma}{\gamma+1}} & 0 \geq \frac{p2}{p1} \leq 0.528 \end{cases} \tag{30}$$

1.5 Pneumatic actuator model

p_A, p_B	Pa	pressure in chamber A, B
$\dot{m_A}, \dot{m_B}$	$kg \cdot s^{-1}$	mass flow on way to chamber A, B
S_A, S_B	m^2	piston area
V_A, V_B	m^3	volume of chamber A,B
V_{0A}, V_{0B}	m^3	"dead" volume of chamber A,B
m	kg	piston mass
F_{load}	N	load
x	m	piston position
l	m	maximum piston position

Mechanical equation, where F_f represent all friction and viscous forces.

$$m\ddot{x} = (p_A S_A - p_B S_B) - F_f \tag{31}$$

Volumes of chambers:

$$V_A = S_A x + V_{0A} \tag{32}$$
$$V_B = S_B(l - x) + V_{0B} \tag{33}$$
$$\dot{V}_A = S_A \dot{x} \tag{34}$$
$$\dot{V}_B = -S_B \dot{x} \tag{35}$$

1.5.1 Pressure models

Isothermal model

$$m = \rho V \tag{36}$$
$$\dot{m} = \dot{\rho} V + \rho \dot{V} \tag{37}$$

Applying 6:

$$\rho = \frac{p}{RT} \tag{38}$$
$$\dot{\rho} = \frac{\dot{p}}{RT} \tag{39}$$

Finally get 40:

$$\dot{p} = -\frac{p}{V}\dot{V} + \frac{RT}{V}\dot{m} \tag{40}$$

Adiabatic model For simple adiabatic model following equation can be used 41:

$$\dot{p} = -\frac{\gamma p}{V}\dot{V} + \frac{\gamma RT}{V}\dot{m} \tag{41}$$

$$\dot{p_A} = \frac{\gamma}{S_A x + V_{0A}} \left(-p_A S_A \dot{x} + RT_A \dot{m_A}\right) \tag{42}$$

$$\dot{p_B} = \frac{\gamma}{S_B(l - x) + V_{0B}} \left(p_B S_B \dot{x} + RT_B \dot{m_B}\right) \tag{43}$$

T_A, T_B calculated from 6, or in adiabatic model this parameters can remain constant same as atmospheric temperature.

1.5.2 Input/Output mass flows

$$\dot{m}T = \dot{m_{in}} T_s - \dot{m_{out}} T_{A/B} \tag{44}$$

1.5.3 Differential equation for Temperature change

$$T = \int (\gamma T_s - T_{A/B}) \frac{R\dot{m}_{A/Bin}}{p_{A/B} V_A} T_{A/B} - (\gamma - 1) \frac{R\dot{m}_{A/Bout}}{p_{A/B} V_{A/B}} T_{A/B}^2 - (\gamma - 1) \frac{\dot{V}_{A/B}}{V_{A/B}} T_{A/B} \tag{45}$$

1.5.4 Valve model

S_{eq}	m^2	Equivalent cross section
S_{max}	m^2	Maximum cross section
Cd	$-$	Coefficient of contraction
u	$-$	Regulation variable

Valve flow model with simply input control signal For regulation flow this model used input control signal directly without spool mechanics.

Coefficient of contraction 46:

$$C_d = \frac{S_{eq}}{S_{max}} \tag{46}$$

For flow control regulation $u \in \langle -1, 1\rangle$ can be used.

$$u = \begin{cases} u \in \langle -1, 0) & \text{discharge the chamber} \\ u = 0 & \text{valve closed} \\ u \in (0, 1\rangle & \text{filling the chamber} \end{cases} \tag{47}$$

$$\dot{m} = uS_{max}C_d p_1 \sqrt{\frac{2}{RT_1}} \cdot \psi\left(\frac{p_2}{p_1}\right) \tag{48}$$

For filling the chamber:

- $p_1 = p_s$
- $p_2 = p_A$ or p_B
- $T_1 = T_s$

For discharge the chamber:

- $p_1 = p_A$ or p_B
- $p_2 = p_0$
- $T_1 = T_A, T_B$

where p_s is supply pressure. p_0 atmospheric pressure. As T_i - atmospheric temperature using according to isothermal process.

$$\dot{m}_A = \begin{cases} uS_v C_d p_s \sqrt{\frac{2}{RT_s}} \cdot \psi\left(\frac{p_A}{p_s}\right) & , u \in (0, 1\rangle \\ 0 & , u = 0 \\ uS_v C_d p_A \sqrt{\frac{2}{RT_A}} \cdot \psi\left(\frac{p_0}{p_A}\right) & , u \in \langle -1, 0) \end{cases} \tag{49}$$

$$\dot{m}_B = \begin{cases} uS_v C_d p_s \sqrt{\frac{2}{RT_s}} \cdot \psi\left(\frac{p_B}{p_s}\right) & , u \in (0, 1\rangle \\ 0 & , u = 0 \\ uS_v C_d p_A \sqrt{\frac{2}{RT_B}} \cdot \psi\left(\frac{p_0}{p_B}\right) & , u \in \langle -1, 0) \end{cases} \tag{50}$$

Valve flow with spool mechanic included With respect to valve spool modeled as 1DOF system 1 and mechanical and geometrical properties following equation were used.

