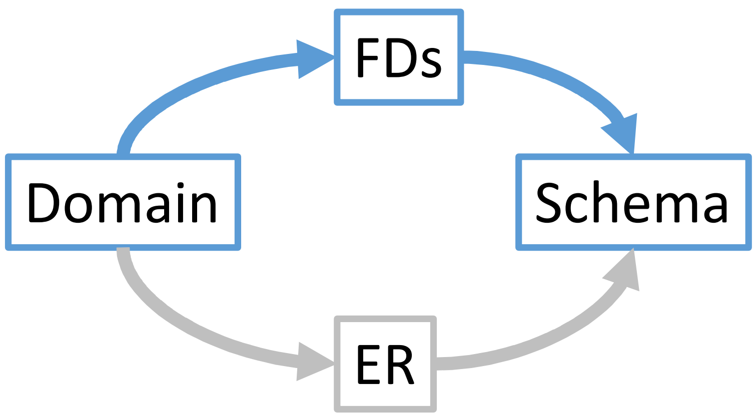
TDA357/DIT621 – Databases

Lecture 6 and 7 – Design using Functional Dependencies and normal forms

Jonas Duregård

Another high level design approach

- This week we will look at functional dependencies (FDs) and normal forms
- This is an alternative (and to some degree complementary) approach to ER that we studied last week
  - We start in a domain description and end in a database schema
- Two lectures, and Friday exercises as usual



Normalisation in a nutshell

- Extract a bunch of formal statements from the domain description
- Compute a normalised database schema from the formal statements

Domain $\xrightarrow{\text{Modelling}}$ Attributes + Bunch of facts $\xrightarrow{\text{Normalization}}$ Ready-to-use schema

- Highly systematic, almost mechanical process
- By a carefully constructed normalization algorithm, the normal form the schema ends up in will satisfy some important properties

Functional Dependencies (FDs)

- A functional dependency is written as <set of attributes> → <attribute>
- Example: room time→course

Do not confuse with references!

- Pronounced "room and time determines course"
- It is a statement that can be true or false
- A few ways of understanding the meaning of the statement above:
  - If we know room and a time, we can uniquely determine course
  - There can be at most one course value for each (room,time)-pair
  - There exists a partial function *f* that takes a room and a time and yields a course
- In a domain it might have said something like "courses can book rooms at any free times"

Three ways we can use functional dependencies

- Check if they hold for a specific data set
- Check if a design ensures they hold for all data sets
- Express desired properties of a design

- I will explain each of these in turn

Functional dependency as a property of data

- One way of formally defining functional dependency $x_1\ x_2 \ldots \rightarrow y$:
  For $R(y\ x_1\ x_2 \ldots)$, if two rows agree on x_1, x_2 … they must also agree on y
- In other words, there cannot exist two rows where the left-hand side attributes are the same, but the right-hand side attribute differs
  - "$X \rightarrow y$ = rows that agree on X must agree on y"

Two rows "agreeing on x" just means the x-column(s) have the same value

- Which FDs hold for this data?

Table: Bookings

courseCode	name	day	timeslot	room	seats
TDA357	Databases	Tuesday	0	GD	236
TDA357	Databases	Tuesday	1	GD	236
ERE033	Reglerteknik	Tuesday	0	HB4	224
ERE033	Reglerteknik	Friday	0	GD	236

courseCode → name?

- Yes! (TDA357 maps to Databases, ERE033 to Reglerteknik)

day → timeslot?

- No! (Tuesday maps to both 0 and 1)

LHS = Left-hand side (of arrow)
RHS = Right-hand side

day timeslot room → courseCode?

- Yes! (There are no rows where all three LHS columns match)

seats → room?

- Yes! 236 for GD, 224 for HB4,
- This might not be intentional given what we know of the domain…

FDs on small tables

- Relation R(a,b,c) has a single row. Does a → b hold for that table?
  - Yes! In fact, all FDs hold for tables with fewer than two rows.
- Consider: For an FD not to hold on a relation, there needs to be two conflicting rows.
- More general: For fd x_1 x_2 … x_n → y, there needs to be two rows that agree on all the x_i values (but not on y) for the FD to not hold.

Thinking about attributes in FD analysis

- Consider this relational schema with courses and teachers:

```
Teachers(email, tname)
Courses(code, cname, teacher)
  teacher -> Teachers.email
```

- The domain only has four attributes, teacher emails and names and course codes and names.
- The teacher attribute of Courses and email attribute of Teachers are two names for the same attribute, not two independent attributes

FDs as a properties of designs

- Knowing what it means for an FD to hold for a data set, we can determine if a design (schema) guarantees that it holds for all valid data sets
- Example: Does the schema below guarantee that …

code → cname?

  - Yes! (by primary key constraint in courses)

cname → code?

