Problema 4 Dades aparellades

Interval de confiança per a mostres aparellades per a la mitjana de la variable $D = X - Y$, μ_D.

$D \sim N(\mu_D, \sigma_D^2)$ σ_D^2 desconeguda

X = Abans	96	102	108	89	85
Y = Després	104	112	112	93	89
$D = X - Y$	-8	-10	-4	-4	-4

Interval de confiança al nivell γ per μ_D

$$\left[\bar{D} - t_{n-1,\gamma} \frac{S_D}{\sqrt{n-1}} , \bar{D} + t_{n-1,\gamma} \frac{S_D}{\sqrt{n-1}} \right]$$

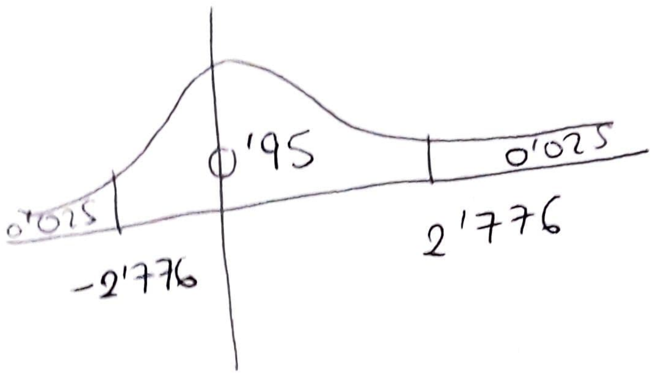
$\bar{D} = \frac{-30}{5} = -6$

$S_D^2 = \frac{8^2+10^2+4^2+4^2+4^2}{5} - 6^2$

$= 6'4$ $\qquad S_D^2 =$

$S_D = \sqrt{6'4} = 2'53$

$qt(0'975, 4) = 2'776$



Interval de confiança al 95 % per a μ_D

$\left[-6 - 2'776 \cdot \frac{2'53}{\sqrt{4}}, -6 + 2'776 \cdot \frac{2'53}{\sqrt{4}}\right]$

$= [-6 - 3'51, -6 + 3'51]$

$= [-9'51, -2'49]$

Com $0 \notin [-9'51, -2'49]$

hi ha diferències.

Problema 5

Dades no aparallades

a) Dades no aparallades amb σ_1^2 i σ_2^2 conegudes

$X \sim N(\mu_1, \sigma_1^2)$

$Y \sim N(\mu_2, \sigma_2^2)$

$X - Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

$$\frac{\bar{X} - \bar{Y} - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

ja que $\bar{X} \sim N\left(\mu_1, \frac{\sigma_1^2}{n_1}\right)$

$\bar{Y} \sim N\left(\mu_2, \frac{\sigma_2^2}{n_2}\right)$

$\bar{X} - \bar{Y} \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$

Interval de confiança per a $\mu_1 - \mu_2$

$$\left[\bar{x} - \bar{y} - u_\gamma \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}, \bar{x} - \bar{y} + u_\gamma \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right.$$

$n_1 = n_2 = 6$

$\bar{x} = 12'73$

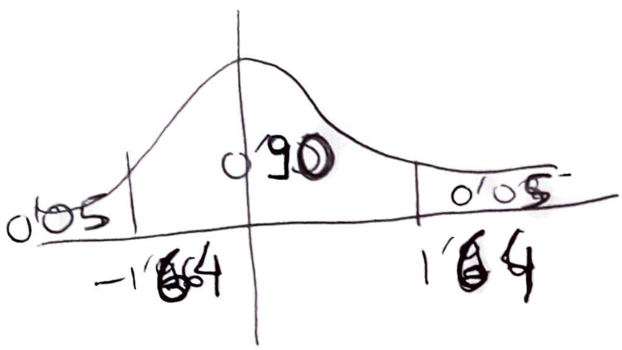
$\bar{y} = 12'9$

$\bar{x} - \bar{y} = -0'1667$

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{25}{6} + \frac{25}{6}} = \sqrt{\frac{50}{6}}$$

$\gamma = 0'9 \qquad U_{0'9} = \text{qnorm}(0'95)$

$= 1'64$



$$\left[-0'1667 - 1'64\sqrt{\frac{50}{6}},\ -0'1667 + 1'64\sqrt{\frac{50}{6}}\right]$$

$$= \left[-0'1667 - 4'73,\ -0'1667 + 4'73\right]$$

$$= \left[-4'9,\ 4'563\right]$$

Com $0 \in$ Interval no hi ha diferències podem suposar $\mu_1 - \mu_2 = 0$

Problema 5

b) Dades no aparellades amb $\sigma_1^2 = \sigma_2^2$ desconegudes

$X \sim N(\mu_1, \sigma^2)$

$Y \sim N(\mu_2, \sigma^2)$

$$\frac{\overline{X} - \overline{Y} - (\mu_1 - \mu_2)}{\sqrt{n_1 S_1^2 + n_2 S_2^2}} \sqrt{\frac{n_1 n_2 (n_1+n_2-2)}{n_1+n_2}}$$

$\sim t_{n_1+n_2-2}$ t-student amb n_1+n_2-2 graus llibertat

Interval per a $\mu_1 - \mu_2$ al nivell γ

$$\left[\bar{x} - \bar{y} - t_{n_1+n_2-2} \frac{\sqrt{n_1+n_2}\sqrt{n_1 S_1^2 + n_2 S_2^2}}{\sqrt{n_1 n_2 (n_1+n_2-2)}} \right.,$$

$$\left. \bar{x} - \bar{y} + t_{n_1+n_2-2} \frac{\sqrt{n_1+n_2}\sqrt{n_1 S_1^2 + n_2 S_2^2}}{\sqrt{n_1 n_2 (n_1+n_2-2)}} \right]$$

$n_1 = n_2 = 6$

$\bar{x} - \bar{y} = -0'1661$

$s_1^2 = 5'0671$ $\quad s_2^2 = 1'6833$

$$\sqrt{(6 s_1^2 + 6 s_2^2)} \frac{\sqrt{12}}{\sqrt{6^2 \cdot 10}} = \frac{\sqrt{2}}{\sqrt{10}} \sqrt{s_1^2 + s_2^2}$$

$$= \frac{\sqrt{s_1^2 + s_2^2}}{\sqrt{5}} = \frac{\sqrt{2'6}}{\sqrt{5}} = 0'7194$$

$$qt(0'95, 10) = 1'8125$$

$$[\bar{x} - \bar{y} - 1'8125 \cdot 0'7194 , \bar{x} - \bar{y} + 1'8125 \cdot 0'7194]$$

$$= [-0'1667 - 1'304 , -0'1667 + 1'304]$$

$$= [-1'4706, 1'1373]$$

Com $0 \in$ Interval podem acceptar que no hi ha diferències

Observació: Hem suposat que $\sigma_1^2 = \sigma_2^2$ i desconegut. Hauriem de fer abans un interval per $\frac{\sigma_1^2}{\sigma_2^2}$.