(2)

$$\lim_{n\to+\infty} \frac{2^2+5^4+\cdots+(n^2+1)^{2n}}{2+17^2+\cdots+(n^4+1)^n}$$

$A_n = \sum_{k=1}^{n} (k^2+1)^{2k}$ successió en $\mathbb{R}$

$B_n = \sum_{k=1}^{n} (k^4+1)^{k}$ successió estrict. creixent amb $\lim_{n\to+\infty} B_n = +\infty$

$$\underset{\text{STOLZ}}{=} \lim_{n\to+\infty} \frac{(n^2+1)^{2n}}{(n^4+1)^n} = \lim_{n\to+\infty} \left(\frac{n^4+1+2n^2}{n^4+1}\right)^n \underset{\text{Indeterminació!}}{=} 1^\infty$$

STOLZ: Estudiem $\lim \frac{A_n - A_{n-1}}{B_n - B_{n-1}}$

En efecte

$$\lim_{n\to+\infty} \left(\frac{n^4+1+2n^2}{n^4+1}\right)^n = \lim_{n\to+\infty} \left(1 + \frac{2n^2}{n^4+1}\right)^n$$

$$= \lim_{n\to+\infty} \left(\left[\left(1 + \frac{1}{\frac{n^4+1}{2n^2}}\right)^{\frac{n^4+1}{2n^2}}\right]^{\frac{2n^2}{n^4+1}}\right)^n$$

$$= \lim_{n\to\infty} e^{\frac{2n^3}{n^4+1}} = e^0 = 1. \Rightarrow \lim_{n\to+\infty} \frac{2+5^4+\cdots+(n^2+1)^{2n}}{2+17^2+\cdots+(n^4+1)^n}$$

//

$$\lim_{n\to+\infty} \frac{n\sin(n^2)}{\log(n^n)} = \lim_{n\to+\infty} \frac{\cancel{n}\sin(n^2)}{\cancel{n}\log(n)} = 0. \quad //$$

Teorema/Lema Sandwich.

$$-\frac{1}{\log(n)} \le \frac{\sin(n^2)}{\log(n)} \le \frac{1}{\log(n)}$$

$$-\frac{1}{\log(n)} \xrightarrow{n\to+\infty} 0 \qquad \frac{1}{\log(n)} \xrightarrow{n\to+\infty} 0$$