4.11 Subespais de $\mathbb{R}^4$: → $F = \langle (1,0,1,0), (0,2,1,0) \rangle$

→ $G = \{ (x,y,z,t); \; x-z = y = 0 \}$.

Trobar dimensions, bases i equacions implícites dels subespais F, G, $F \cap G$ i $F+G$.

F: dimensió ⇒ $\dim(F) = 2$.

base ⇒ $(1,0,1,0)$, $(0,2,1,0)$ són linealment independents i generadors. base $= \{(1,0,1,0), (0,2,1,0)\}$.

equacions:
$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ x & y & z & t \end{pmatrix} \xrightarrow{(-x)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & y & z-x & t \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2x+y-2z & 2t \end{pmatrix} \begin{matrix} \\ \cdot y \\ \cdot(-2) \end{matrix} \rightarrow$$

$$\xrightarrow{(1/2)} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & x+\frac{y}{2}-z & t \end{pmatrix} \quad \Rightarrow \quad F = \begin{cases} x + \dfrac{y}{2} - z = 0 \\ t = 0 \end{cases}$$

$x+\frac{y}{2}-z \overset{\underbrace{\quad}}{=} 0 \qquad t \overset{\underbrace{\quad}}{=} 0$

G: equacions: $G = \begin{cases} x - z = 0 \\ y = 0 \end{cases}$

base: agafem z i t com a variables lliures:

$$\begin{array}{cccc} x & y & z & t \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \Rightarrow \text{base} = \{(1,0,1,0), (0,0,0,1)\}$$

dimensió: $\dim(G) = 2$.

F+G: generadors de $F+G = \langle (1,0,1,0), (0,2,1,0), (1,0,1,0), (0,0,0,1) \rangle$

base:
$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow \text{base} = \{(1,0,1,0), (0,2,1,0), (0,0,0,1)\}.$$

dimensió: $\dim(F+G) = 3$.

equacions: $ax + by + cz + dt = 0$

$$\begin{cases} a & & +c & & = 0 \\ & 2b & +c & & = 0 \\ & & & d & = 0 \end{cases}$$

⇒ Si agafem $c = 1$ com a variable lliure:

$(-1, -1/2, 1, 0)$

equació: $-x - \dfrac{1}{2}y + z = 0$

F∩G: dimensió: (Graumann): $\dim(F) + \dim(G) = \dim(F+G) + \dim(F\cap G) \Rightarrow \dim(F\cap G) = 1$.

$\qquad\qquad\qquad\quad (2) \qquad\quad (2) \qquad\quad (3) \qquad\quad (1)$

equacions:

$$\begin{array}{c} \left.\begin{cases} x & & -z & = 0 \\ & y & & = 0 \end{cases}\right] G \\ \left.\begin{cases} 2x & +y & -2z & = 0 \\ & & & t = 0 \end{cases}\right] F \end{array} \xrightarrow{F_3 = F_3 - 2F_1} \begin{cases} x-z & = 0 \\ y & = 0 \\ y & = 0 \\ t & = 0 \end{cases} \Rightarrow \text{equacions } F\cap G: \begin{cases} x-z = 0 \\ y = 0 \\ t = 0 \end{cases}$$

base: agafam com a variable $z = 1$.

$$\{(1, 0, 1, 0)\} \Rightarrow \text{base de } F \cap G.$$