LAB. 4 ICD 2020 - 2021

1. (a) $\lim_{x\to+\infty} \underbrace{\left(x - \sqrt{x^2-2x}\right)}_{L} = [\infty - \infty]$

$$L = \lim_{x\to+\infty} \frac{\left(x-\sqrt{x^2-2x}\right)\left(x+\sqrt{x^2-2x}\right)}{x+\sqrt{x^2-2x}} =$$

$$= \lim_{x\to+\infty} \frac{x^2-(x^2-2x)}{x+\sqrt{x(x-2)}} = \lim_{x\to+\infty} \frac{2x}{x+\sqrt{x}\sqrt{x-2}} =$$

$$= \lim_{x\to+\infty} \frac{x\cdot 2}{x\left(1+\frac{1}{\sqrt{x}}\sqrt{x-2}\right)} = \lim_{x\to+\infty} \frac{2}{1+\sqrt{\underbrace{\frac{x-2}{x}}_{\to 1}}}$$

$$= \frac{2}{1+\sqrt{1}} = \boxed{1}$$

(b) $\lim_{x\to 2} \frac{2x-6}{x^2+x-12} =: L$

$$\left[x^2+x-12=0 \iff x = \frac{-1\pm\sqrt{1-4\cdot(-12)\cdot 1}}{2} = \frac{-1\pm\sqrt{49}}{2} = \frac{-1\pm 7}{2}\right] \quad \begin{matrix} 3 & -4 \end{matrix}$$

NO CAL

$$x^2+x-12 \xrightarrow[x\to 2]{} 2^2+2-12 = -6 \neq 0.$$

$$L = \frac{2\cdot 2-6}{2^2+2-12} = \frac{-2}{-6} = \boxed{\frac{1}{3}}$$

(c) $\lim_{x\to 2} \frac{\sqrt{x^2+x+1}-\sqrt{2x^2-x+1}}{\sqrt{x^2+3}-\sqrt{x^2+x+1}} = \frac{\sqrt{7}-\sqrt{7}}{\sqrt{7}-\sqrt{7}} = \left[\frac{0}{0}\right]$

$$= \lim_{x\to 2} \frac{\left(\sqrt{x^2+x+1}-\sqrt{2x^2-x+1}\right)\left(\sqrt{x^2+x+1}+\sqrt{2x^2-x+1}\right)\left(\sqrt{x^2+3}+\sqrt{x^2+x+1}\right)}{\left(\sqrt{x^2+x+1}+\sqrt{2x^2-x+1}\right)\left(\sqrt{x^2+3}-\sqrt{x^2+x+1}\right)\left(\sqrt{x^2+3}+\sqrt{x^2+x+1}\right)}$$

$$= \lim_{x\to 2} \frac{\left(x^2+x+1-(2x^2-x+1)\right)\left(\sqrt{x^2+3}+\sqrt{x^2+x+1}\right)}{\left(\sqrt{x^2+x+1}+\sqrt{2x^2-x+1}\right)\left(x^2+3-(x^2+x+1)\right)} =$$

$$= \lim_{x\to 2} \frac{(-x^2+2x)\left(\sqrt{x^2+3}+\sqrt{x^2+x+1}\right)}{\left(\sqrt{x^2+x+1}+\sqrt{2x^2-x+1}\right)(2-x)} =$$

$$= \lim_{x\to 2} \frac{x\left(\sqrt{x^2+3}+\sqrt{x^2+x+1}\right)}{\left(\sqrt{x^2+x+1}+\sqrt{2x^2-x+1}\right)} =$$

$-x^2+2x =$
$= x(-x+2)$

$$= \frac{2\left(\sqrt{4+3}+\sqrt{4+2+1}\right)}{\sqrt{4+2+1}+\sqrt{8-2+1}} = \frac{2\cdot 2\sqrt{7}}{2\sqrt{7}} = \boxed{2}$$

(d) $$\lim_{x\to+\infty} \frac{x^2+x^{5/2}+1}{x\underbrace{\sqrt[3]{x^2}}_{=x^{2/3}}-x^2\sqrt{x}+3x} = \lim_{x\to+\infty} \frac{x^2+x^{5/2}+1}{x^{1+2/3}-x^{2+1/2}+3x}$$

$$= \lim_{x\to+\infty} \frac{x^2+x^{5/2}+1}{x^{5/3}-x^{5/2}+3x} = \lim_{x\to+\infty} \frac{x^{5/2}\left(x^{-1/2}+1+x^{-5/2}\right)}{x^{5/2}\left(x^{-5/6}-1+3x^{-3/2}\right)}$$

$\frac{5}{2}$ és la potència + gran.

