

1D pipeline model

State equation:

$$P = \rho R \theta, \quad \begin{array}{l} P - \text{pressure, } \rho - \text{density, } R - \text{gas} \\ \text{constant, } \theta - \text{temperature} \end{array}$$

Motion equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = - \frac{1}{\rho} \frac{\partial P}{\partial x} - H, \quad \begin{array}{l} u - \text{air velocity} \\ x - \text{position} \end{array}$$

$$H = \frac{\lambda}{2D} |u|u - \text{pressure loss}$$

$$\lambda = \begin{cases} \frac{64}{Re} & \text{if } Re \leq 2300 \\ 0,3164 Re^{-0,25} & \text{if } Re > 2300 \end{cases}$$

$$T = \frac{4h(\theta - \theta_a)}{\rho C_v D}$$

Heat transfer between pipe and air.

$$h = 0,046 Re^{0,8} Pr^{0,4} \frac{k}{D}$$

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial u} = 0$$

Pr - Prandtl number

$$k = 7,95 \cdot 10^5 \theta + 2,0465 \cdot 10^{-3}$$

Thermal conductivity

1st Law of thermodynamics:

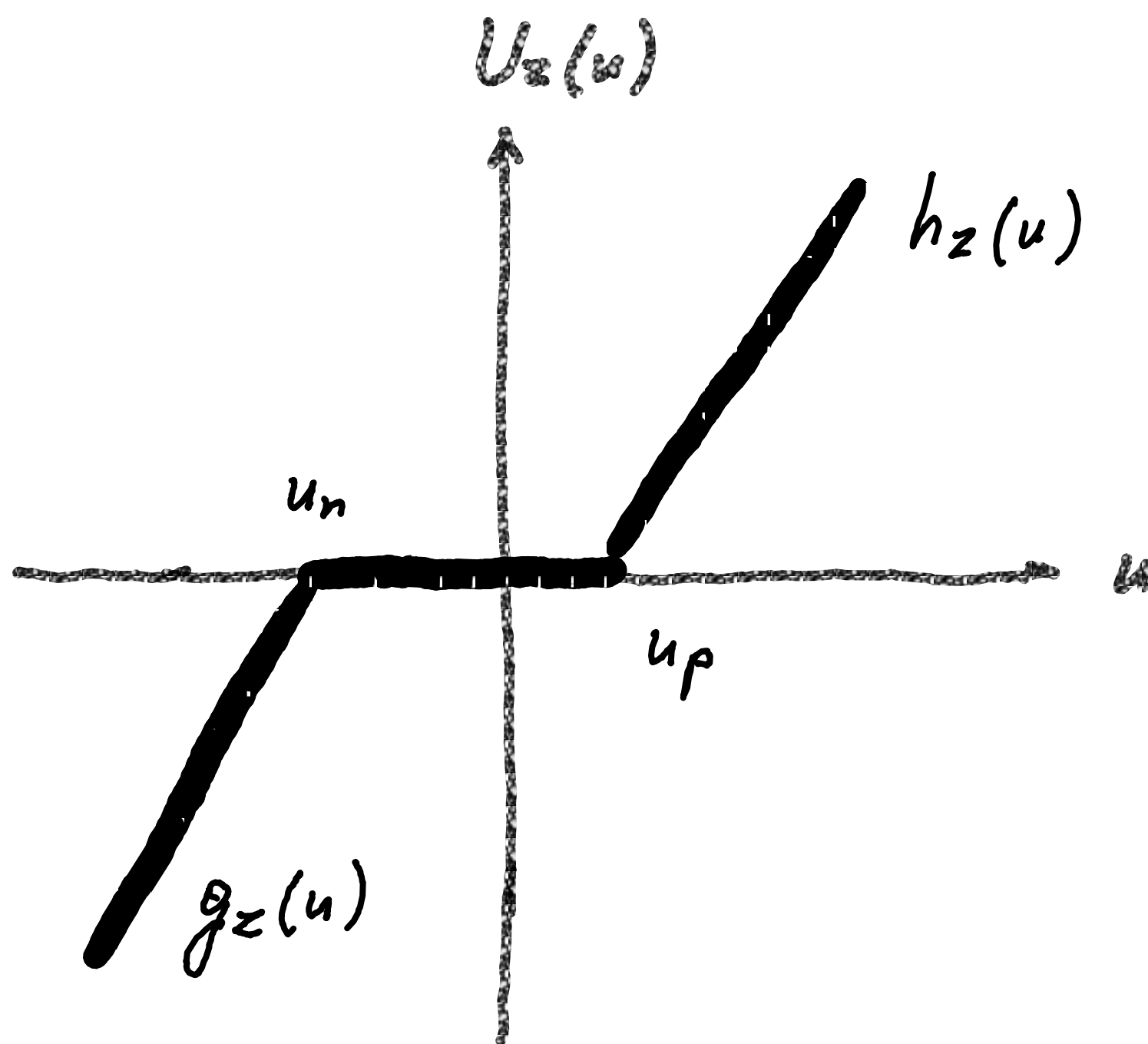
$$\frac{\partial \theta}{\partial t} = \frac{4h(\theta_a - \theta)}{\rho C_v D} - u \frac{\partial \theta}{\partial x} - \frac{R\theta}{C_v} \frac{\partial u}{\partial x} + \frac{1}{C_v} \frac{\lambda |u| u^2}{2D}$$

h - heat transf. coefficient, θ_a - atmosphere pressure

D - diameter of pipelines, λ - coefficient of pipeline friction

Valve Deadzone :

$$U_z(u) = \begin{cases} g_z(u) < 0 \\ 0 \\ h_z(u) > 0 \end{cases} \quad \begin{matrix} u \leq u_n \\ u_n < u < u_p \\ u \geq u_p \end{matrix}$$



Valve spool dynamic

Mechanic system

$$\ddot{x} = \frac{1}{m} (S_A p_A - S_B p_B - S_O p_O - F_f)$$

where

$$F_f = \begin{cases} C \cdot \dot{x} + [f_c + (f_s - f_c) e^{-(\dot{x}/v_s)^s}] \operatorname{sign}(\dot{x}) & \text{if } \dot{x} \leq v_e \\ \mu \cdot \dot{x} & \text{if } \dot{x} > v_e \end{cases}$$

where C - viscous friction coefficient

f_c - Coulumb friction

f_s - maximum static friction

v_s - Stribeck velocity (0,1 - 0,0001 m/s)

s - arbitrary index (0,5 - 2)

v_e - critical velocity.

μ - dynamic friction factor.