Valve flow with spool In this model we accept a spool displacement x_s, controlled by input voltage u.

$$\dot{m}(P_u, P_d) = \begin{cases} C_f A_v \left(\frac{\gamma}{R}\left(\frac{2}{\gamma-1}\right)\right)^{\frac{1}{2}} \cdot \frac{P_u}{\sqrt{T}}\left(\frac{P_d}{P_u}\right)^{\frac{1}{\gamma}} \cdot \sqrt{1-\left(\frac{P_d}{P_u}\right)^{\frac{\gamma-1}{\gamma}}} & \text{, if } \frac{P_d}{P_u} > P_{cr} \text{ (subsonic)} \\ C_f A_v \frac{P_u}{\sqrt{T}} \cdot \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} & \text{, if } \frac{P_d}{P_u} \le P_{cr} \text{ (sonic)} \end{cases} \tag{51}$$

where C_f is discharge coefficient, A_v is the effective are of valve orifice.

$$A_v = \frac{\pi x_s^2}{4} \tag{52}$$

$$x_s = C_v u \tag{53}$$

where C_v is the valve constant.

Valve model by Endler Require fitting constants and generally system identification. Mass flow rates are given by following equations:

$$\begin{aligned} \dot{m}_A(u, p_A) = g_1(p_A, sign(u)) arctg(2u) \\ \dot{m}_B(u, p_B) = g_2(p_B, sign(u)) arctg(2u) \end{aligned} \tag{54}$$

where g_1, g_2 are signal functions given:

$$\begin{aligned} g_1(p_A, sign(u)) = \beta\Delta p_A = \begin{cases} (p_s - p_A)\beta^{ench} & \text{, if u} \ge 0 \\ (p_A - p_0)\beta^{esv} & \text{, if u} < 0 \end{cases} \\ g_2(p_B, sign(u)) = \beta\Delta p_B = \begin{cases} (p_s - p_B)\beta^{ench} & \text{, if u} < 0 \\ (p_B - p_0)\beta^{esv} & \text{, if u} \ge 0 \end{cases} \end{aligned} \tag{55}$$

where $\beta^{ench}, \beta^{evs}$ are constant coefficients. For fitting model stop piston (speed of piston is null). This mean that volume is constant. We can measure flow rate $\dot{m}$ versus input voltage u with given pressure difference.

Valve dead-zone For more precision control and modeling of the valve system, valve dead-zone can be used 56.

$$u_z = \begin{cases} g_z(u) < 0 & \text{, if } u \le u_n \\ 0 & \text{, if } u_n < u < u_p \\ h_z(u) > 0 & \text{, if } u \ge u_p \end{cases} \tag{56}$$

1.6 Mechanical assembly

Mechanical assembly basically represented by following equation 57.

$$\ddot{x} = \frac{1}{m}\left(S_A p_A - S_B p_B - S_0 p_0 - F_f\right) \tag{57}$$

where F_f is a friction force. Friction force can be modeled in the different ways.

As an example of possible model is following equation. That consist from complex friction forces including viscous friction and Coulomb friction 58.

$$F_f = \begin{cases} C\dot{x} + \left(f_c + (f_s - f_c)e^{-\left(\frac{\dot{x}}{v_s}\right)^{\delta}}\right) sign(\dot{x}) & \text{, if } \dot{x} \le v_e \\ \mu\dot{x} & \text{, if } \dot{x} > v_e \end{cases} \tag{58}$$

where C - viscous friction coefficient, f_c - Coulomb friction, f_s - maximum static friction, μ - dynamic friction factor, v_s - Stribeck velocity, δ - arbitrary index, v_e critical velocity.

2 Models based on approximation

Generally with dataset of input-output signals approximation model can be fit. Using System Identification Toolbox and modeled as Black-Box or Gray-Box models. This section attempted to fit some models using data from SimScape and Equation model presented before.

Fit approximation model make sense only if we know what to fit. Using signal process techniques and identify dominant signals that providing best classification features we will train models with respect to this signals.

Demonstration scripts are done and waiting for signals :)

2.1 State-space model

2.2 ARX model

3 Models comparison

3.1 Model based on equations

This model 2 was developed with respect to equations represented in previous section.