  - No! (Counterexample: any two courses with the same name)

code → email

  - Yes! (teacher is just another name for email)

code → tname

  - Yes! (by primary key + reference)

```
Teachers(email, tname)
Courses(code, cname, teacher)
  teacher -> Teachers.email
```

Bookings(<u>courseCode</u>, name, <u>day</u>, <u>timeslot</u>, room, seats)
(day, timeslot, room) UNIQUE

- Does the schema above guarantee …
- day timeslot room → courseCode
  - Yes (through UNIQUE constraint)
- day timeslot room coursCode → seats
  - Yes (through primary key and/or UNIQUE)
- room → seats
  - No ☹
- courseCode → name
  - No ☹

Counterexample of room → seats and courseCode → name

<u>courseCode</u>	**name**	**<u>day</u>**	**<u>timeslot</u>**	**room**	**seats**
CC1	N1	Tuesday	0	R1	0
CC1	N2	Tuesday	1	R1	1

Different timeslot, so no key violation

Creating counterexamples

- A good exercise (and exam question) for FDs is to given a schema with constraints and one or several FDs, show that the FDs are not guaranteed by the schema
- We need to construct a relation that has:
  - For each FD at least two rows that agree on its left hand side
  - … but disagree on its right hand side
  - … and respects the constraints of the schema
- We'll need at least two rows, possibly more

- Given the relation ***R(<u>a</u>, <u>b</u>, c)***, construct a counterexample to both a→c and b→c
- This doesn't work because it violates the key constraint:

a1 b1 c1
a1 b1 c2

- This doesn't work because b→c holds:

a1 b1 c1
a1 b2 c2

- This is a valid counterexample:

a1 b1 c1
a1 b2 c2
a2 b2 c1

a1 related to both c1 and c2, same for b2

No two rows have the same value for both a and b

So far we can …

- Verify/disprove that an FD holds for a specific data set
- Verify/disprove that an FD always hold for all valid data in a schema

FDs as intention for designs

- Since we can verify that an FD holds for a schema, we can also use them to specify desired properties of our schema
- This is what makes FDs a design tool
- A sentence like "every course has a teacher" can be modelled as the FD course $\rightarrow$ teacher (or whatever attributes we use)
  - If this FD does not hold for our design, maybe the design is bad?

Side note: Single/Multiple FDs

Notation: I use lowercase x/y/z for single attributes and uppercase X/Y/Z for attribute sets

- It is common to have multiple attributes on the right-hand side of FDs
  x y z → a b c
- This means exactly the same as these three FDs:
  x y z → a
  x y z → b
  x y z → c
- I find it most useful to think of the first as a convenient way of writing multiple FDs, rather than thinking of it as single FD with multiple attributes
- It is <u>not</u> the same with the left-hand side! x y → a does <u>not</u> mean x → a!

Formal properties of FDs

Warning: Things may get slightly mathsy from this point

- FDs have lots of interesting mathematical properties
- I will explain some of the more useful ones:
  - Transitivity
  - Augmentation
  - Reflexivity

- These three are commonly referred to as Armstrong's axioms and they can be formulated in a few different but equivalent ways*

*But the way I formulate them is -of course- the best way

Recall: $X \rightarrow y$ = rows that agree on X must agree on y

Transitivity of functional dependencies

- Functional dependency is a transitive relation
  - This means that if $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

- Note that Y is an attribute set here, so $X \rightarrow Y$ may be multiple FDs with the same LHS

- Proof sketch: Look at any rows that agree on X. Since $X \rightarrow Y$, they must also agree on Y, and since $Y \rightarrow Z$ they must further agree on Z.
  Thus $X \rightarrow Z$.

Recall: $X \rightarrow y$ = rows that agree on X must agree on y

Augmentation

- If $x_1\ x_2 \ldots \rightarrow y$, then for all z:
  $z\ x_1\ x_2 \ldots \rightarrow y$
- Intuitively: You can add any attributes you want to the LHS of a valid FD and still get a valid FD
  - Think: "knowing an extra attribute never prevent us from finding y"
- Proof sketch: Since all rows that agree on the xs must agree on y, then particularly all rows that agree on z as well as the xs must do so.

Recall: $X \rightarrow y$ = rows that agree on X must agree on y

Reflexivity and trivial FDs

- For all x: $x \rightarrow x$
  (x determines itself)

- By augmentation, $X \rightarrow y$ whenever $y \in X$
  - Example: $a\ b\ c \rightarrow b$
  - We call these depencies trivial
  - Rule of thumb: Ignore trivial dependencies

- Proof sketch: Any values that agree on x will agree on x ☺

Example: Deriving functional dependencies

- For any attributes x, y, z, w, and q:
  x → y, z → w, and y w → q implies x z → q
- Proof:
  x → y implies x z → y (by augmentation)
  z → w implies x z → w (by augmentation)
  x z → y w and y w → q implies x z → q (by transitivity)

$$\frac{\begin{array}{r} x \rightarrow y \\ z \rightarrow w \\ y\,w \rightarrow q \end{array}}{x\,z \rightarrow q}$$

- Note that in the third step we merge x z → y and x z → w into x z → y w (See slide on Single/Multiple FDs)

Minimal basis (a.k.a. minimal cover)

- A minimal basis F^- of a set of functional dependencies F is a set equivalent to F but with the following conditions:
    - F^- has no trivial dependenices
    - No dependency in F^- follow from other dependencies in F^- through transitivity or augmentation

