COMS W4160— Computer Graphics Spring 2018

Paper Homework1

out: Feb. 1, Thursday

**due: 10PM, Feb. 8 (Thursday), Submit electronically on Coureworks**

**Problem 1**

Problem 8 in Chapter 2 of the textbook: Show by counterexample that it is not always true that for 3D vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$

**Problem 2**

(a) Given three 2D points $\tilde{p}_1$, $\tilde{p}_2$, and $\tilde{p}_3$ defining the three vertices of a 2D triangle, *derive* an expression of computing the area of the triangle.

(b) Given three 3D vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, *derive* an expression of computing the volume of the tetrahedron with side vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ (see Figure 1).

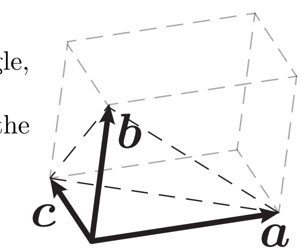


Figure 1: **a tetrahedron**

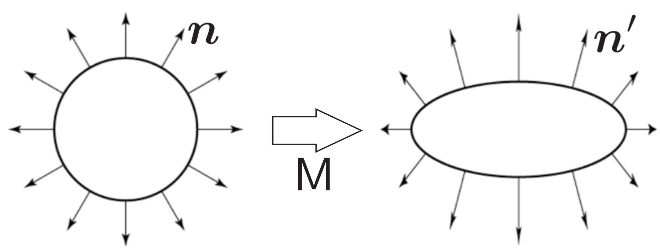


Figure 2: **Problem 3**

**Problem 3**

During the class, we discussed how an arbitrary shape (in 2D or 3D) is deformed under an affine transformation M (see Figure 2): every point $\boldsymbol{v}$ on the shape is transformed to a new position $\mathsf{M}\boldsymbol{v}$. We invited you to think about how the surface normal direction should be transformed under the same transformation matrix M. Now, you need to prove the result I mentioned in class: a surface normal $\boldsymbol{n}$ at any surface point is transformed into a new normal direction $\boldsymbol{n}'$,

$$\boldsymbol{n}' = \mathsf{M}_n \boldsymbol{n}, \text{ where } \mathsf{M}_n = \mathsf{M}^{-T},$$

where $\mathsf{M}_n$ is the inverse of the transpose of M.

**Problem 4**

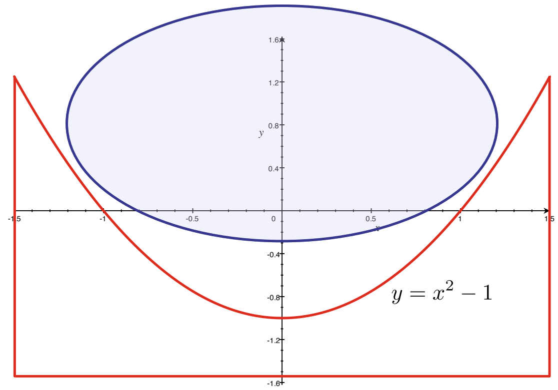


Figure 3: **Problem 5**

Given the equation of a straight line

$$Ax + By = C.$$

We can write it using vector doc product

$$[A \;\; B] \cdot [x \;\; y] = C.$$

Prove that an affine transformation using the matrix

$$\mathsf{M} = \begin{bmatrix} \mathsf{T}_{2\times 2} & \boldsymbol{u} \\ \mathbf{0}_{1\times 2} & 1 \end{bmatrix}$$

preserves (i) straight lines, (ii) parallel lines, and (iii) ratios of length along lines.

**Problem 5**

In class, we discussed the explicit and implicit representation of curves and now we will use them to consider the interaction of two shapes in 2D. in a 2D rigid body animation, there are two objects (see Figure 3), a fixed concave supporter (outlined by red curves) whose supporting curve is described by an explicit representation

$$y = x^2 - 1, \; x \in [-1.5, 1.5],$$

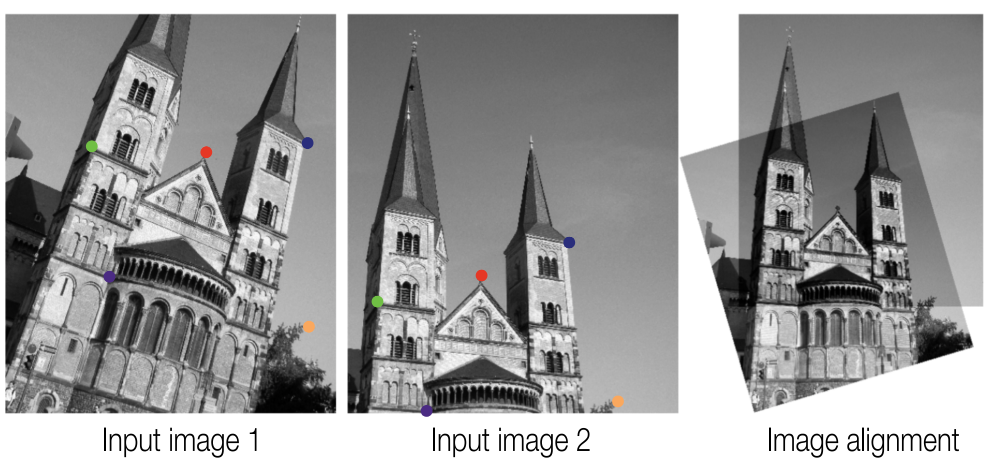


Figure 4: **Problem 6**

and an elliptic object whose shape is described by an implicit representation

$$f(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0,\ b \leq a < 1.5,$$

This description assumes the elliptic object is centered at $(0,0)$. However, since the elliptic object is not fixed, initially in the animation it is placed at $(0,H)$ and starts to fall down due to gravity. It then hits the supporter and eventually reaches a stable equilibrium. When the elliptic object settles stably on the supporter, its center position position is at $(0,h)$. Derive a formula to compute $h$ using the elliptic parameters $a$ and $b$.

**Problem 6**

We now consider a problem encountered in image processing. Suppose you had a tour at a historic church, and took many pictures (see Figure 4). You want to align some of the pictures together so you can combine them into a larger picture of the church (Figure 4-right). To this end, on one image (Figure 4-left) you identified five points that label certain feature points of the church. On another image (Figure 4-middle) you identify the same set of feature points and also label them. As a result, you know the correspondence of the five feature points between the two images (see colored dots on Figure 4- left and middle). Denote the five points on the first image as $\tilde{\boldsymbol{a}}_i \in \mathbb{R}^2$, $i = 1 \cdots 5$, and the corresponding points on the second image as $\tilde{\boldsymbol{b}}_i \in \mathbb{R}^2$, $i = 1 \cdots 5$.

Derive an affine transformation $\mathsf{T}$ that transforms the first image in order to align with the second image. You don't need to compute $\mathsf{T}$ exactly, just describe equations you need to solve to obtain $\mathsf{T}$.