# LAB 8 ICD 2020-2021

1. Dem. que si $x > 0 \Rightarrow x + \frac{1}{x} \geq 2$.

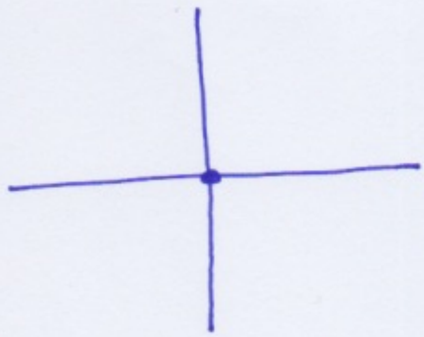
$$g(x) = x + \frac{1}{x} - 2 = \frac{x^2+1-2x}{x} = \frac{x^2-2x+1}{x} = \frac{(x-1)^2}{x}$$

V.v. $g(x) \geq 0 \quad \forall x > 0$, evident ja que

$$g(x) = \frac{\overbrace{(x-1)^2}^{\geq 0}}{\underbrace{x}_{>0}} \geq 0 .$$

. També podem fer representació gràfica de g (estudiar creixement, etc. i veure $g(x) \geq 0$.)

2. $f(x) = \dfrac{x}{\sqrt{4x^2+1}}$



· Domini: $\underbrace{\underbrace{4x^2}_{\geq 0} + \underbrace{1}_{\geq 1}}_{\geq 1} > 0$ ~~[illegible]~~ $\Rightarrow D(f) = \mathbb{R}$

$1 > 0$ OK.

· TALLS EIXOS:

· $x = 0 \in D(f)$ : $f(0) = 0 \rightarrow$ Punt $(0, \underset{f(0)}{0})$

· $f(x) = 0 \Leftrightarrow x = 0 \longrightarrow$ Punt $(0,0)$

· SIMETRIES / PERIODICITAT: $\rightarrow$ NO

$$f(-x) = \frac{-x}{\sqrt{4(-x)^2+1}} = \frac{-x}{\sqrt{4x^2+1}} = -f(x) \rightarrow \text{SENAR}$$

· ASSÍMPTOTES:

→ VERTICALS: No n'hi ha (domini $=\mathbb{R}$)

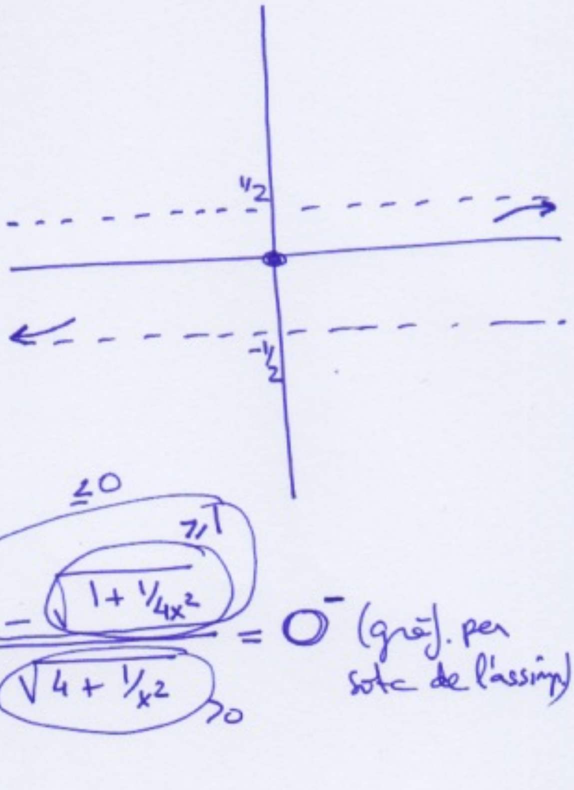
→ HORITZONTALS:

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to+\infty} \frac{x}{\sqrt{4x^2+1}} = \lim_{x\to+\infty} \frac{x}{\sqrt{x^2(4+1/x^2)}}$$

$$= \lim_{x\to\pm\infty} \frac{x}{|x|\sqrt{4+1/x^2}} = \begin{cases} \lim_{x\to+\infty} \dfrac{x}{x\sqrt{4+1/x^2}} = \lim_{x\to+\infty} \dfrac{1}{\sqrt{4+1/x^2}} = \dfrac{1}{2} \\[2ex] \lim_{x\to-\infty} \dfrac{x}{(-x)\sqrt{4+1/x^2}} = \dfrac{-1}{2} \end{cases}$$

Així, $y=\frac{1}{2}$ és A.H. quan $x\to+\infty$

$y=-\frac{1}{2}$ és A.H. quan $x\to-\infty$




Posició:

$$\lim_{x\to+\infty}\left(f(x)-\frac{1}{2}\right) = \lim_{x\to+\infty}\left(\frac{x}{\sqrt{4x^2+1}}-\frac{1}{2}\right)$$

$$= \lim_{x\to+\infty} \frac{2x-\sqrt{4x^2+1}}{2\sqrt{4x^2+1}} =$$

$$= \lim_{x\to+\infty} \frac{2x-|2x|\sqrt{1+1/4x^2}}{2\sqrt{4x^2+1}} =$$

$$\underset{\uparrow}{=} \lim_{x\to+\infty} \frac{2x\left(1-\sqrt{1+1/4x^2}\right)}{2x\sqrt{4+1/x^2}} = \lim_{x\to+\infty} \frac{\overbrace{1-\overbrace{\sqrt{1+1/4x^2}}^{\geq 1}}^{\leq 0}}{\underbrace{\sqrt{4+1/x^2}}_{>0}} = 0^- \text{ (gràf. per sota de l'assímp)}$$

$|2x| = 2x$
$\uparrow$
$x\to+\infty \Rightarrow x>0$

$$\lim_{x\to-\infty}\left(f(x)+\frac{1}{2}\right) = \lim_{x\to-\infty}\left(\frac{x}{\sqrt{4x^2+1}}+\frac{1}{2}\right) = \lim_{x\to-\infty} \frac{2x+\sqrt{1+4x^2}}{2\sqrt{4x^2+1}}$$

$$= \lim_{x\to-\infty} \frac{2x+|2x|\sqrt{1+1/4x^2}}{2|x|\sqrt{4+1/x^2}} \underset{\uparrow}{=} \lim_{x\to-\infty} \frac{2x-2x\sqrt{1+1/4x^2}}{-2x\sqrt{4+1/x^2}} =$$

$|2x| = -2x$
$x\to-\infty \Rightarrow x<0$

$$= \lim_{x\to-\infty} \frac{\overbrace{1-\overbrace{\sqrt{1+1/4x^2}}^{\geq 1}}^{\leq 0}}{\underbrace{-\sqrt{4+1/x^2}}_{<0}} = 0^+ \text{ (gràf. per sobre de l'assímptota)}$$

→ OBLÍQÜES : NO N'HI HA pq hi ha horitzontals

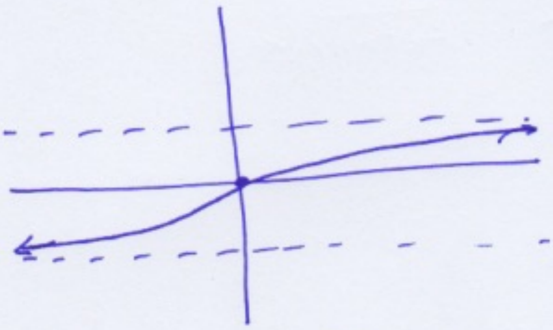
· MONOTONIA : $D(f)=\mathbb{R}$

$$f'(x)=\frac{\sqrt{4x^2+1}-x\cdot\frac{8x}{2\sqrt{4x^2+1}}}{4x^2+1}=\frac{\cancel{4x^2}+1-\cancel{4x^2}}{\sqrt{4x^2+1}\,(4x^2+1)}=\frac{1}{(4x^2+1)^{3/2}}$$

$f'(x)\neq 0$ sempre $\Rightarrow$ No hi ha punts crítics.


$$f'(0)=\frac{1}{1}=1>0$$

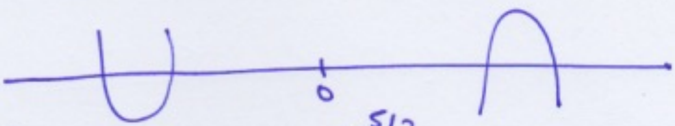
$f$ és creixent en tot $\mathbb{R}$.