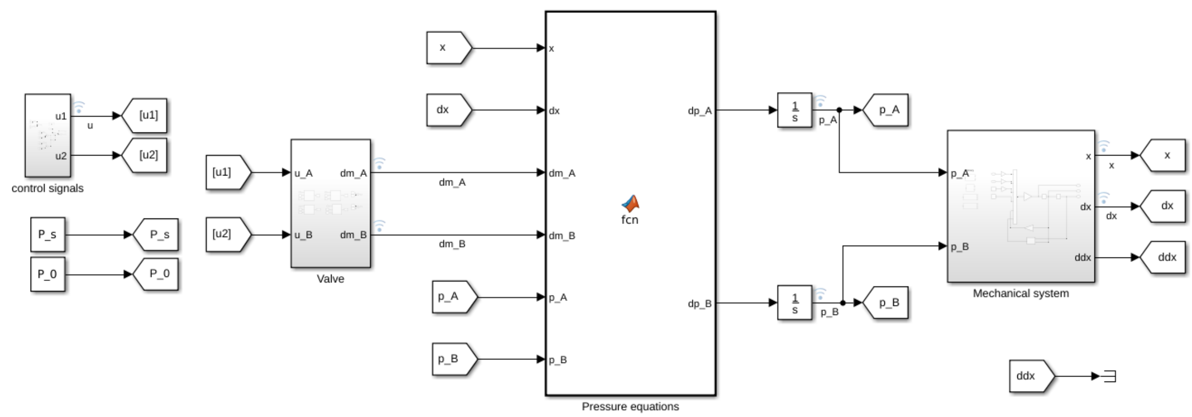

Figure 2: Simulink model based on equations

3.2 Model Simscape

Model 3 was developed using SimScape toolbox.

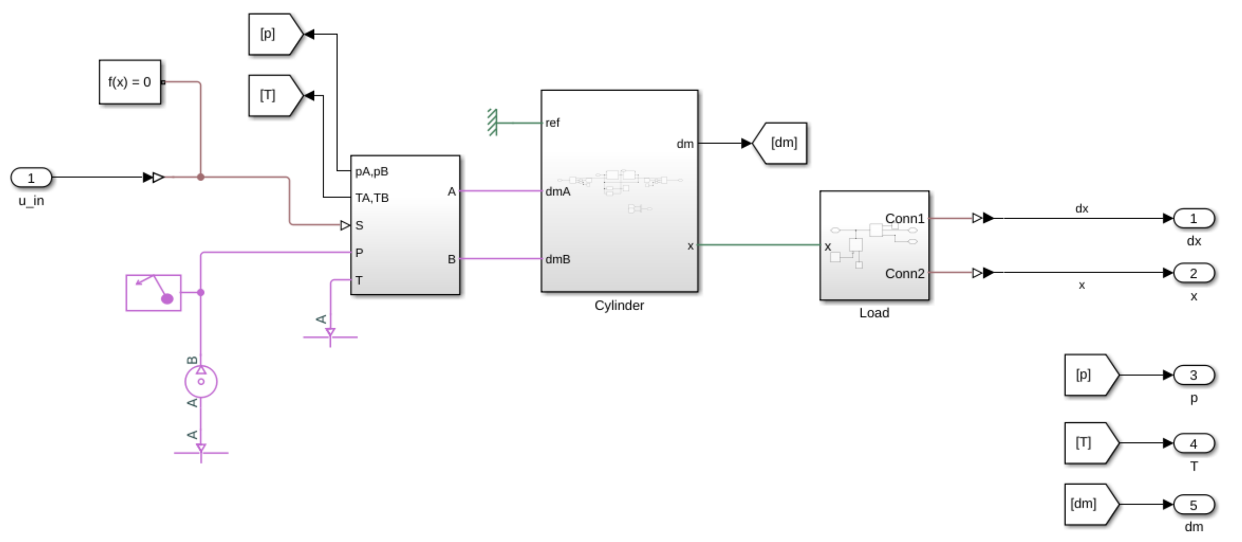

Figure 3: Simulink model using SimScape Toolbox

3.3 Comparison

Following figure 4 represent comparison of 2 models (Simscape and based on equations) using same parameters for simulation: There is slight difference between models causing Valve dynamics simplifications in model based on equations.

