SOLUCIÓ PROVA 2 (GRUP TF).

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- Demostreu que la successió $\left\{a_n = \left(\frac{n+5}{2n+1}\right)\right\}_{n\in\mathbb{N}}$ es convergent a $\frac{1}{2}$.

Hem de veure que $\forall \varepsilon > 0$ prou petit, $\exists n_0(\varepsilon) \in \mathbb{N}$ tq

$$|a_n - \tfrac{1}{2}| < \varepsilon \quad \forall n \geq n_0(\varepsilon).$$

$$\left|\frac{n+5}{2n+1} - \frac{1}{2}\right| = \left|\frac{9}{4n+2}\right| = \frac{9}{4n+2} \underset{\uparrow}{<} \varepsilon$$

Ho volem.

$$\frac{9}{4n+2} < \varepsilon \iff n > \left(\frac{9}{\varepsilon} - 2\right)/4$$

Per tant, prenent $n_0 = \left[\left(\frac{9}{\varepsilon} - 2\right)/4\right] + 1 \in \mathbb{N}$.

- Considereu $\varepsilon = 0.05$, doneu $n_0(\varepsilon) \in \mathbb{N}$.

$$n_0(0.05) = \left[\left(\frac{9}{0.05} - 2\right)/4\right] + 1 = [44'5] + 1 = 45$$

- Doneu el valor de la cota superior mes petita:

$(a_n)_n$ es estrictament decreixent ja que $a_{n+1} < a_n$.

$$a_{n+1} < a_n \iff \frac{(n+1)+5}{2(n+1)+1} < \frac{n+5}{2n+1} \iff \frac{n+6}{2n+3} < \frac{n+5}{2n+1}$$

$$\iff (n+6)(2n+1) < (n+5)(2n+3)$$

$$\iff \cancel{2n^2} + \cancel{n} + \cancel{12n} + 6 < \cancel{2n^2} + \cancel{3n} + \cancel{10n} + 15$$

$$\iff 6 < 15 \quad \text{ok}.$$

Per tant, la cota superior més petita sera $a_1 = \frac{6}{3} = 2$.