Problema 16

$X \sim \exp(\lambda) \qquad \lambda = 2$

$f(x) = \lambda e^{-\lambda x} \mathbb{1}_{(0,\infty)}(x)$

$$F(x) = \begin{cases} 0 & x < 0 \\ \int_0^x \lambda e^{-\lambda u}\, du = -e^{-\lambda u}\Big]_0^x = 1 - e^{-\lambda x} & x \geq 0 \end{cases}$$

$P(X > 2) = 1 - P(X \leq 2) = 1 - F(2)$

$= e^{-2 \cdot 2} = e^{-4}$

$= 0'018315$

$P(X > 2) = \int_2^\infty 2e^{-2x}\, dx = -e^{-2x}\Big]_2^\infty$

$= e^{-4}$

$P(X = 3) = F(3) - F(3^-) = 0$

$= P(X \leq 3) - P(X < 3)$

$P(X < 3 \mid X \in (0,4)) = P(X < 3 \mid 0 < X < 4)$

$$= \frac{P(X<3,\ 0<X<4)}{P(0<X<4)} = \frac{P(0<X<3)}{P(0<X<4)}$$

$$= \frac{F(3)}{F(4)} = \frac{1-e^{-2\cdot 3}}{1-e^{-2\cdot 4}} = \frac{1-e^{-6}}{1-e^{-8}} = \frac{0'9975}{0'9997}$$

$$= 0'9978$$

$$E(X) = \int_0^{\infty} x \cdot 2e^{-2x}\, dx$$

$$= -x \cdot e^{-2x} \Big]_0^{\infty} + \int_0^{\infty} e^{-2x}\, dx$$

$$= \frac{e^{-2x}}{-2} \Big]_0^{\infty} = \frac{1}{2}$$

$$E(X) = \frac{1}{\lambda} = \frac{1}{2}$$