- Used for a lot of algorithms and to express a set of FDs in a compact form

Minimal basis

- Suppose we are given this set of FDs (5 total), what is a minimal basis?
  a → b
  b → c
  a d → ~~b~~ ~~c~~ ~~d~~

- a d → d is removed because it is trivial
- a d → b is removed because it is implied by a → b (augmentation)
- a d → c is removed because it is implied by a → b and b → c (transitivity and augmentation)
- Final set: a → b, b → c

Transitive closure

- The transitive closure X^+ of a set of attributes X, is the set of attributes that can be functionally determined by X
- In other words, X^+ = All attributes y such that $X \rightarrow y$
  - Includes ALL derived functional dependencies
  - Includes trivial dependencies
  - X^+ is closed in the sense that attributes in X^+ never determine attributes outside X^+
- Can be computed by a simple algorithm from any set of FDs:
  - Start with $X^+ = X$ (an under-approximation)
  - Repeat until done: For any FD $Y \rightarrow z$ such that $Y \subseteq X^+$, add z to X^+

Transitive closure, example

$x \rightarrow y$
$y\ w \rightarrow q$
$z \rightarrow w$
$q \rightarrow x$
$q\ r \rightarrow s$

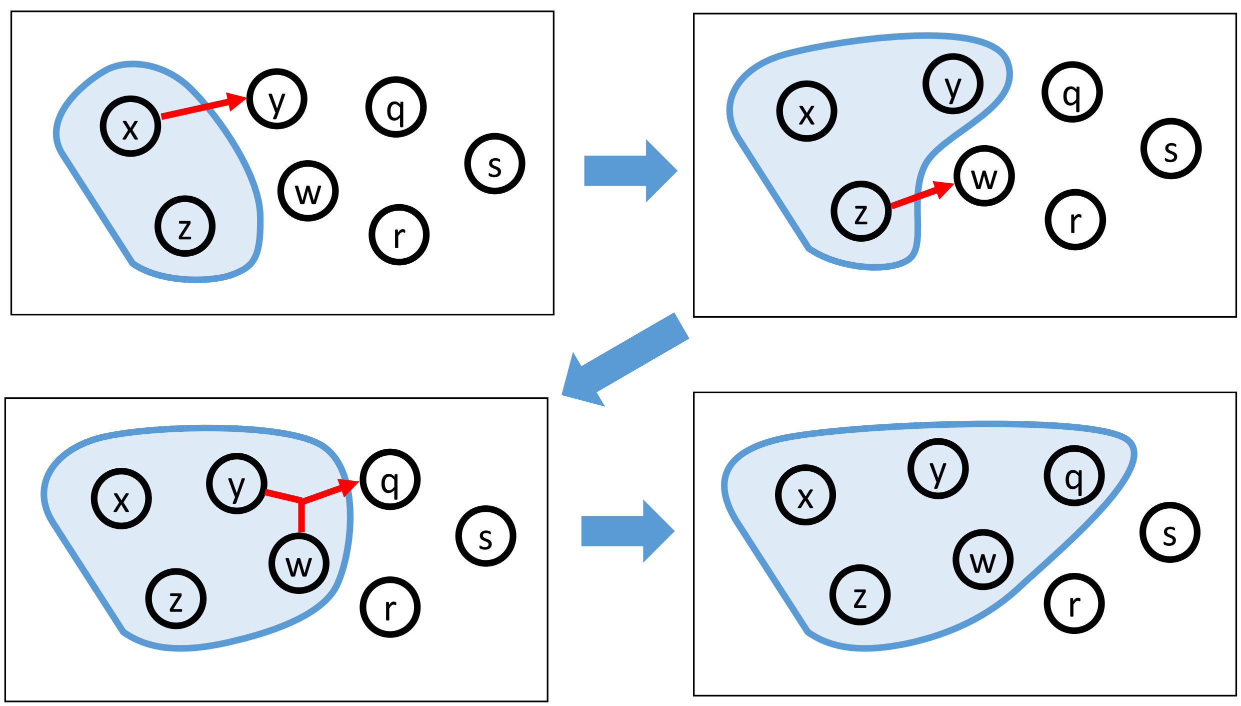
- Given these FDs, compute the closure $\{x,z\}^+$
- Initially we know $\{x,z\} \subseteq \{x,z\}^+$ (from trivial FDs)
- Add y because $x \rightarrow y$ and $\{x\} \subseteq \{x,z\}^+$ $\quad \{x,z,y\} \subseteq \{x,z\}^+$
- Add w because $z \rightarrow w$ and $\{z\} \subseteq \{x,z\}^+$ $\quad \{x,z,y,w\} \subseteq \{x,z\}^+$
- Add q because $y\ w \rightarrow q$ and $\{y,w\} \subseteq \{x,z\}^+$ $\quad \{x,z,y,w,q\} \subseteq \{x,z\}^+$
- No more FDs add attributes, so $\{x,z\}^+ = \{x,z,y,w,q\}$ is our result
- This proves all these non-trivial FDs:

