Problema 5 Llista 3

$X \sim N(87, 10^2)$

$\mu = 87$
$\sigma = 10$
$\sigma^2 = 100$

$n = 150$

$P(\overline{X}_{150} < 83)$?

$$\overline{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\overline{X}_{150} \sim N\left(87, \frac{100}{150}\right)$$

$$P(\overline{X}_{150} < 83) = \text{pnorm}\left(83, 87, \text{sqrt}\left(\frac{100}{150}\right)\right)$$

$$= \text{pnorm}\left(\frac{83-87}{\sqrt{\frac{100}{150}}}\right) = \text{pnorm}(-4'8989)$$

$$= 1 - \text{pnorm}(4'8989) = 4'81 \cdot 10^{-7} \simeq 0$$

P 6] Llista 3

$$X \sim N(\mu, \sigma^2) \qquad \sigma^2 \text{ desconeguda}$$

$$S = 0'2 \qquad S^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{X})^2}{n}$$

Teoria: $T = \dfrac{\bar{X} - \mu}{S} \sqrt{n-1} \sim T_{n-1}$ distribució T de student

$n = 17$

$n-1 = 16$

$$P(|\bar{X} - \mu| > 0'1) = 1 - P(|\bar{X} - \mu| \leq 0'1)$$

$$= 1 - P(-0'1 \leq \bar{X} - \mu \leq 0'1)$$

$$= 1 - P\left(-\frac{0'1}{S} \leq \frac{\bar{X} - \mu}{S} \leq \frac{0'1}{S}\right)$$

$$= 1 - P\left(-\frac{0'1}{0'2}\sqrt{16} \leq \frac{\bar{X} - \mu}{0'2}\sqrt{16} \leq \frac{0'1}{0'2}\sqrt{16}\right)$$

$$= 1 - P(-2 \leq T_{16} \leq 2)$$

$$= 2(1 - P(T_{16} \leq 2))$$

$$= 2(1 - pt(2,16)) = 0'06278$$

$$pt(2,16) = 0'968614 \simeq pnorm(2) = 0'9772$$

Problema 7

$X =$ " Temps en arreglar un ordinador"

$$\frac{100}{60} = \frac{10}{6} = \frac{5}{3}$$

$$\mu = 3$$
$$\sigma = \frac{5}{3} \qquad \sigma^2 = \frac{25}{9}$$

$$X \sim N\left(3, \frac{5}{3}\right) \qquad S_n \sim N(n\mu, n\sigma^2)$$

$n = 100$

$$S_{100} = X_1 + X_2 + \dots + X_{100} \sim N\left(300, 100 \cdot \frac{25}{9}\right)$$

7 treballadors

40 hores treball setmanal

$$7 \cdot 40 = 280$$

$$P(S_{100}' < 280)$$

$$= pnorm\left(280, 300, 10 \cdot \frac{5}{3}\right)$$

$$= pnorm\left(\frac{280-300}{\frac{50}{3}}\right)$$

$$= pnorm(-1'2) = \boxed{0'1150697}$$

Problema 8

$$S_{1000} \sim B\left(1000, \frac{1}{6}\right)$$

S_{1000} = "numero de sisos al tirar un dau 1000 vegades"

és una binomial amb $n = 1000$, $p = \frac{1}{6}$

Aproximem la binomial per la normal utilitzant el teorema del límit central

$$B(n,p) \underset{\substack{\uparrow \\ \text{aproximadament} \\ \text{per "n gran"}}}{\sim} N(np, n \cdot p(1-p))$$

$$\frac{S_n - np}{\sqrt{n \cdot p(1-p)}} \underset{\substack{\uparrow \\ \text{aproximadament}}}{\sim} N(0,1)$$

$$P(S_{1000} > 150) = 1 - P(S_{1000} \leq 150)$$

$$\simeq 1 - P\left(\frac{S_{1000} - 1000 \cdot \frac{1}{6}}{\sqrt{1000 \cdot \frac{1}{6} \cdot \frac{5}{6}}} \leq \frac{150 - 1000 \cdot \frac{1}{6}}{\sqrt{1000 \, \frac{1}{6} \, \frac{5}{6}}}\right)$$

$$\simeq 1 - \text{pnorm}\left(\frac{150 - 166'7}{11'785}\right) = 1 - \text{pnorm}(-1'417)$$

$$= \text{pnorm}(1'417) = \boxed{0'9217}$$