

① Solució del sistema a $\mathbb{Z}/14$

$$3^2 \cdot x + y = 1$$

$$20 \cdot x - 23 \cdot y = 3$$

$$m = 14 = 2 \cdot 7 \Rightarrow \varphi(14) = 1 \cdot 6 = 6.$$

$$37 \equiv 1 \text{ mod } \varphi(14) = 6 \Rightarrow 3^{37} \equiv 3 \text{ mod } (14)$$

El sistema és equivalent a:

$$\begin{cases} 3x + y = 1 \\ 6x + y = 3 \end{cases} \quad (\text{obs } 28 = 27 + 1 \Rightarrow -27 \equiv 1 \text{ a } \mathbb{Z}/14)$$

$$3x \equiv 2$$

$$\text{lucd}(3, 14) = 1 \Rightarrow 3 \text{ té invers a } \mathbb{Z}/14$$

$$14 = 4 \cdot 3 + 2 \quad 1 = 3 - 4 \cdot 2 = 3 - 4(14 - 4 \cdot 3)$$

$$3 = 1 \cdot 2 + 1$$

$$= 5 \cdot 3 - 14$$

L' invers de 3 a $\mathbb{Z}/14$ és $\overline{5}$

$$x \equiv 2 \cdot 5 \equiv 10$$

$$\begin{cases} x \equiv 2 \cdot 5 \equiv 10 \\ y \equiv 1 - 30 \equiv -29 \equiv -1 \equiv 13 \end{cases}$$

②

$$x \equiv 2 \text{ mod } 5$$

$$x \equiv 3 \text{ mod } 7$$

$$x \equiv 1 \text{ mod } 2.$$

$$N_1 = 14, N_2 = 10, N_3 = 35.$$

$$y_1 \text{ invers de } 14 \text{ a } \mathbb{Z}/5 \Rightarrow y_1 = -1 \equiv 4$$

$$y_2 \text{ " de } 10 \text{ a } \mathbb{Z}/7 \Rightarrow y_2 = -2 \equiv 5$$

$$y_3 \text{ " de } 35 \text{ a } \mathbb{Z}/2 \Rightarrow y_3 = 1$$

$$x \equiv x_0 = 2 \cdot 14 \cdot 4 + 3 \cdot 10 \cdot 5 + 1 \cdot 35 \cdot 1 = 297 \equiv 17 \text{ mod } 70$$

$$x = 70 \cdot y + 17$$

$$\Rightarrow \boxed{x = 157}$$