  $x\ z \rightarrow y \qquad x\ z \rightarrow w \qquad x\ z \rightarrow q$

Closure $\{x,z\}^+$, visually

Red arrow indicates a "non-closed" FD

$x \rightarrow y$
$y\ w \rightarrow q$
$z \rightarrow w$
$q \rightarrow x$
$q\ r \rightarrow s$

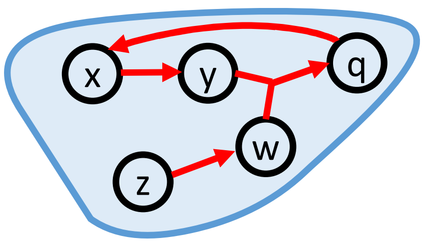


q r → s is irrelevant since r is not in $\{x,z\}^+$

$\{x,z\}^+ = \{x,z,y,w,q\}$

What it means to be closed

- Study the set $\{x,z\}^+ = \{x,z,y,w,q\}$
- If we "follow the arrow" for any FD on attributes, we end up back in $\{x,z\}^+$.



$x \to y$
$y\ w \to q$
$z \to w$
$q \to x$
$q\ r \to s$

Keys and superkeys

- We can define the property of being a key of a relation using FDs
- Intuitively: A set of attributes is a *superkey* if it determines all other attributes
- Formally: The attribute set X is a *superkey* of R if X^+ contains all attributes of R

- X is a (minimal) key if removing any attribute from X makes it a non-superkey
  - Saying only "key" usually means minimal key
  - Each superkey is a superset of at least one minimal key
  - Each key is a superkey (but not the other way around)
  - Adding any attribute to a superkey makes a new superkey

Finding a minimal key

- Find a minimal key for R(x,y,z,w,q,r,s) with the listed FDs
- Start with K={x,y,z,w,q,r}, it's trivially a superkey (K^+=R)
- Remove one element from K at a time, and compute the closure of K, if it is no longer R, return the removed element
- When no element can be removed, we have a minimal key
- {x,z,r} is one such key, since $\{x,z,r\}^+$ = R and neither $\{x,z\}^+$, $\{x,r\}^+$ nor $\{z,r\}^+$ are equal to R
- {q,z,r} is another minimal key, and possible result of the procedure

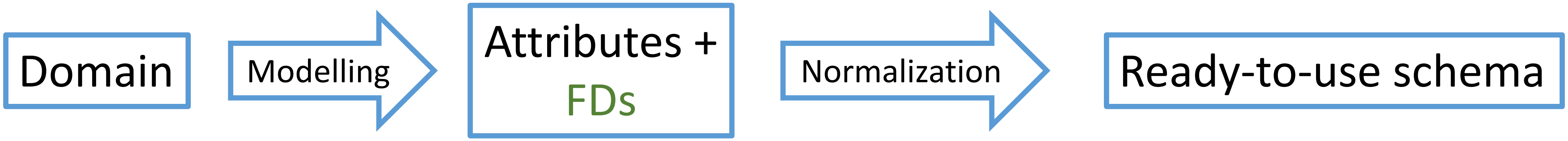
$x \rightarrow y$
$y\ w \rightarrow q$
$z \rightarrow w$
$q \rightarrow x$
$r \rightarrow s$

Clever tricks for finding multiple keys

- We know r and z will be in ALL minimal keys, because no FD determines them
  (imagine removing r from K, how is it still in K^+?)
- This means w and s will be in NO minimal keys
  (they are in $\{r,z\}^+$ so in K^+ for any K s.t. $\{r,z\} \subseteq K$)

$x \rightarrow y$
$y\ w \rightarrow q$
$z \rightarrow w$
$q \rightarrow x$
$r \rightarrow s$

Normal forms and normalization



Normal forms and normalization

- Normal form is a very important concept in database design
- Identify all the attributes in the domain and place them in one big relation D(x, y, z, …), collect FDs, then normalize D to get your design
- Normalizing is a recursive procedure, to normalize relation R:
  - Check if R is already a normal form, if it is we are done
  - Otherwise decompose R into relations R_1 and R_2 and normalize both
- Note: A normal form is not the same as a canonical form, there may be multiple normal forms derived from the same initial domain

BCNF, the Boyce-Codd Normal Form

Probably the most well-known normal form

BCNF Normalisation algorithm

To normalize relation R:

This FD is referred to as a BCNF-violation

Find a non-trivial FD $X \rightarrow y$ such that $X^+ \neq R$ (X is not a superkey)

If there is no such FD you are done, R is already in BCNF

Otherwise decompose R into $R_1(X^+)$ and $R_2(X \cup (R - X^+))$ and normalize them

Note: R is replaced by R_1 and R_2 (so R is not present in the final schema)

Example

1. Find violation
2. Decompose
3. Repeat

courseCode → name
room → seats
day timeslot courseCode → room
day timeslot room → courseCode

- Normalise this relation using the FDs above:

~~R~~`(courseCode, name, day, timeslot, room, seats)`

$R_1(X^+)$ $\quad$ $R_2(X \cup (R - X^+))$

Decompose on courseCode → name
$\text{courseCode}^+ = \{\text{courseCode, name}\}$

R_1`(courseCode, name)` $\quad$ $\cancel{R_2}$`(courseCode, day, timeslot, room, seats)`

$R_{21}(X^+)$ $\quad$ $R_{22}(X \cup (R - X^+))$

Decompose on room → seats
$\text{room}^+ = \{\text{room, seats}\}$

R_{21}`(room, seats)` $\quad$ R_{22}`(courseCode, day, timeslot, room)`

All of R_1, R_{21}, and R_{22} are now BCNF!

