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Simulation, animation and program support for a high performance pneumatic force actuator system

Djordje Dihovicni*, Novak Nedic

Department of Control Systems, Visa Tehnicka Skola, Bulevar AVNOJ-a 152 a, 11070 New Belgrade, Serbia
Department of Control Systems and Fluid, Mechanical Faculty University of Kragujevac, Dositejeva 19, 36000 Kraljevo, Serbia

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Abstract

Program support, simulation and the animation of dual action pneumatic actuators controlled with proportional spool valves are developed. Various factors are involved, such as time delay in the pneumatic lines, leakage between chambers, air compressibility in cylinder chambers as well as non-linear flow through the valve. Taking into account the complexity of the model, and the fact that it is described by partial different equations, it is important to develop the program support based on numerical methods for solving this kind of problems. Simulation and program support in Maple and Matlab programming languages are conducted, and the efficiency of the results is shown from the engineering point of view.

Keywords: Pneumatic actuator; Simulation; Animation; Program support

1. Introduction

Detailed mathematical model of dual action pneumatic actuator system, controlled by proportional spool valves, is shown in paper [13], and the effects of non-linear flow through the valve, leakage between chambers, time delay, attenuation and other effects are carefully considered.

These pneumatic systems have a lot of advantages if we compare them with the same hydraulic types; they are suitable for clean environments, and much safer. In accordance with project and space conditions, valves are positioned at relatively large distance from pneumatic cylinder [2].

Considering real pneumatic systems, it is crucial to describe them with time delay, non-linearities, with attempt of not creating only academic model [5]. Despite these problems, the development of fast algorithms and using numerical methods for solving partial different equations, as well as enhanced simulation and animation techniques have become necessary [1]. Various practical stability approaches, for solving complex partial equations, have used similar algorithms [6].

* Corresponding author at: Department of Control Systems, Visa Tehnicka Skola, Bulevar AVNOJ-a 152 a, 11070 New Belgrade, Serbia.
E-mail address: ddihovic@EUnet.yu (D. Dihovicni).

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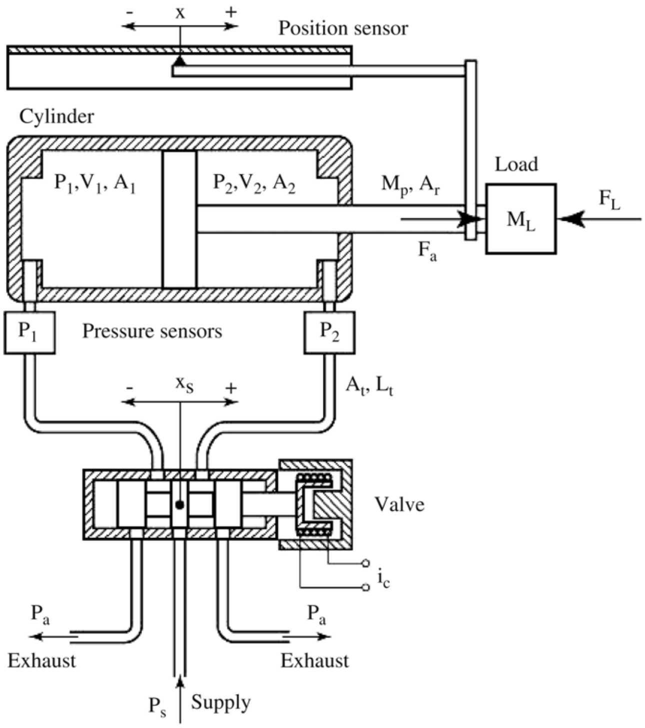

Fig. 1. Schematic representation of the cylinder–valve system.

2. Representation of pneumatic cylinder–valve system

Typical pneumatic system includes pneumatic cylinder, command device, force, position and pressure sensors, as well as connecting tubes.

In Fig. 1, a schematic representation of the pneumatic actuator system is shown.

