3 The data in the table are the yearly rainfall totals in Scranton, Pa., for the years 1951–1984.

Rainfall Totals (inches) for Scranton, Pa., 1951–1984

21.3	28.8	17.6	23.0	27.2	28.5	32.8	28.2	25.9	22.5	27.2	33.1	28.7	24.8	24.3	27.1	30.6
26.8	18.9	36.3	28.0	17.9	25.0	27.5	27.7	32.1	28.0	30.9	20.0	20.2	33.5	26.4	30.9	33.2

a Make a 95% confidence interval for the median.

b Make 90% confidence intervals for the 20th and 80th percentiles.

c The confidence interval procedure assumes that the observations are independent and identically distributed. Do you think this is a reasonable assumption for the rainfall data? If not, what could cause this assumption to be invalid?

4 Suppose we test the hypotheses H_0: $\theta_{.5} = 75$ versus H_a: $\theta_{.5} > 75$ and, regardless of the data, we reject H_0. What is the probability of a Type I error? What is the power of the test for values of $\theta_{.5} > 75$?

5 Suppose we assume that the population distribution under H_0 is symmetric so that $\theta_{.5} = \mu$. Without looking at the data to check the validity of this assumption, we apply the binomial test and the CLT test. Suppose it turns out that 39 data values that are equal to 75.1 and the 40th one is equal to 90.

a What decision is reached using the binomial test to test H_0: $\theta_{.5} = 75$ versus H_a: $\theta_{.5} > 75$?

b What decision is reached using the CLT test to test H_0: $\mu = 75$ versus H_a: $\mu > 75$, where the statistic is computed using the sample standard deviation S in place of the unknown population standard deviation σ?

c Based on the results of parts a and b, what types of distributions that satisfy the alternative hypothesis are particularly easy for the binomial test to detect in comparison to the CLT test?

d Replace 90 by other values such as 80, 78, and 76 that are closer to the null hypothesis. Note what happens to the value of Z_μ. Does this correspond to intuition?

6 Refer to Section 1.3.3. No computations are required to answer the following questions.

a What is the value of the power of the binomial test when $\mu = 75$?

b What happens to the power as μ gets large?

c How does increasing the sample size affect the power of the binomial test?

7 Suppose we test H_0: $\theta_{.5} = \theta_H$ versus $H_a > \theta_H$ using the binomial test with a sample size $n = 10$.

a If we reject H_0 when $B \geq 8$, use the binomial Table A1 to determine the exact probability of a Type I error.

b Suppose we observe a value of $B = b_{obs}$. The p-value is the probability that $B \geq b_{obs}$ given that H_0 is true. Find the p-values for $b_{obs} = 5, 6, 7, 8, 9, 10$.

Theory and Complements

8 Refer to the derivations of the power functions in Section 1.3. Evaluate and sketch the power functions of the statistics Z_μ and Z_B for values of the mean between 75 and 77 assuming that the populations have normal distributions. Using your sketch, determine the maximum difference between the power functions. Repeat this procedure for the Laplace population distribution.