Wait, why not split on day timeslot course → room?

R_{22}(courseCode, day, timeslot, room)

day timeslot courseCode → room

day timeslot room → courseCode

Recall: Find a non-trivial FD X → y such that $X^+ \neq R$ (X is not a superkey)

$\{day, timeslot, courseCode\}^+ = \{ day, timeslot, courseCode, room\} = R_{22}$

$\{day, timeslot, room\}^+ = \{ day, timeslot, room, courseCode\} = R_{22}$

Both {day, timeslot, courseCode} and {day, timeslot, room} are keys!

What about keys?

courseCode → name
room → seats
day timeslot courseCode → room
day timeslot room → courseCode

- Keys can be determined using FDs (and closures) after decomposing
- Much of it is already done as part of the algorithm (we found two keys for R_{22} for instance)

R_1 (<u>courseCode</u>, name)

R_{21} (<u>room</u>, seats)

R_{22} (<u>courseCode</u>, <u>day</u>, <u>timeslot</u>, room)
(day, timeslot, room) UNIQUE

Multiple keys: Use one as primary key, the other(s) UNIQUE

What about references?

- In this case it's fairly easy to see that these are sensible references:

```
R1(courseCode, name)
R21(room, seats)
R22(courseCode, day, timeslot, room)
     (day, timeslot, room) UNIQUE
     courseCode -> R1.courseCode
     room -> R21.room
```

- General pattern: When decomposing R, add a reference X -> R_1.X to R_2
- This will not always work, particularly if R_1 or R_2 is later decomposed ☹

Decomposition of data

No redundancy, no anomalies ☺

Table: Bookings

courseCode	name	day	timeslot	room	seats
TDA357	Databases	Tuesday	0	GD	236
TDA357	Databases	Tuesday	1	GD	236
ERE033	Reglerteknik	Tuesday	0	HB4	224
ERE033	Reglerteknik	Friday	0	GD	236

Table: R_1 (Courses)

courseCode	name
TDA357	Databases
ERE033	Reglerteknik

Table: R_{22} (Bookings)

courseCode	day	timeslot	room
TDA357	Tuesday	0	GD
TDA357	Tuesday	1	GD
ERE033	Tuesday	0	HB4
ERE033	Friday	0	GD

Table R_{21} (Rooms)

room	seats
HB4	224
GD	236

Lossless join

- Note that if we join along the references, we get the original table

Table: R_1 (Courses)

courseCode	name
TDA357	Databases
ERE033	Reglerteknik

Table: R_{22} (Bookings)

courseCode	day	timeslot	room
TDA357	Tuesday	0	GD
TDA357	Tuesday	1	GD
ERE033	Tuesday	0	HB4
ERE033	Friday	0	GD

Table R_{21} (Rooms)

room	seats
HB4	224
GD	236

Joins USING (courseCode) and USING (room)

Query: R_1 NATURAL JOIN R_{22} NATURAL JOIN R_{21}

courseCode	name	day	timeslot	room	seats
TDA357	Databases	Tuesday	0	GD	236
TDA357	Databases	Tuesday	1	GD	236
ERE033	Reglerteknik	Tuesday	0	HB4	224
ERE033	Reglerteknik	Friday	0	GD	236

- Means we did not loose any data in the decomposition

Another example

- Normalizing R(a,b,c,d,e)
- Decompose R on a b $\rightarrow$ c, $\{a,b\}^+=\{a,b,c,d\}$
  - R_1(a,b,c,d)
    R_2(a,b,e)
- Further normalize R_1 on c $\rightarrow$ d, $\{c\}^+=\{c,d\}$
  - R_{11}(c,d)
    R_{12}(a,b,c)
- End result with keys:
  - $R_{11}(\underline{c},d)$
    $R_{12}(\underline{a},\underline{b},c)$
    $R_2(\underline{a},\underline{b},\underline{e})$

a b $\rightarrow$ c
c $\rightarrow$ d

Note: Not just (a,b,c)!