The motion equation for the piston–rod assembly is described as:

$$\left(M_L + M_p\right) \cdot \frac{\mathrm{d}}{\mathrm{d}t}\dot{x} + \beta \cdot \dot{x} + F_f + F_L = P_1 \cdot A_1 - P_2 \cdot A_2 - P_a \cdot A_r \tag{1}$$

where M_L is the external mass, M_p is the piston and rod assembly mass, x represents the piston position, β is the viscous friction coefficient, F_f is the Coulomb friction force, F_L is the external force, P_1 and P_2 are the absolute pressure in actuator chambers, P_a is the absolute ambient pressure, A_1 and A_2 are the piston effective areas, and A_r is the cross-sectional area of the rod.

The general model for a volume of gas consists of state equation, the conservation of mass, and the energy equation. The equation for each chamber can be written using the assumptions that the gas is ideal, the pressure and temperature within the chamber are homogeneous, and the kinetic and potential energy terms are negligible. Taking into account a control volume V,density ρ, mass m, pressure P, and temperature T, an equation for ideal gas is described as:

$$P = \rho \cdot R \cdot T \tag{2}$$

where R is the gas constant. Mass flow is given as:

$$\dot{m} = \frac{\mathrm{d}}{\mathrm{d}t} \cdot (\rho \cdot V) \tag{3}$$

and it can be expressed as:

$$\dot{m}_{ul} - \dot{m}_{iz} = \dot{\rho} \cdot V + \rho \cdot \dot{V}, \tag{4}$$

where $\dot{m}_{ul}\dot{m}_{iz}$ are the input and output mass flow. Energy equation is described as:

$$q_{ul} - q_{iz} + k \cdot C_v \cdot (\dot{m}_{ul} \cdot T_{\text{in}} - \dot{m}_{iz} \cdot T) - W = \dot{U}, \tag{5}$$

where q_{ul} and q_{iz} are the heat transfer terms, k is the specific heat ratio, C_v is the specific heat at constant volume, T_{in} is the temperature of the incoming gas flow, W is the rate of change in the work, and $\dot{U}$ is the change of the internal energy. The total change of the internal energy is given as:

$$\dot{U} = \frac{\mathrm{d}}{\mathrm{d}t}(C_v \cdot m \cdot T) = \frac{1}{k-1} \cdot \frac{\mathrm{d}}{\mathrm{d}t}(P \cdot V) = \frac{1}{k-1}\left(V \cdot \dot{P} + P \cdot \dot{V}\right) \tag{6}$$

where, $C_v = R/(k-1)$. If we use the term $\dot{U} = P \cdot \dot{V}$ and substitute Eq. (6) into Eq. (5), it yields:

$$q_{ul} - q_{iz} + \frac{k}{k-1} \cdot \frac{P}{\rho \cdot T}(\dot{m}_{ul} \cdot T_{ul} - \dot{m}_{iz} \cdot T) - \frac{k}{k-1} \cdot P \cdot \dot{V} = \frac{1}{k-1} \cdot V \cdot \dot{P}. \tag{7}$$

If we assume that input flow is on the gas temperature in the chamber, which is considered here for analysis, then we have:

$$\frac{k}{k-1} \cdot (q_{ul} - q_{iz}) + \frac{1}{\rho} \cdot (\dot{m}_{ul} - \dot{m}_{iz}) - \dot{V} = \frac{V}{k \cdot P} \cdot \dot{P}. \tag{8}$$

Further simplification may be made by analyzing the terms of heat transfer in Eq. (8).

If we consider that the process is adiabatic ($q_{ul} - q_{iz} = 0$), the pressure in the chamber can be derived as:

$$\dot{P} = k \cdot \frac{P}{\rho \cdot V} \cdot (\dot{m}_{ul} - \dot{m}_{iz}) - k \cdot \frac{P}{V} \cdot \dot{V} \tag{9}$$

and if we substitute ρ from Eq. (2), then it yields:

$$\dot{P} = k \cdot \frac{R \cdot T}{V} \cdot (\dot{m}_{ul} - \dot{m}_{iz}) - k \cdot \frac{P}{V} \cdot \dot{V} \tag{10}$$

