(3) Considerem una successió definida per $x_1 \geq 0$ i la recurrència $x_{n+1} = \sqrt{5x_n^2+1}$

- Estudiem la monotonia.

$$x_{n+1} - x_n = \sqrt{5x_n^2+1} - x_n$$

$$\underset{\substack{\uparrow \\ \text{Multipliquem} \\ \text{pel conjugat}}}{=} \frac{\left(\sqrt{5x_n^2+1} - x_n\right)\left(\sqrt{5x_n^2+1} + x_n\right)}{\left(\sqrt{5x_n^2+1} + x_n\right)}$$

$$= \frac{5x_n^2+1 - x_n^2}{\sqrt{5x_n^2+1} + x_n} = \frac{\overbrace{4x_n^2+1}^{>0}}{\underbrace{\sqrt{5x_n^2+1} + x_n}_{>0}} > 0.$$

$\leftarrow x_n \geq 0 \ \forall n \geq 1$

$\Rightarrow x_{n+1} > x_n$ (Monòtona Creixent).

- Estudiem la convergència:

Suposem $\lim_{n \to +\infty} x_n = l$.

$$x_{n+1} = \sqrt{5x_n^2+1}$$

$$\downarrow \qquad\qquad \downarrow$$

$$l = \sqrt{5l^2+1} \quad ; \quad l^2 = 5l^2+1 \Rightarrow l = \sqrt{\frac{-1}{4}} \nexists\, l \in \mathbb{R}.$$

No es convergent en $\mathbb{R}$.

- Calculem $\lim_{n \to +\infty} \frac{x_{n+1}}{x_n} = \lim_{n \to +\infty} \frac{\sqrt{5x_n^2+1}}{x_n}$.

$$= \lim_{n \to +\infty} \sqrt{5 + \frac{1}{x_n}} \underset{\uparrow}{=} \sqrt{5}$$

$\lim_{n \to +\infty} x_n = +\infty$.