Try splitting on c $\rightarrow$ d first, for this example that should give the same result

Finding all FDs

- Consider this simple situation with four attributes R(x,y,z,w) and two functional dependencies: x → z and y z → w
- When normalizing R it may be important to know that there is another FD that can be derived from these: x y → w
- In principle, you should consider all non-trivial derived FDs but sometimes this a large set and it is easy to miss FDs
  - Essentially you have to consider every LHS and compute closures

Example where derived FDs matter

- R(a,b,c,d,e)
- Split on a → b:

  R_1(a,b)
  R_2(a,c,d,e)
- Now a c → d is a BCNF violation in R_2!
- We cannot entirely ignore b c → d even though b is not in R_2.

a → b
b c → d

A flaw of BCNF

- Consider this example (again): $R(x,y,z,w)$ where $x \rightarrow z$ and $y\ z \rightarrow w$
- If we decompose on $x \rightarrow z$ ($\{x\}^+ = \{x,z\}$) we get
  - $R_1(\underline{x},z)$ $\{x\}$ is the only key
  - $R_2(\underline{x},\underline{y},w)$ $\{x,y\}$ is the only key

Because $x\ y \rightarrow w$

- Both of these relations are in BCNF w.r.t. the given FDs
- But now $y\ z \rightarrow w$ is not guaranteed by the schema ☹
  - Exercise: Construct a counterexample
- There is a weaker normal form called third normal form (3NF) that does not have this problem, but it has other problems instead…
  - There is no "silver bullet" for design work

Yet another issue with BCNF

- This relation has no non-trivial functional dependencies, so is in BCNF:

Table: Courses

course	**book**	**author**	**teacher**
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Ana
Reglerteknik	RTB 1	AuthorX	TeacherX
Reglerteknik	RTB 2	AuthorX	TeacherX

Deletion anomaly: Deleting all course books also deletes all teachers

Update anomaly: Changing some value can cause inconsistencies

- The domain said something like "each course has a number of teachers and a number of books with one or more authors" (no FDs at all!)
- The data above says Databases has one book and two teachers, and Reglerteknik has two books and one teacher
- Clearly there is redundancy here, and potential for anomalies

Looks like we need another normal form!

- This one is called the fourth normal form (4NF)
- Since the problematic table had no FDs at all, this form will need some additional source of facts
- We call these facts multivalued dependencies (MVDs)*
- We write $x_1\ x_2\ x_3 \ldots \twoheadrightarrow y_1\ y_2\ y_3 \ldots$
    - Note that both sides are sets of values and we <u>cannot</u> split the RHS

*The term multivalued dependency is really quite unfortunate, but it is what it is

Multivalued dependencies, informally

- For our example, we would say course ↠ teacher
- This means that each course value has a set of teacher values that is independent from all other values (author and book)

Table: Courses

course	**book**	**author**	**teacher**
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Ana
Databases	DTCB	Widom	Jonas
Databases	DTCB	Widom	Ana
Reglerteknik	RTB 1	AuthorX	TeacherX
Reglerteknik	RTB 2	AuthorX	TeacherX

Databases has (independently):
One book with two authors
Two teachers

Reglerteknik has two books by the same author and one teacher

- This is exactly the same as saying course ↠ book author

Multivalued dependencies, formally

- The claim that X↠Y holds for relation R means:

For every pair of rows row t and u in R that agree on X we can find a row v s.t:

v agrees with both t and u on X

v agrees with t on Y

v agrees with u on R – X – Y (all attributes not in the MVD)

- Example: course ↠ teacher

If we remove any row, the MVD won't hold

Table: Courses

course	book	author	teacher
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Ana
Databases	DTCB	Widom	Jonas
Databases	DTCB	Widom	Ana

Row t

Row u

Row v:
v.(book,author)=u.(book,author)
v.teacher = t.teacher

MVDs, informally

- course $\twoheadrightarrow$ teacher means that every course has a list of teachers that is independent from its list of (book,author)-pairs
- Jonas and Ana are the teachers here, and if Jonas occurs with the row (DTCB, Ullman), Ana must do so as well.

Table: Courses

course	**book**	**author**	**teacher**
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Ana
Databases	DTCB	Widom	Jonas
Databases	DTCB	Widom	Ana

If we remove this row, Jonas has a different book list than Ana, and the MVD does not hold

Verifying MVDs on data is hard

- To check if an FD holds: Just group values up by the LHS and check that all rows in each group have the same value for the RHS
- To check if an MVD holds: Check every individual pair of values with identical LHS and search for a row with correct values
- I find a more intuitive way of thinking is this: For $X \twoheadrightarrow Y$, every X needs to have every possible combination of Y and other attributes (R–X–Y)
  - Essentially the rows for a given X must be a cartesian product!
  - If teacher Jonas occurs with one book/autor, it must occur with all book/author combinations for that course
  - This is what makes (book,author) independent from teacher

MVDs and cartesian product

- Note how the rows for databases is the cartesian product of these sets:
  - {(DTCB, Ullman), (DTCB, Widom)}
  - {Jonas, Ana}
- This means that the MVD course ↠ teacher holds for these rows (and equivalently, course ↠ book author)

Table: Courses

course	**book**	**author**	**teacher**
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Ana
Databases	DTCB	Widom	Jonas
Databases	DTCB	Widom	Ana