and if we consider that the process is isothermal ($T =$ constant), then the change in the internal energy can be written as:

$$\dot{U} = C_v \cdot \dot{m} \cdot T \tag{11}$$

and hence Eq. (8) can be written as:

$$q_{\text{in}} - q_{\text{out}} = P \cdot \dot{V} - \frac{P}{\rho} \cdot (\dot{m}_{\text{in}} - \dot{m}_{\text{out}}) \tag{12}$$

and then:

$$\dot{P} = \frac{R \cdot T}{V} \cdot (\dot{m}_{\text{in}} - \dot{m}_{\text{out}}) - \frac{P}{V} \cdot \dot{V}. \tag{13}$$

Comparing Eq. (10) with Eq. (13), it can be shown that the only difference is in the heat transfer factor term k. Then both the equations can be given as:

$$\dot{P} = \frac{R \cdot T}{V} \cdot (\alpha_{ul} \cdot \dot{m}_{\text{in}} - \alpha_{iz} \cdot \dot{m}_{\text{out}}) - \alpha \cdot \frac{P}{V} \cdot \dot{V} \tag{14}$$

where α, α_{ul} and α_{iz} can take values 1, i k, respectively, in accordance with the heat transfer during the process.

If we choose the origin of piston displacement at the middle of the stroke, the volume equation can be expressed as:

$$V_i = V_{0i} + A_i\left(\frac{1}{2} \cdot L \pm x\right) \tag{15}$$

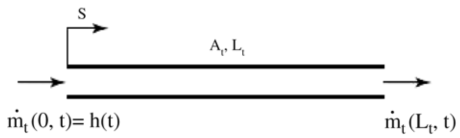

Fig. 2. Pneumatic tube notations.

where $i = 1, 2$ is cylinder chambers, index V_{0i} is the non-active volume at the end of the stroke, A_i is the effective piston area, L is the piston stroke, and x is the position of the piston. If we substitute Eq. (15) into Eq. (14), and the pressure time derivation in pneumatic cylinder chambers, then it yields:

$$\dot{P}_i = \frac{R \cdot T}{V_{oi} + A_i \cdot \left(\frac{1}{2} \cdot L \pm x\right)} \cdot (\alpha_{ul} \cdot \dot{m}_{ul} - \alpha_{iz} \cdot \dot{m}_{iz}) - \alpha \cdot \frac{P \cdot A_i}{V_{oi} + A_i \left(\frac{1}{2} \cdot L \pm x\right)} \cdot \dot{x}. \tag{16}$$

3. Mathematical model of cylinder–valve system

In Schuder, 1959 and [3], two basic equations were derived which consider the flow change in pneumatic systems. They are:

$$\frac{\partial P}{\partial s} = -R_i \cdot u - \rho \cdot \frac{\partial w}{\partial t} \tag{17}$$

$$\frac{\partial u}{\partial s} = -\frac{1}{\rho \cdot c^2} \cdot \frac{\partial P}{\partial t} \tag{18}$$

where P is the pressure through the tube, u is the velocity, ρ is the air density, c is the sound speed, s is the tube axis coordinate, and R_t is the tube resistance. If we use mass flow through the tube, $\dot{m}_t = \rho \cdot A_t \cdot w$, where A_t is cross-sectional area, finally it yields:

$$\frac{\partial P}{\partial s} = -\frac{1}{A_t} \cdot \frac{\partial \dot{m}_t}{\partial t} - \frac{R_t}{\rho \cdot A_t} \cdot \dot{m}_t \tag{19}$$

$$\frac{\partial \dot{m}_t}{\partial s} = -\frac{A_t}{c^2} \cdot \frac{\partial P}{\partial t}. \tag{20}$$

The overall analyzes are based on the turbulent flow in the tube, which is shown in Fig. 2.

