LAB. 5 ICD 2020-2021

1.

(a) $\lim_{x\to+\infty}\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)=[\infty-\infty]=$

$$=\lim_{x\to+\infty}\frac{\left(\sqrt{x^2+x}-\sqrt{x^2-x}\right)\left(\sqrt{x^2+x}+\sqrt{x^2-x}\right)}{\sqrt{x^2+x}+\sqrt{x^2-x}}=$$

$$=\lim_{x\to+\infty}\frac{\cancel{x^2}+x-(\cancel{x^2}-x)}{\sqrt{x^2+x}+\sqrt{x^2-x}}=\lim_{x\to+\infty}\frac{2x}{\sqrt{\underbrace{x^2+x}_{x^2(1+1/x)}}+\sqrt{\underbrace{x^2-x}_{x^2(1-1/x)}}}$$

$$=\lim_{x\to+\infty}\frac{2x}{\underbrace{|x|}_{\substack{\| \leftarrow x\to+\infty\\ x\ \ \Rightarrow x>0}}\left(\sqrt{1+1/x}+\sqrt{1-1/x}\right)}=\lim_{x\to+\infty}\frac{2}{\sqrt{\underbrace{1+1/x}_{\downarrow\ 1}}+\sqrt{\underbrace{1-1/x}_{\downarrow\ 1}}}=\frac{2}{2}=\boxed{1}$$

(b) $\lim_{x\to2}\frac{\sin(x-2)}{\ln(x-1)}=\left[\frac{0}{0}\right]=$

$$=\lim_{x\to2}\frac{\sin(x-2)}{x-2}\cdot\frac{x-2}{\ln(\underbrace{x-1}_{\substack{\|\\ 1+(x-2)}})}=\lim_{x\to2}\frac{\sin(x-2)}{\underbrace{(x-2)}_{\substack{\|\\ y}}}\cdot\frac{x-2}{\ln(1+(x-2))}$$

$$\underset{y=x-2\xrightarrow[x\to2]{}0}{=}\lim_{y\to0}\underbrace{\frac{\sin(y)}{y}}_{\downarrow\ 1}\cdot\frac{1}{\underbrace{\frac{\ln(1+y)}{y}}_{\downarrow\ 1}}=1\cdot\frac{1}{1}=\boxed{1}$$

(c) $\lim_{x\to+\infty}\underbrace{x^{-1}}_{\downarrow\ 0}\underbrace{\sin(\underbrace{e^x}_{\downarrow\ +\infty})}_{\text{afitat}}=\boxed{0}$

0. acotada = 0

(d) $\lim_{x\to0}\frac{(1-e^{-x})^2\sin(x^2)}{(\cos(x)-1)\ln(1+x^3)}=\left[\frac{0\cdot0}{0\cdot0}\right]$

Farem servir: $\lim_{y\to0}\frac{\sin y}{y}=1$, $\lim_{y\to0}\frac{\ln(1+y)}{y}=1$

$\lim_{y\to0}\frac{1-\cos y}{y^2/2}=1$

$$(1-e^{-x})^2 = \left(1-\frac{1}{e^x}\right)^2 = \frac{(e^x-1)^2}{e^{2x}}$$

$$\frac{(1-e^{-x})^2\sin(x^2)}{(\cos x-1)\ln(1+x^3)} = \frac{(e^x-1)^2}{e^{2x}}\cdot\frac{\sin x^2}{x^2}\cdot x^2\cdot\frac{(-x^2/2)}{\cos x-1}\,\frac{1}{\left(\frac{-x^2}{2}\right)}\,\frac{x^3}{\ln(1+x^3)}\cdot\frac{1}{x^3}$$

$$= \frac{(e^x-1)^2}{e^{2x}}\left(\frac{\sin x^2}{x^2}\right)\frac{1}{\left(\frac{\cos x-1}{-x^2/2}\right)}\cdot\frac{1}{\left(\frac{\ln(1+x^3)}{x^3}\right)}\cdot\underbrace{\frac{\cancel{x^2}}{\frac{-x^2}{2}\cdot x^3}}_{=\frac{-2}{x^3}}$$

$$= \underbrace{\frac{-2}{e^{2x}}}_{\downarrow x\to 0\ \ \frac{-2}{e^0}=-2}\cdot\underbrace{\frac{(e^x-1)^2}{x^3}}_{\downarrow (*)\ x\to 0\ \ \infty}\cdot\underbrace{\frac{\sin x^2}{x^2}}_{\downarrow x\to 0\ \ 1}\cdot\frac{1}{\underbrace{\left(\frac{\cos x-1}{\frac{-x^2}{2}}\right)}_{\downarrow x\to 0\ \ 1}}\cdot\frac{1}{\underbrace{\frac{\ln(1+x^3)}{x^3}}_{\downarrow x\to 0\ \ 1}}$$

(*) $\lim_{x\to 0}\frac{(e^x-1)^2}{x^3} = \left[\frac{0}{0}\right] \underset{\text{Hôp.}}{=} \lim_{x\to 0}\frac{2(e^x-1)\cdot e^x}{3x^2} = \left[\frac{0}{0}\right]$

$$\underset{\text{Hôp.}}{=} \lim_{x\to 0}\frac{2e^x(e^x-1)+2e^x\cdot e^x}{6x} = \frac{0+2}{0} = \frac{2}{0} = \infty.$$

|| De fet $\begin{cases} -\infty, & x\to 0^- \\ +\infty, & x\to 0^+ \end{cases}$

$$\Longrightarrow \lim_{x\to 0}\underbrace{\frac{(1-e^{-x})^2\sin(x^2)}{(\cos x-1)\ln(1+x^3)}}_{f(x)} = \infty.$$

De fet, si mirèssim signe:

$\lim_{x\to 0^-} f(x) = +\infty$ i $\lim_{x\to 0^+} f(x) = -\infty$.

2. Estudiar en quins punts $x \in \mathbb{R}$ $f(x) = \dfrac{x+1}{1+2e^{-1/x}}$

és contínua.

- $x+1$ polinomi $\Rightarrow$ cont. a $\mathbb{R}$.
- $1+2e^{-1/x}$ té prob en $x=0$, en la resta de punts és cont.

~~Aqu~~. $1+2e^{-1/x} = 1 + \dfrac{2}{e^{1/x}} = \dfrac{e^{1/x}+2}{e^{1/x}} = 0 \Leftrightarrow$

$$\Leftrightarrow e^{1/x}+2=0 \Leftrightarrow \underbrace{e^{1/x}}_{}=\underbrace{-2}_{<0}$$ IMPOSSIBLE pq $e^y > 0$.

Així, f és contínua a $\mathbb{R}\setminus\{0\}$ pels arguments anteriors.

Estudiem $x=0$:

$$\lim_{x\to 0^-} f(x) = \lim_{\substack{x\to 0\\ x<0}} \frac{x+1}{1+2e^{\underbrace{-1/x}_{-\infty}}} = \frac{1}{+\infty} = 0.$$

$$e^{+\infty} = +\infty$$

$$\lim_{x\to 0^+} f(x) = \lim_{\substack{x\to 0\\ x>0}} \frac{x+1}{1+2e^{\underbrace{-1/x}_{+\infty}}} = \frac{1}{1+2\cdot 0} = \frac{1}{1} = 1$$

$$e^{-\infty} = 0$$

f mai podrà ser contínua a $x=0$ ja que $\nexists$

$$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x).$$

$\Rightarrow f \in \mathcal{C}(\mathbb{R}\setminus\{0\})$.