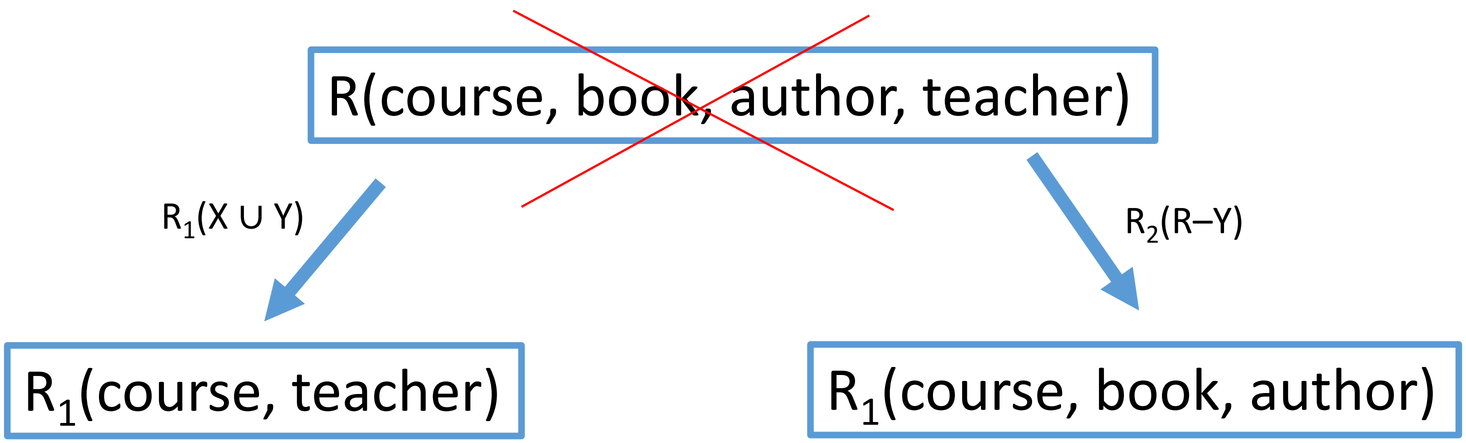
Fourth normal form

- For a relation R to be in fourth normal:
  - R must be in BCNF
  - For all non-trivial MVDs $X \twoheadrightarrow Y$ on R, X is a superkey of R
- If $X \twoheadrightarrow Y$ and X is not a superkey, we say $X \twoheadrightarrow Y$ is a 4NF violation

- To normalize: Find a violation $X \twoheadrightarrow Y$ and break R into
  - $R_1(X \cup Y)$ (”every attribute in the MVD”)
  - $R_2(R - Y)$ (”LHS and every attribute <u>not</u> in the MVD”)
  - Then normalize both R_1 and R_2

4NF normalisation

- Normalizing R(course, book, author, teacher) on course ↠ teacher
  - Here X={course}, Y={teacher}



Normalising the data

Table: Courses

course	book	author	teacher
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Ana
Databases	DTCB	Widom	Jonas
Databases	DTCB	Widom	Ana
Reglerteknik	RTB 1	AuthorX	TeacherX
Reglerteknik	RTB 2	AuthorX	TeacherX

Exercise: Find another MVD here?

Table: R_1 (a.k.a. CourseTeacher)

course	teacher
Databases	Jonas
Databases	Ana
Reglerteknik	TeacherX

Table: R2 (a.k.a. CourseBooks)

course	book	author
Databases	DTCB	Ullman
Databases	DTCB	Widom
Reglerteknik	RTB 1	AuthorX
Reglerteknik	RTB 2	AuthorX

Lossless join

Note that if we join the two tables using course …

Table: R_1 (a.k.a. CourseTeacher)

course	teacher
Databases	Jonas
Databases	Ana
Reglerteknik	TeacherX

Table: R2 (a.k.a. CourseBooks)

course	book	author
Databases	DTCB	Ullman
Databases	DTCB	Widom
Reglerteknik	RTB 1	AuthorX
Reglerteknik	RTB 2	AuthorX

NATURAL JOIN

course	book	author	teacher
Databases	DTCB	Ullman	Jonas
Databases	DTCB	Ullman	Ana
Databases	DTCB	Widom	Jonas
Databases	DTCB	Widom	Ana
Reglerteknik	RTB 1	AuthorX	TeacherX
Reglerteknik	RTB 2	AuthorX	TeacherX

We get the original table back!