Differentiating Eq. (19), and Eq. (20) with respect to s, the equation of mass flow through the tube is given:

$$\frac{\partial^2 \dot{m}_i}{\partial t^2} - c^2 \cdot \frac{\partial^2 \dot{m}_i}{\partial s^2} + \frac{R_i}{\rho} \cdot \frac{\partial \dot{m}_i}{\partial t} = 0. \tag{21}$$

The equation represents generalized wave equation, with new terms, and can be solved by using the form (Chester, 1971):

$$\dot{m}_t\,(s, t) = \phi\,(t) \cdot \xi\,(s, t) \tag{22}$$

where $\xi\ (s, t)$ and $\phi\ (t)$ are new functions. Substituting Eq. (22) into Eq. (21) yields:

$$\varphi \frac{\partial^2 \xi}{\partial t^2} - c^2 \cdot \phi \cdot \frac{\partial^2 \xi}{\partial s^2} + \left(\phi \cdot \frac{R_t}{\rho} + 2 \cdot \phi'\right) \cdot \frac{\partial \xi}{\partial t} + \left(\phi' \cdot \frac{R_t}{\rho} + \phi''\right) \cdot \xi = 0. \tag{23}$$

The function $\phi(t)$ is determined by simplifying the equation for ξ, and after substituting it into Eq. (23), the remaining part of the equation ξ, does not contain the terms of first order, so:

$$2 \cdot \phi' + \phi \cdot \frac{R_t}{\rho} = 0 \tag{24}$$

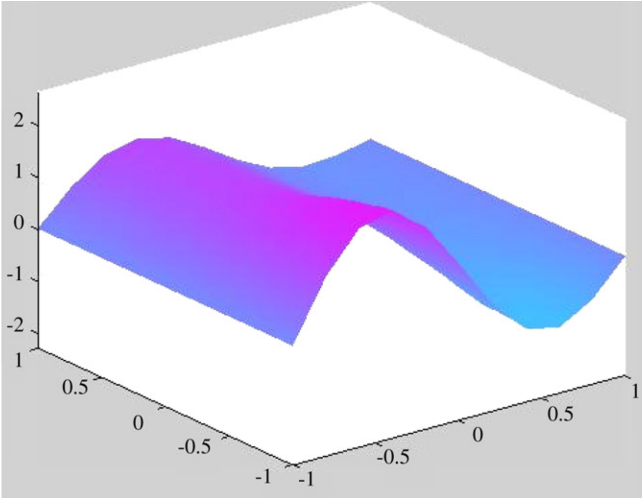

Fig. 3. The animation created in Matlab programming language.

and that yields to:

$$\phi(t) = \mathrm{e}^{\frac{-R_t}{2\cdot\rho}\cdot t}. \tag{25}$$

In that case the resultant equation for ξ will be:

$$\frac{\partial^2 \xi}{\partial t^2} - c^2 \cdot \phi \frac{\partial^2 \xi}{\partial s^2} + \frac{R_t^2}{4\cdot\rho^2} \cdot \xi = 0 \tag{26}$$

which represents the dispersive hyperbolic equation. Tubes are usually not so long, so it might be assumed that the dispersion is small, and hence can be neglected. On using that assumption it yields:

$$\frac{\partial^2 \xi}{\partial t^2} - c^2 \cdot \phi\, \frac{\partial^2 w}{\partial s^2} = 0 \tag{27}$$

which represents the classical one-dimensional wave equation, which can be solved by using specific boundary and initial conditions:

$$\begin{cases} \xi\,(s,0) = 0 \\ \dfrac{\partial \xi}{\partial t}\,(s,0) = 0 \\ \xi\,(0,t) = h\,(t)\,. \end{cases} \tag{28}$$

The solution for the problem of boundary-initial values is given in (Chester, 1971) and can be expressed as:

$$\xi\,(s,t) = \begin{cases} 0 & \text{ako jet} < s/c \\ h\left(t - \dfrac{s}{c}\right) & \text{ako jet} > s/c. \end{cases} \tag{29}$$