· CONVEXITAT:

$$f''(x)=\frac{-3}{2}(4x^2+1)^{-\frac{3}{2}-1}\cdot 8x=-12x\,(4x^2+1)^{-5/2}$$

$f''(x)=0 \Leftrightarrow x=0$ → ~~punt d'inflex~~




$$f''(-1)=12\,(4+1)^{-5/2}>0$$

$$f''(1)=-12(4+1)^{-5/2}<0$$

$f$ és convexa en $(-\infty,0)$ i còncava en $(0,+\infty)$
$f$ té un punt d'inflexió en $x=0$

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· Recorregut : $\left(-\frac{1}{2},\frac{1}{2}\right)$

$$f(x) = \frac{x}{x^2+x-2}$$

· DOMINI: Racional $\Rightarrow D(f) = \mathbb{R} \setminus \{$zeros den.$\}$

$$x^2+x-2=0 \iff x = \frac{-1 \pm \sqrt{1-4\cdot1\cdot(-2)}}{2\cdot1} = \frac{-1\pm3}{2} \begin{cases} 1 \\ -2 \end{cases}$$

$$\Rightarrow \boxed{D(f) = \mathbb{R}\setminus\{-2, 1\}}$$

· TALLS EIXOS:

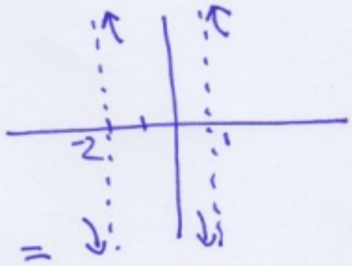
- $x=0 \in D(f) \Rightarrow f(0) = 0 \rightarrow$ Punt $(0, f(0)) = (0,0)$
- $f(x) = 0 \iff x = 0 \rightarrow$ Punt $(0,0)$.

· SIMETRIES / PERIODICITAT $\rightarrow$ NO

$$f(-x) = \frac{-x}{(-x)^2+(-x)-2} = -\frac{x}{x^2-x-2} \neq \begin{cases} f(x) \\ -f(x) \end{cases} \Rightarrow \text{NO SIMETRIES}$$

· ASSÍMPTOTES:

· VERTICALS: $x=-2$ i $x=1$.




$$\lim_{x\to-2} f(x) = \infty \quad \frac{-2}{0} = \infty$$

Posició: $\lim\limits_{x\to-2^\pm} f(x) = \lim\limits_{x\to-2^\pm} \frac{x}{(x-1)(x+2)} =$

$$= \begin{cases} \dfrac{-2}{-3\cdot0^+} = +\infty, & x\to-2^+ \ (x+2>0) \\ \dfrac{-2}{-3\cdot0^-} = -\infty, & x\to-2^- \ (x+2<0) \end{cases}$$

$$\lim_{x\to1} f(x) = \frac{1}{0} = \infty$$

Posició: $\lim\limits_{x\to1^\pm} \frac{x}{(x-1)(x+2)} = \begin{cases} \dfrac{1}{0^+\cdot3} = +\infty, & x\to1^+ \ \ x-1>0 \\ \dfrac{1}{0^-\cdot3} = -\infty, & x\to1^- \ \ x-1<0 \end{cases}$

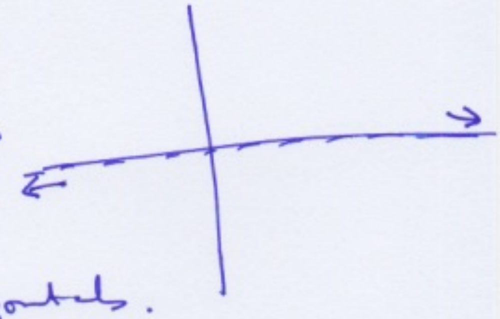
· HORITZONTALS:

$$\lim_{x\to\pm\infty} f(x) \underset{\substack{\uparrow \\ \text{grau num} = 1 \\ \text{grau den} = 2}}{=} 0 \implies y=0 \text{ és A.H.}$$