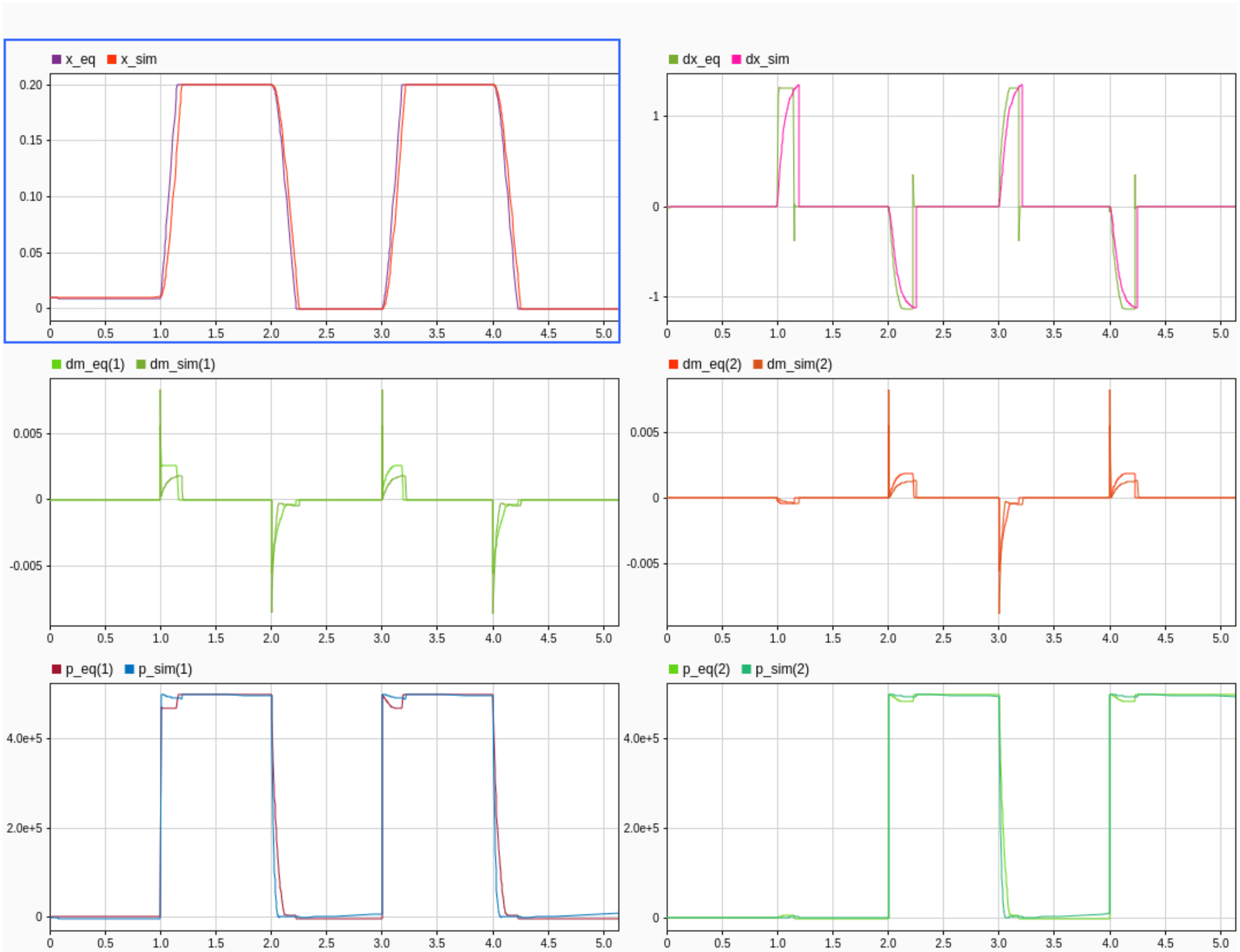

Figure 4: Comparison of simscape and model based on equations

4 Parameter identification

4.1 Mechanical assembly

In mechanical system there is F_f force represented by frictions accruing in the system. This force can be modeled by different friction models with respect to 1.6. Friction force parameters can be estimated using "gray-box" method. Using $\dot{m}$ mass flow data versus x position measured on real assembly and use these data as an input and output, we can fit F_f. Simplify model can contain TODO:

- F_C static friction
- C_v viscous
- C_p Pressure difference

4.2 Cylinder

Dead volume: $p_1 V_1^n = p_2 V_2^n$ or datasheet.

4.3 Valve

For valve system there are two parameters that need to be estimated. According to equation 59 with constant p_1 (pressure supply) and p_2 (atmospheric pressure), we can estimate $\boldsymbol{C}$ if we neglect Valve Spool dynamic. If in experiment we determine that spool dynamic necessary to include. We provide same experiment with spool model including to "Gray-box" fitting model.

$$\dot{m} = \boldsymbol{u}(x_s)\boldsymbol{C} p_1 \sqrt{\frac{2}{RT_1}} \cdot \psi\left(\frac{p_2}{p_1}\right) \tag{59}$$