

**Exercici 18**

Resoleu el sistema de congruències:

$$3x \equiv 2 \pmod{4}, 4x \equiv 7 \pmod{15}, 5x \equiv -1 \pmod{17}$$

doneu totes les solucions.

**Solució 18.**

Com s'observa  $\text{mcd}(4, 15) = \text{mcd}(15, 17) = \text{mcd}(4, 17) = 1 \Rightarrow \exists$  una solució al sistema de congruències.

$$\begin{aligned} \text{Com } 3x &\equiv 2 \pmod{4} \Rightarrow [3x]_4 = [2]_4 \\ &\Rightarrow [x]_4 = ([3]_4)^{-1}[2]_4 \\ &\Rightarrow [x]_4 = [3]_4[2]_4 = [6]_4 \\ &\Rightarrow x = 4l + 2 \end{aligned}$$

$$\begin{aligned} \text{Com } 4x &\equiv 7 \pmod{15} \Rightarrow [4x]_{15} = [4(4l + 2)]_{15} = [7]_{15} \\ &\Rightarrow [16l + 8]_{15} = [7]_{15} \\ &\Rightarrow [16l]_{15} = [-1]_{15} \\ &\Rightarrow [l]_{15} = ([16]_{15})^{-1}[-1]_{15} \\ &\Rightarrow [l]_{15} = [1]_{15}[-1]_{15} = [-1]_{15} \\ &\Rightarrow l = 15q - 1 \Rightarrow x = 4(15q - 1) + 2 = 60q - 2 \end{aligned}$$

$$\begin{aligned} \text{Com } 5x &\equiv -1 \pmod{17} \Rightarrow [5x]_{17} = [5(60q - 2)]_{17} = [-1]_{17} \\ &\Rightarrow [300q - 10]_{17} = [-1]_{17} \\ &\Rightarrow [300q]_{17} = [9]_{17} \\ &\Rightarrow [q]_{17} = ([300]_{17})^{-1}[9]_{17} \\ &\Rightarrow [q]_{17} = [14]_{17}[9]_{17} = [126]_{17} \\ &\Rightarrow q = 17r + 126 \Rightarrow x = 60(17r + 126) - 2 = 1020r + 7558 \end{aligned}$$

Càlcul de  $([300]_{17})^{-1} \Rightarrow 300x + 17y = 1$

$$\begin{aligned} 300 &= 17 \times 17 + 11 \\ 17 &= 11 \times 1 + 6 \\ 11 &= 6 \times 1 + 5 \\ 6 &= 5 \times 1 + 1 \\ 5 &= 1 \times 5 + 0 \end{aligned}$$

$$\begin{aligned} 1 &= 6 - 5 = 6 - (11 - 6) = 2 \times 6 - 11 \\ 1 &= 2(17 - 11) - 11 = -2 \times 17 + 3 \times 11 \\ 1 &= -2 \times 17 + 3 \times (300 - 17 \times 17) \\ 1 &= 53 \times 17 - 3 \times 300 \end{aligned}$$

Per tant:

$$([300]_{17})^{-1} = [-3]_{17} = [14]_{17}$$

Totes les solucions del sistema són:  $x = 1020r + 7558 \forall r \in \mathbb{Z}$  o  $x \equiv 418 \pmod{1020}$