Sanity check:
We did not loose any information

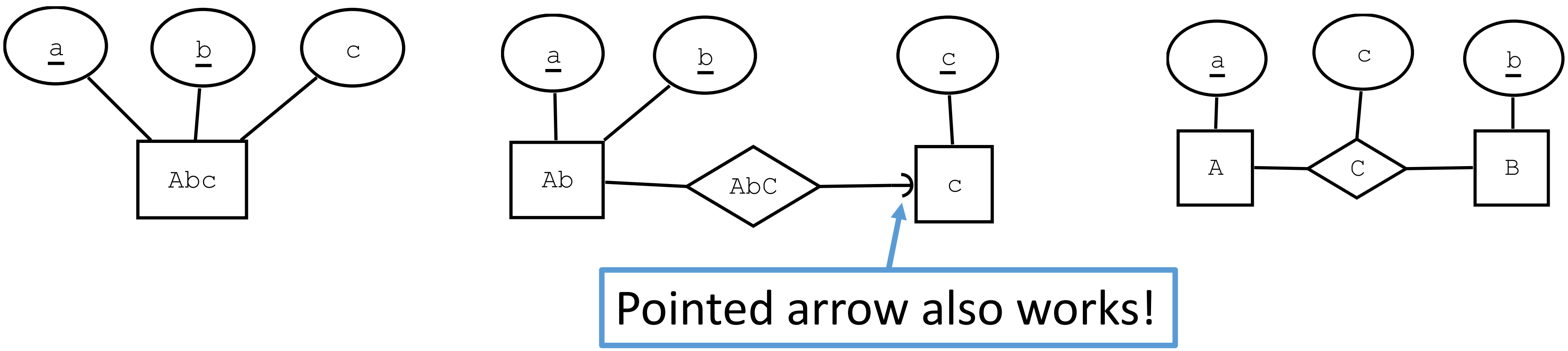
Functional dependencies vs. ER-design

- FDs can find some things that ER cannot find
- ER can find a lot of things that FDs cannot find
  - Most many-to-many relationships cannot be expressed using FDs
  - Sentences like ”students can register for courses” do not express any FDs (but possibly some MVDs?)
- The two approaches complement eachother, and confirm eachother (or sometimes contradict eachother which may indicate a problem)
- So doing both an ER-design and a FD analysis may be useful
  - This is what you will do in Task 2

Expressiveness of ER vs FDs

- What does a b → c mean formulated in ER?

Maybe this? Or this? Or even this?



- All of these ER-designs satisfy a b → c, but differ in what is considered an independent entity

Practical use of FDs combined with ER

- FDs can be used to verify the correctness of an ER-design
  - Is the result in BCNF w.r.t. the dependencies you have identified?
  - Are the primary keys you identified sensible from your FDs?
  - If not there may be an error in your ER-translation or your understanding/modelling of the domain
- Sometimes FDs can be used to patch things up in your ER-design, particularly they are useful for finding secondary keys (UNIQUE constraints)
  - Every (minimal) key of each relation should be either the primary key or unique

Mining attributes (and FDs) from ER-design

- If you already have an ER-design, that may help you determine a useful set of attributes
- Looking at the relational schema is less helpful, because it contains multiple attributes that have different names but are conceptually the same (because of references)
- You can also extract some FDs by studying the diagram/schema, but that sort of misses the point of finding them since you will never find any FDs that can improve your design
  - We want to find FDs that express things our ER-design is missing
  - We should look for FDs in the domain description

Finding functional dependencies

- Determine all attributes
- Discover FD's either by looking at each attribute and ask "what do i need to know to determine this?" and by looking at each fact in the domain description and asking "does this express a dependency?"
- You can find multiple FDs determining the same attribute

Other normal forms

- There is a whole little hierarchy of normal forms

Higly simplified:

- 1NF: basically means "only has actual tables"
- 2NF: 1NF + no FDs from key attributes to non-key attributes
- 3NF: 2NF + no FDs between non-key attributes
- BCNF a.k.a. 3½NF: 3NF + attributes depend only on keys
- 4NF: BCNF + No violating MVDs
- 5NF, 6NF, DK/NF ...: Outside the scope of this course

I expect you to know how to normalize to BCNF and 4NF

First normal form – 1NF

- First normal form means that table cells do not contain tables or other complex data structures

<table>
<tr><th>col1</th><th>stuff</th></tr>
<tr><td>x</td><td><table><tr><th>a</th><th>b</th></tr><tr><td>a1</td><td>b1</td></tr><tr><td>a2</td><td>b2</td></tr></table></td></tr>
<tr><td>y</td><td><table><tr><th>a</th><th>b</th></tr><tr><td>a3</td><td>b3</td></tr></table></td></tr>
</table>

col1	**a**	**b**
x	a1	b1
x	a2	b2
y	a3	b3

Second normal form - 2NF

- A table is in second normal form if there is no FD from a set of prime attributes (attributes that are a proper subset of any key) to a non-prime attribute
- Example:

Suppose R(a,b,c,d) has FDs a b → c and a → d

Here (a, b) is the only minimal key ($\{a,b\}^+ = R$)

That means a is a prime attribute and d is not, so a → d is a 2NF violation

Third normal form - 3NF

- 3NF has this condition on non-trivial FDs:
  The LHS must be a superkey, unless the RHS is a prime attribute
- If you remove the "unless"-part, you get BCNF
  - BCNF is stronger than 3NF – some BCNF violations are not 3NF violations
- Example:

Suppose R(a,b,c,d) has FDs a b → c and c d -> a

Both FDs are BCNF violations, but neither are 3NF violations since c and a are both prime attributes (since {a,b,d} and {b,c,d} are minimal keys)

Non-exhaustive checklist for the exam

- You have a general understanding of what an FD or an MVD claims about a relation

You know how to:

- Construct counterexamples to an FD
- Find a minimal basis F^- for a set of FDs F
- Compute the closure X^+ of an attribute set X
- Normalize to BCNF
- Check if an MVD holds and construct counterexamples
- Normalize to 4NF