

$$H(a,b,c,d) = (c \oplus a)((\bar{d} + bd) + \overline{(a + b + c)})$$

XOR

$$c \oplus a = \bar{a}c + a\bar{c}$$

distributiva $x + yz = (x + y)(x + z)$

$$\bar{d} + bd = (\bar{d} + b)(\bar{d} + d) = (\bar{d} + b)$$

$$\uparrow x + \bar{x} = 1$$

De Morgan $\overline{(x + y)} = \bar{x} \cdot \bar{y}$

$$\overline{(a + b + c)} = \bar{a} \bar{b} \bar{c}$$

$$x \cdot \bar{x} = 0$$

$$H(a,b,c,d) = (c \oplus a) \left((\bar{d} + bd) + \overline{(a + b + c)} \right) = (\bar{a}c + a\bar{c})(\bar{d} + b) + \bar{a} \bar{b} \bar{c} = \bar{a}c\bar{d} + a\bar{c}\bar{d} + \bar{a}cb + a\bar{c}b + \bar{a}c\bar{a} \bar{b} \bar{c} + a\bar{c} \bar{a} \bar{b} \bar{c} = \bar{a}c\bar{d} + a\bar{c}\bar{d} + \bar{a}bc + ab\bar{c}$$

$$\bar{\bar{x}} = x$$

De Morgan $\overline{(x + y)} = \bar{x} \cdot \bar{y}$

$$H(a,b,c,d) = \bar{a}c\bar{d} + a\bar{c}\bar{d} + \bar{a}bc + ab\bar{c} = \overline{\overline{\bar{a}c\bar{d}} + \overline{a\bar{c}\bar{d}} + \overline{\bar{a}bc} + \overline{ab\bar{c}}} = \overline{\bar{a}c\bar{d} \cdot a\bar{c}\bar{d} \cdot \bar{a}bc \cdot ab\bar{c}} = \overline{\bar{a}c\bar{d} \cdot a\bar{c}\bar{d} \cdot \bar{a}bc \cdot ab\bar{c}}$$

NAND