$\frac{5}{3}-\frac{5}{2}=\frac{10-15}{6}=-\frac{5}{6}$

$$= \lim_{x\to+\infty} \frac{1+\overset{\to 0}{x^{-1/2}}+\overset{\to 0}{x^{-5/2}}}{-1+\underset{\downarrow 0}{x^{-5/6}}+3\underset{\downarrow 0}{x^{-3/2}}} = \frac{1}{-1} = \boxed{-1}$$

(e) $$\lim_{x\to+\infty} \frac{x\log x-e^{2x}+x^3}{x^2\sqrt{\log x}+\sqrt{e^x}} = \lim_{x\to+\infty} \frac{\frac{x\log x}{e^{2x}}-\frac{e^{2x}}{e^{2x}}+\frac{x^3}{e^{2x}}}{\frac{x^2\sqrt{\log x}}{e^{2x}}+\frac{\sqrt{e^x}}{e^{2x}}}$$

logx creix
+ lent que exponencials
Dividem tot per e^{2x}

$$= \lim_{x\to+\infty} \frac{\frac{x\log x}{e^{2x}}-1+\frac{x^3}{e^{2x}}}{\frac{x^2\sqrt{\log x}}{e^{2x}}+\underbrace{e^{\frac{x}{2}-2x}}_{=e^{-\frac{3}{2}x}}} = \frac{0-1+0}{0+0} = \frac{-1}{0} = \boxed{\infty}$$

$e^{-\frac{3}{2}x} \to e^{-\infty}=0$

$\lim_{x\to+\infty} \frac{x^n}{e^{mx}} = 0$ $n,m>0$ $\lim_{x\to+\infty} \frac{x\log x}{e^{2x}} = 0$ $\lim_{x\to+\infty} \frac{x^2\sqrt{\log x}}{e^{2x}} = 0$

creix + ràpid creix + ràpid

Amb Hôpital:

(*) $$\lim_{x\to+\infty} \frac{x^3}{e^{2x}} = \left[\frac{\infty}{\infty}\right] \underset{\text{H.ôp.}}{=} \lim_{x\to+\infty} \frac{3x^2}{2e^{2x}} = \left[\frac{\infty}{\infty}\right] =$$

$$\underset{\text{Hôp.}}{=} \lim_{x\to+\infty} \frac{6x}{4e^{2x}} = \left[\frac{\infty}{\infty}\right] \underset{\text{Hôp.}}{=} \lim_{x\to+\infty} \frac{6}{8\underbrace{e^{2x}}_{\downarrow \atop +\infty}} = \boxed{0}.$$

- $$\lim_{x\to+\infty} \frac{x\log x}{e^{2x}} = \left[\frac{\infty}{\infty}\right] \underset{\text{Hôp.}}{=} \lim_{x\to+\infty} \frac{\log x + x\cdot\frac{1}{x}}{2e^{2x}}$$

$$= \lim_{x\to+\infty} \frac{\log x + 1}{2e^{2x}} = \left[\frac{\infty}{\infty}\right] \underset{\text{Hôp.}}{=} \lim_{x\to+\infty} \frac{\frac{1}{x}}{4e^{2x}} =$$

$$= \lim_{x\to+\infty} \frac{1}{\underbrace{4xe^{2x}}_{\downarrow \atop +\infty}} = \frac{1}{\infty} = \boxed{0}$$

