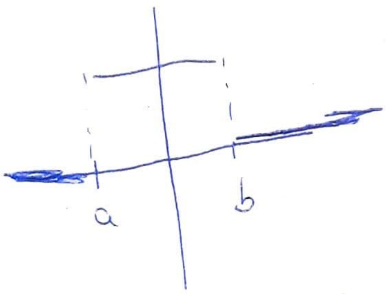
Problema 13

$$f(x) = K(x^2-1)\,\mathbb{1}_{(1,3)}(x)$$

$$f(x) = \begin{cases} 0 & x \leq 1 \\ K(x^2-1) & 1 \leq x < 3 \\ 0 & x \geq 3 \end{cases}$$

ja que $\mathbb{1}_{(a,b)}(x) = \begin{cases} 0 & x \leq a \\ 1 & a < x < b \\ 0 & x \geq b \end{cases}$



a) Per tal que f sigui densitat ha de cumplir

- $f \geq 0$
- $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{1} \underbrace{f(x)}_{0}dx + \int_{1}^{3} \underbrace{f(x)}_{K(x^2-1)}dx + \int_{3}^{\infty} \underbrace{f(x)}_{0}dx$$

$$= \int_1^3 K(x^2-1)\,dx = K\left(\frac{x^3}{3} - x\right]_1^3 = K\left(9-3-\frac{1}{3}+1\right)$$

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$$= K\left(9 - \frac{1}{3} - (3-1)\right)$$

$$K\left(\frac{20}{3}\right) = 1$$

$$K = \frac{3}{20}$$

$$f(x) = \frac{3}{20}(x^2-1)\,\mathbb{1}_{(1,3)}(x)$$

b)

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \dfrac{3}{20}\displaystyle\int_1^x (u^2-1)\,du & 1 \le x < 3 \\ 1 & x \ge 3 \end{cases}$$

$$\frac{3}{20}\int_1^x (u^2-1)\,du = \frac{3}{20}\left(\left.\frac{u^3}{3}\right]_1^x - \left.u\right]_1^x\right)$$

$$= \frac{3}{20}\left(\frac{x^3}{3} - \frac{1}{3} - (x-1)\right)$$

$$= \frac{3}{20}\left(\frac{x^3}{3} - x + \frac{2}{3}\right)$$

d)

$$P(X \leq 2) = F(2) = \frac{3}{20}\left(\frac{8}{3} - 2 + \frac{2}{3}\right)$$

$$= \frac{3}{20}\left(\frac{4}{3}\right) = \frac{4}{20} = \boxed{\frac{1}{5}}$$

$$P(X > 3) = 1 - F(3) = 1 - 1 = 0$$

$$P(X < 3) = F(3) = 1$$

$$P(X = 1) = 0$$

$$P(X \geq 0) = 1 - F(0) = 1 - 0 = 1.$$

$$E(X) = \frac{3}{20}\int_1^3 x\,(x^2-1)\,dx$$

$$= \frac{3}{20}\int_1^3 x^3 - x \; dx$$

$$= \frac{3}{20}\left(\frac{x^4}{4}\Big]_1^3 - \frac{x^2}{2}\Big]_1^3\right)$$

$$= \frac{3}{20}\left(\frac{81}{4} - \frac{1}{4} - \frac{9}{2} + \frac{1}{2}\right]$$

$$= \frac{3}{20}\left(\frac{80}{4} - 4\right) = \frac{3}{20}(16)$$

$$= \frac{3}{5} \cdot 4 = \boxed{\frac{12}{5}}$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$E(X^2) = \frac{3}{20}\int_1^3 x^2(x^2-1)\,dx$$

$$= \frac{3}{20}\int_1^3 x^4 - x^2\,dx$$

$$= \frac{3}{20}\left(\frac{x^5}{5}\Big]_1^3 - \frac{x^3}{3}\Big]_1^3\right)$$

$$= \frac{3}{20}\left(\frac{243}{5} - \frac{1}{5} - 9 + \frac{1}{3}\right)$$

$$\begin{array}{r} 81 \\ \times\ 3 \\ \hline 243 \end{array}$$

$$= \frac{3}{20}\left(\frac{242}{5} - \frac{26}{3}\right)$$

$$= \frac{3}{20}\left(\frac{726 - 130}{15}\right)$$

$$= \frac{3}{20}\left(\frac{596}{15}\right)$$

$$\cancel{= 298}$$

$$= \frac{149}{25}$$

596 | 4
19 149
36

$$Var(X) = \frac{149}{25} - \frac{144}{25} = \frac{5}{25} = \boxed{\frac{1}{5}}$$

Problema 14 (1)

$$f(x) = K\,(2x+3)\,\mathbb{1}_{(0,1)}(x)$$

a) $K \int_0^1 (2x+3)\,dx = K \left. 2\frac{x^2}{2} + 3x \right]_0^1$

$= K(4) = 1$

$K = \frac{1}{4}$

b) $F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4}(x^2+3x) & 0 \leq x < 1 \\ 1 & x \geq 1. \end{cases}$

c) $P(X \leq 2) = F(2) = 1$

$P(X > 0'5) = 1 - P(X \leq 0'5)$

$= 1 - F(0'5)$

$= 1 - \frac{1}{4}(0'5^2 + 3 \cdot 0'5)$

$P(X \geq 0'3) = 1 - F(0'3) = 1 - \frac{1}{4}(0'3^2 + 3 \cdot 0'3)$

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d) $E(X) = \int_0^1 x \frac{1}{4}(2x+3)\, dx$

$= \frac{1}{4}\int_0^1 2x^2 + 3x \, dx$

$= \frac{1}{4} \left[2\frac{x^3}{3} + 3\frac{x^2}{2} \right]_0^1$

$= \frac{1}{4}\left(\frac{2}{3} + \frac{3}{2} \right)$

$= \frac{1}{4}\left(\frac{13}{6} \right) = \frac{13}{24}$

$E(X^2) = \int_0^1 x^2 \frac{1}{4}(2x+3)\, dx$

$= \frac{1}{4} \left[2\frac{x^4}{4} + 3\frac{x^3}{3} \right]_0^1$

$= \frac{1}{4}\left(\frac{2}{4} + 1 \right)$

$= \frac{1}{4}\left(\frac{1}{2} + 1 \right) = \frac{1}{4}\left(\frac{3}{2} \right) = \frac{3}{8}$

$Var(X) = \frac{3}{8} - \left(\frac{13}{24} \right)^2$