Problema 1

Ruleta francesa:

Té 37 números del 1 al 36 i el 0.

Si apostem a un número entre el 1 i el 36 es paga 35 vegades l'aposta.

Si surt el 0 no és paga. (i s'ho queda la banca)

A = "surt el número"

$P(A) = \frac{1}{37}$

X = "guany casino"

$P(X = 1) = \frac{36}{37}$

$P(X = -35) = \frac{1}{37}$

$E(X) = 1 \cdot \frac{36}{37} - 35 \cdot \frac{1}{37} = \frac{1}{37} = 0'0\widehat{27}$

$$Var(X) = \left(1 - \frac{1}{37}\right)^2 \cdot \frac{36}{37} + \left(-35 - \frac{1}{37}\right)^2 \cdot \frac{1}{37}$$

$$= 34'080$$

Ruleta americana

Té dos zeros $\quad P(A) = \frac{1}{38}$

$$P(X=1) = \frac{37}{38}$$

$$P(X=-35) = \frac{1}{38}$$

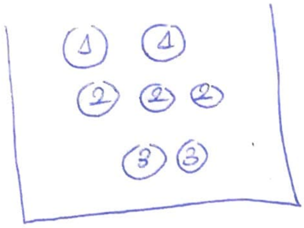
$$E(X) = 1 \cdot \frac{37}{38} - 35 \cdot \frac{1}{38} = \frac{2}{38} = 0'0526315$$

$$Var(X) = \left(1 - \frac{2}{38}\right)^2 \frac{37}{38} + \left(-35 - \frac{2}{38}\right)^2 \cdot \frac{1}{38}$$

$$= 33'207768$$

Perquè fos un joc just s'houria de pagar a 36 l'aposta a la francesa i 37 a l'americana.

Problema 2



Traiem 2 boles amb reposició

$X =$ "suma de les 2 boles"

$p_1 = p(1) = \frac{2}{7}$

$p_2 = p(2) = \frac{3}{7}$

$p_3 = p(3) = \frac{2}{7}$

$P(X=2) = p_1 \cdot p_1 = \left(\frac{2}{7}\right)^2 = \frac{4}{49}$

$P(X=3) = p_1 p_2 + p_2 p_1 = 2 \cdot \frac{2}{7} \cdot \frac{3}{7} = \frac{12}{49}$

$P(X=4) = 2 p_1 p_3 + p_2 p_2 = 2 \cdot \frac{2}{7} \cdot \frac{2}{7} + \left(\frac{3}{7}\right)^2 = \frac{17}{49}$

$P(X=5) = 2 p_2 p_3 = 2 \cdot \frac{3}{7} \cdot \frac{2}{7} = \frac{12}{49}$

$P(X=6) = p_3 \cdot p_3 = \left(\frac{2}{7}\right)^2 = \frac{4}{49}$

Comprovem que la suma és 1 $\qquad \frac{4+12+17+12+4}{49}$

$E(X) = 2 \cdot \frac{4}{49} + 3 \cdot \frac{12}{49} + 4 \cdot \frac{17}{49} + 5 \cdot \frac{12}{49} + 6 \cdot \frac{4}{49}$

$= \frac{196}{49} = \boxed{4}$

$$Var(X) = (2-4)^2 \cdot \frac{4}{49} + (3-4)^2 \frac{12}{49} +$$

$$+ (4-4)^2 \frac{17}{49} + (5-4)^2 \frac{12}{49} + (6-4)^2 \cdot \frac{4}{49}$$

$$= \frac{56}{49} = 1'142857$$

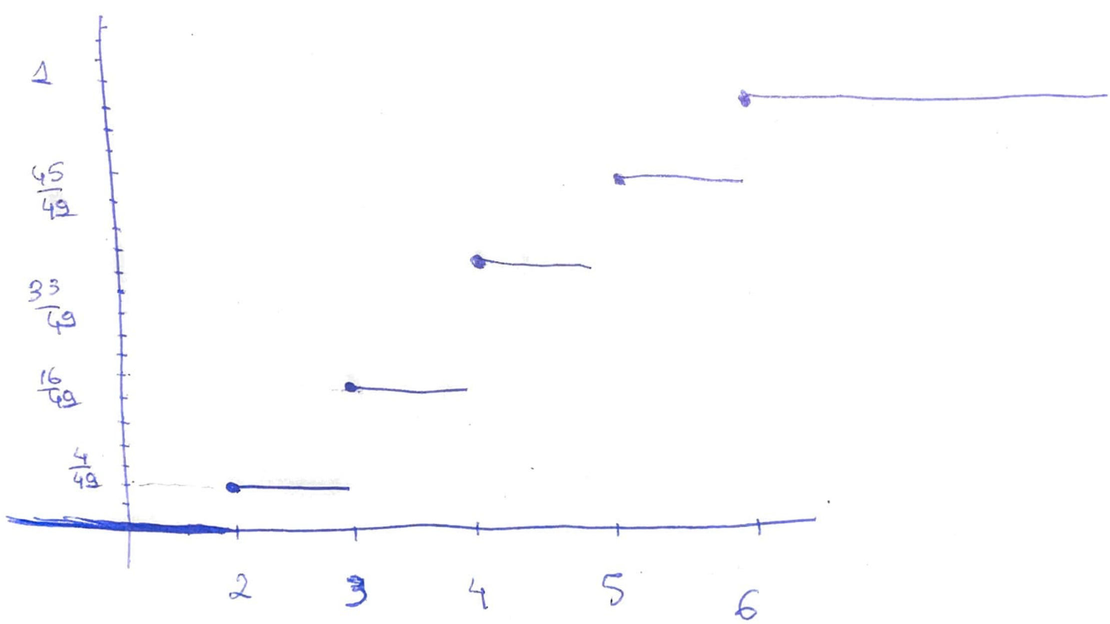
També es pot calcular:

$$Var(X) = 2^2 \cdot \frac{4}{49} + 3^2 \frac{12}{49} + 4^2 \frac{17}{49} + 5^2 \cdot \frac{12}{49} + 6^2 \cdot \frac{4}{49}$$

$$-4^2$$

$$= \frac{840}{49} - 16 = \frac{840-784}{49} = \frac{56}{49}$$

$$F(x) = \begin{cases} 0 & x < 2 \\ \frac{4}{49} & 2 \le x < 3 \\ \frac{16}{49} & 3 \le x < 4 \\ \frac{33}{49} & 4 \le x < 5 \\ \frac{45}{49} & 5 \le x < 6 \\ 1 & x \ge 6 \end{cases}$$



Problema 3

$X =$ "nombre de boles blanques"

$X \in \{0, 1, 2, 3\}$

$$P(X=0) = \frac{\binom{3}{0}\binom{10}{5}}{\binom{13}{5}} = \frac{10}{13} \cdot \frac{9}{12} \cdot \frac{8}{11} \cdot \frac{7}{10} \cdot \frac{6}{9} = 0'1958$$

$$P(X=1) = \frac{\binom{3}{1}\binom{10}{4}}{\binom{13}{5}} = 0'4895$$

$$P(X=2) = \frac{\binom{3}{2}\binom{10}{3}}{\binom{13}{5}} = 0'2797$$

$$P(X=3) = \frac{\binom{3}{3}\binom{10}{0}}{\binom{13}{5}} = 0'035$$

Podem calcular amb R

```
k <- 0:3
p <- choose(3, k) * choose(10, 5-k)/
       choose(13, 5)
p
```

ja que $P(X=k) = \dfrac{\binom{3}{k}\binom{10}{5-k}}{\binom{13}{5}}$

$k \in \{0, 1, 2, 3\}$

X és una variable hipergeometrica amb

$r = 3$
$N = 13$
$n = 5$
$N - r = 10$

també podriem utilizar

```
k <- 0:3
dhyper(k, 3, 10, 5)
```

$E(X) = 1'153846$ $\quad$ $Var(X) = 0'591716$

$= n\frac{r}{N} = 5 \cdot \frac{3}{13} =$ $\quad$ $= npq \frac{N-n}{N-1}$

$= np$ $\quad$ $p = \frac{r}{N}$

Problema 3

$r = 3$ $N - r = 10$
$N = 13$
$n = 5$

Hipergeomètrica (3, 10, 5)

$$P(X=k) = \frac{\binom{3}{k}\binom{10}{5-k}}{\binom{13}{5}} \qquad k \in \{0,1,2,3\}$$

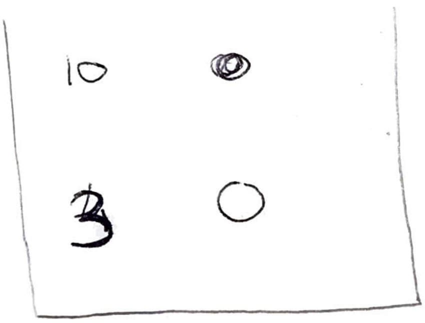
k <- 0:3

= dhyper(k, 3, 10, 5)

$E(X) = 1'153846$

$Var(X) = 0'591716$

Problema 4



5 extraccions amb reemplaçament

X = "nombre de boles blanques"

$X \in \{0, 1, 2, 3, 4, 5\}$

$$P(X=k) = \binom{5}{k}\left(\frac{3}{13}\right)^{k}\left(\frac{10}{13}\right)^{5-k}$$

X és una Binomial amb $n=5$

i $p=\frac{3}{13}$

$$P(X=0)=\left(\frac{10}{13}\right)^5$$

$$P(X=1)=5\cdot\frac{3}{13}\left(\frac{10}{13}\right)^4$$

$$P(X=2)=10\cdot\left(\frac{3}{13}\right)^2\left(\frac{10}{13}\right)^3$$

$$P(X=3)=\binom{5}{3}\left(\frac{3}{13}\right)^3\left(\frac{10}{13}\right)^2=10\cdot\left(\frac{3}{13}\right)^3\left(\frac{10}{13}\right)^2$$

$$P(X=4)=\binom{5}{4}\left(\frac{3}{13}\right)^4\frac{10}{13}=5\cdot\left(\frac{3}{13}\right)^4\cdot\frac{10}{13}$$

$$P(X=5)=\left(\frac{3}{13}\right)^5$$

Podem calcular com:

```
k <- 0:5
dbinom(k, 5, 3/13)
```

$E(X)=np=$ 1'153846

$Var(X)=np(1-p)=$ 0'887514