The input wave will reach the end of the tube in a time interval $\tau = L_t/c$. Substituting t with L_t/c in Eq. (24), and ρ from the state equation, it is given that:

$$\phi = \mathrm{e}^{-\frac{R_T\cdot R\cdot T}{2\cdot P}\cdot\frac{L_T}{c}} \tag{30}$$

where P is the pressure. Mass flow at the end of the tube, when $s = L_t$ is given with:

$$\dot{m}_t\,(L_t,t) = \begin{cases} 0 & \text{ako jet} < L_t/c \\ \mathrm{e}^{-\frac{R_T\cdot R\cdot T}{2\cdot P}\cdot\frac{L_T}{c}} \cdot h\left(t - \dfrac{L_t}{c}\right) & \text{ako jet} > \dfrac{L_t}{c}. \end{cases} \tag{31}$$

The tube resistance R_t can be calculated (Munson, 1990) and it is shown by the following equation:

$$\Delta p = f \cdot \frac{L_t}{D} \cdot \frac{\rho \cdot w^2}{2} = R_t \cdot w \cdot L_t \tag{32}$$

where f is the friction factor, and D is the inner diameter of the tube. For laminar flow, $f = 64/Re$, where Re is the Reynolds number. The resistance of the tube then becomes:

$$R_t = \frac{32 \cdot \mu}{D^2} \tag{33}$$

where μ is the dynamic viscosity of the air. If the Blasius formula is used, then it yields:

$$f = \frac{0.316}{Re^{1/4}} \tag{34}$$

so the flow resistance for the turbulent flow then becomes:

$$R_t = 0.158 \cdot \frac{\mu}{D^2} \cdot Re^{3/4}. \tag{35}$$

4. Program support

Program support is developed in Maple programing language.

```
#
# File for simulation
# the hyperbolic equation
# Definition the number of precision
Digits := 3:
# Definition of the hyperbolic equation
pdeq := diff(ξ, t)^2 − c^2 ∗ φ ∗ diff(ω, s)^2;
# Definition of initial conditions
init1 := ξ(s, 0) = 0:
init2 := diff(ξ, t) = 0
init3 := ξ(0, t) = Heaviside(t)
# Creating the procedure for numeric solving
pdsol := pdsolve(pdeq, init1, init2,init3):
#Activating the function for graphic display
# the solutions of the equation
plot(i(t), t = 0...10, i = 0.04...008, axes = boxed, ytickmarks = 6);
```

5. Animation

The animation is created in Matlab programming language (see Fig. 3).

```
% Solving the equation
% ∂²ξ/∂t² − c² · φ ∂²w/∂s² = 0
%
% Problem definition
g = 'squareg'; %
b = 'squareb3'; %0 o
% 0
c = 1;
a = 0;
f = 0;
d = 1;
```

```
% Mesh
[p, e, t] = initmesh('squareg');
% Initial conditions:
% ξ(s, 0) = 0
% ∂ξ/∂t (s, 0) = 0
% ξ(0, t) = h(t)
% Using higher program modes
% Time definition
x = p(1, :)';
y = p(2, :)';
u0 = atan(cos(pi/2 * x));
ut0 = 3 * sin(pi * x). * exp(sin(pi/2 * y));
% Desired solution in 43 points between 0 and 10.
n = 43;
tlist = linspace(0, 10, n);
% Solving of hyperbolic equation
uu = hyperbolic(u0, ut0, tlist, b, p, e, t, c, a, f, d);
% Interpolation of rectangular mesh
delta = −1 : 0.1 : 1;
[uxy, tn, a2, a3] = tri2grid(p, t, uu(:, 1), delta, delta);
gp = [tn; a2; a3];
% Creating the animation
newplot;
M = moviein(n);
umax = max(max(uu));
umin = min(min(uu));
for i = 1 : n, ...
if rem (i, 10) == 0, ...
end, ...
pdeplot(p, e, t, 'xydata',uu(:, i),'zdata', uu(:, i), 'zstyle', 'continuous', ...
'mesh', 'off', 'xygrid', 'on', 'gridparam', gp, 'colorbar', 'off'); ...
axis([−11 − 11 umin umax]); caxis([umin umax]); ...
M(:, i) = getframe; ...
if i == n, ...
if rem (i, 10) == 0, ...
end, ...
% The listing is too long for presentation, thus the most important parts are shown
pdeplot(p, e, t, 'xydata', uu(:, i), 'zdata', uu(:,i), 'zstyle', 'continuous', ...
'mesh', 'off', 'xygrid', 'on', 'gridparam', gp, 'colorbar', 'off'); ...
axis([−11 − 11 umin umax]); caxis([umin umax]); ...
M(:, i) = getframe; ...
if i == n, ...
if rem (i, 10) == 0, ...
end, ...
% Showing the figure
nfps = 5;
movie(M, 10, nfps);
```