- $$\lim_{x\to+\infty} \frac{x^2(\log x)^{1/2}}{e^{2x}} = \left[\frac{\infty}{\infty}\right] \underset{\text{Hôp.}}{=} \lim_{x\to+\infty} \frac{2x(\log x)^{1/2} + x^2\cdot\frac{1}{2}(\log x)^{-1/2}\cdot\frac{1}{x}}{2e^{2x}}$$

$$= \lim_{x\to+\infty} \frac{2x(\log x)^{1/2} + \frac{x}{2}(\log x)^{-1/2}}{2e^{2x}} =$$

$$= \lim_{x\to+\infty} \frac{\frac{x}{2} + 2x\log x}{2e^{2x}(\log x)^{1/2}} \underset{\uparrow \atop \text{Hôp.}}{=} \lim_{x\to+\infty} \frac{\frac{1}{2} + 2\log x + 2}{4e^{2x}(\log x)^{1/2} + \frac{2e^{2x}}{2}\frac{(\log x)^{-1/2}}{x}}$$

$$= \lim_{x\to+\infty} \frac{\frac{5}{2} + 2\log x}{4e^{2x}(\log x)^{1/2} + e^{2x}\frac{(\log x)^{-1/2}}{x}} \underset{\uparrow \atop \text{Hôp.}}{=} \dots = \text{MASSA LLARG.}$$

MÉS FÀCIL: $e^{2x} = e^{x+x} = e^x\cdot e^x$

$$\frac{x^2(\log x)^{1/2}}{e^{2x}} = \underbrace{\frac{x^2}{e^x}}_{\downarrow \text{ COM } (*) \atop 0} \cdot \underbrace{\frac{(\log x)^{1/2}}{e^x}}_{\parallel} = 0\cdot 0 = \boxed{0}$$

$$\left(\frac{\log x}{e^{2x}}\right)^{1/2} \xrightarrow[x\to+\infty]{} 0^{1/2} = 0$$

FER HÔP.

(f) $\displaystyle\lim_{x\to-\infty} \frac{e^x + \log(1+x^3)}{\sqrt[3]{x^2}}$

NO EXISTEIX ja que $1+x^3 \longrightarrow 1-\infty = -\infty$
i log no definit per valors negatius.

2. $f: \mathbb{R} \longrightarrow \mathbb{R}$

$$f(x) = \begin{cases} \dfrac{x^3-1}{1-x^4}, & |x| \neq 1 \quad x \neq \pm 1. \\ -3/4 & , x = 1 \\ 3/4 & , x = -1 \end{cases}$$

f cont. en $x=a$ si $\displaystyle\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$.

Veiem que si $x \neq \pm 1$, $f(x) = \dfrac{x^3-1}{1-x^4}$ és una funció cont. a $\mathbb{R}\setminus\{\pm 1\}$ $(1-x^4=0 \iff x^4=1 \iff x=\pm 1)$.

PUNTS PROBLEMÀTICS : $x = \pm 1$.

i) $\boxed{x=1}$ $f(1) = -3/4$.

<u>OBS</u>: $\cdot\ 1-x^4 = (1-x^2)(1+x^2) = (x^2+1)(1-x)(1+x)$

$= -(x^2+1)(x+1)\underline{(x-1)}$

$\cdot\ x^3-1 = (x-1)(x^2+x+1)$

Així, $\dfrac{x^3-1}{1-x^4} = \dfrac{\cancel{(x-1)}(x^2+x+1)}{-(x^2+1)(x+1)\cancel{(x-1)}} = \dfrac{x^2+x+1}{-(x^2+1)(x+1)}$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2+x+1}{-(x^2+1)(x+1)} = \frac{3}{-2\cdot 2} = -\underset{\underset{f(1)}{\parallel}}{\frac{3}{4}}$$

$\Rightarrow f$ cont. en $x = 1$

ii) $\boxed{x=-1}$ $f(-1) = 3/4$

$$\frac{x^3-1}{1-x^4} \xrightarrow[x\to -1]{} \frac{(-1)^3-1}{0} = \frac{-2}{0} = \infty \;!!!\ldots$$

$\Rightarrow \nexists \lim_{x\to -1} f(x) \Rightarrow \boxed{f \text{ no cont. en } x=-1}$