Posició:

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x}{x^2+x-2} = \lim_{x\to\pm\infty} \frac{1}{\underset{\downarrow \atop \pm\infty}{x} + \underset{\downarrow \atop 1}{\left(1 - \frac{2}{x}\right)}}$$

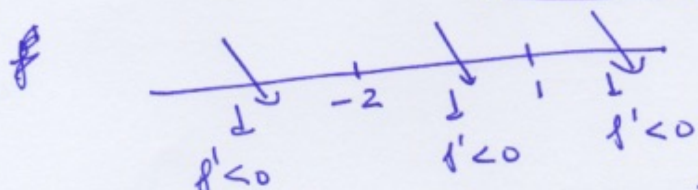
$$= \begin{cases} \dfrac{1}{+\infty} = 0^+ & , x\to+\infty \\[2ex] \dfrac{1}{-\infty} = 0^- & , x\to-\infty \end{cases}$$



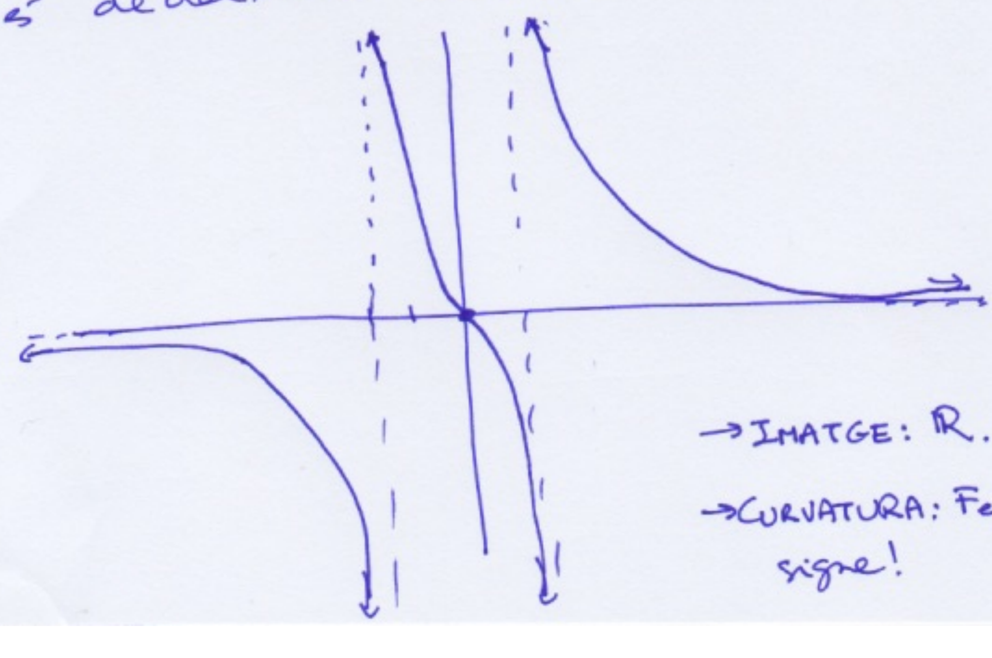
· OBLÍQUES: No hi ha pq hi ha horitzontals.

· MONOTONIA: $f'(x) = \dfrac{x^2+x-2 - x(2x+1)}{(x^2+x-2)^2} = \dfrac{-x^2-2}{(x^2+x-2)^2} =$

$$= -\frac{\overbrace{x^2+2}^{\geq 2 > 0}}{\underbrace{(x^2+x-2)^2}_{>0}} < 0 \text{ . No punts crítics}$$




f és decreixent en $\mathbb{R}\setminus\{-2,1\}$.



→ IMATGE: $\mathbb{R}$.

→ CURVATURA: Fer $f''$ i mirar signe!