6. Conclusions

The complex mathematical model of dual action pneumatic actuators controlled with proportional spool valves [13] is presented. The adequate program support is shown in *Maple* language, based on numerical methods. The simulation and animation are developed in *Matlab* programming language. The simplicity of solving the partial different equations [8], by using this approach and even the partial equations of higher order [4], is crucial in the direction of future development.

7. Discussion

In this scientific paper, disperse hyperbolic equation is solved and a graphic presentation is given. Taking into account that the most modern pneumatic and hydraulic systems should be described by using partial different equations of higher order [9], it is very important to focus on the development of more complex and faster program supports for solving them [11]. Combining time delay component like presented in [10,12,14] and [15], as well as distributed parameter component would open the field for the discussion of applying adequate approaches for solving these kind of problems [7].

References

[1] A.R. Amir-Moez, Extreme properties of eigenvalues of hermitian transformation and singular value of the sum and product of linear transformation, Duke Math. J. 23 (1956) 463–467.
[2] A.M. Al-Ibrahim, D.R. Otis, Transient air temperature and pressure measurements during the charging and discharging processes of an actuating pneumatic cylinder, in: Proceedings of the 45th National Conference on Fluid Power, 1992.
[3] Blaine W. Andersen, The Analysis and Design of Pneumatic Systems, John Wiley and Sond, INC., New York, London, Sydney, 1967.
[4] H.-G. Chen, K.-W. Han, Improved quantitive measures of robustness for multivariable system, IEEE Trans. Automat. Control AC-39 (1994) 807–810.
[5] Lj.D. Debeljkovic, Time delay control systems, Masinski fakultet, Beograd, 1994.
[6] Dj. Dihovicni, N. Nedic, Stability of distributed parameter systems on finite space interval, in: 32-nd Yupiter Conference, Zlatibor 2006, 2006, pp. 3.17–320.
[7] N. Nedic, Dj. Dihovicni, Stability of distributed parameter systems on finite space interval described by equations $\frac{\partial'^3 \underline{x}}{\partial t^3} = f(\frac{\partial \underline{x}}{\partial z})$, in: 32-nd Yupiter Conference, Zlatibor 2006, 2006, pp. 3.21–325.
[8] J. Hale, Theory of Functional Differential Equations, Academic Press, New York, 1977.
[9] A. Hmamed, Further results on the delay independent asymptotic stability of linear systems, Int. J. Syst. Sci. 22 (1991) 1127–1132.
[10] E.W. Kamen, Correction to linear systems with commensurate time delays: Stability and stabilization independent of delay, IEEE Trans. Automat. Control AC-30 (1983) 158–161.
[11] T. Mori, N. Fukuma, M. Kuwahara, On an estimate of the delay rate for stable linear delay systems, Int. J. Control 36 (1982) 95–97.
[12] Z.L. Nenadic, D.L. Debeljkovic, S.A. Milinkovic, On practical stability of time delay systems, in: American Control Conference, Proceedings of the 1997, vol. 5, Issue 4–6, 1997, pp. 3225–3236.
[13] E. Richer, Y. Hurmuzlu, A high performance pneumatic force actuator system, ASME J. Dyn. Syst. Meas. Control (2000) 416–425.
[14] E. Tissir, A. Hmamed, Sufficient stability conditions of interval time delay systems, Adv. Modelling Anal. C45 (1996) 1–16.
[15] S.S. Wang, Further results on stability of $\dot{\underline{x}} = A_0 \cdot \underline{x}(t) + A_1 \underline{x}(t - \tau)$, Syst. Control Lett. 19 (1992) 165–168.