**Homework 1**

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**1.**

For example, vector , and

For these three vectors,

**2.**

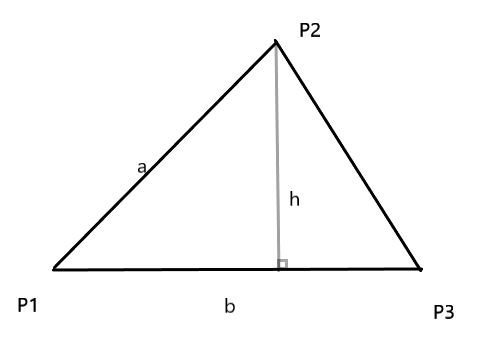
**a.**

let vector be a vector starting from origin point ending at point

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the area of triangle can be described as .

Suppose the follwing example,



The area of triangle is , where is the length of base andis the height.

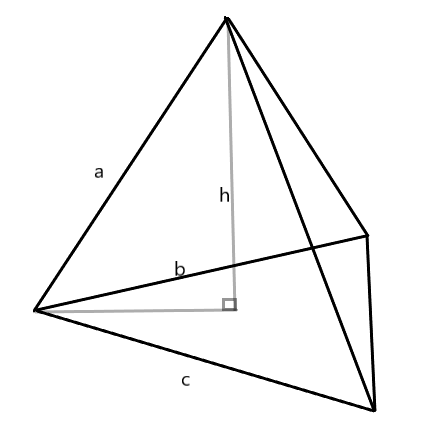
According to the definiation of cross product, . And if we take as the base then, the length of height will be exactly , thus the area of triangle is .

**b.**

the volume of tetrahedron can be described as

as the volume of tetrahedron is , where is the area of base and is the height.

In this case, . And the height, is the dot product of vector and norm of the base (plane formed by and ).



Here, , where is the area of base and is a unit vector along the height direction. The dot product is the projection of on which is the same as the length of height. Thus the volume is .

**3.**

The surface normal is defined as , where is the surface normal and is the surface vertex vector.

Now, suppose in any point of the origin shape, we have .

In the transformed shape, we have .

As the transformation matrix is , then .

Thus, . And as , we have .

**4.**

**i.**

Define the transformed shape as , we have

Then,

Rewrite as the straight line is defined,

Define

We can rewrite the transformed shape as

Thus, the transformed shape is also a straight line.

**ii.**

For two parallel lines and ,

As proved in (i), the transformed line for them are

Define

The two lines can be rewrite as

As , , thus the two transformed lines are also parallel.

**iii.**

Suppose a point lies on a straight line, and is a unit vector along the direction of the straight line (either direction is alright).

Now we have two other points, one point lies on this line and its distance to point is , another point lies on the same line and its distance to is . We can write these two points as

Now that apply the transformation

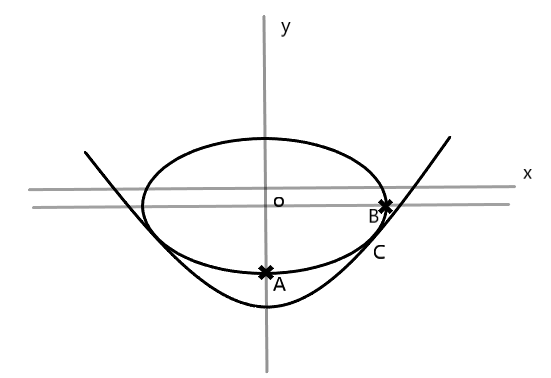
Now the distance between and becomes

And the distance between and becomes .

As

The ratios of length along a same line remains the same.

**5.**



As the graph shows, both the object and supporter are symmetric, thus we just need to analyze half of the shape. Here, I’ll use the right part for analysis.

When the object is stablized, there should be only one intersection on the right part. At the intersection point C, the gradient of both curves are the same. And according to the ellipse shape, the intersection point must occur between point A and point B.

Formula of right-bottom part of ellipse:

Gradient of right-bottom part of ellipse:

Gradient of parabola:

Thus, solving the formula

One possible solution is , and another is

1. When , there are two possible solutions and . In this case, the intersection point would be . And the coordinate of intersection point is . Then, we can compute according to the ellipse function. And .
2. When , there is only one solution . In this case, coordinate of intersection point is ,

**6.**

Consider a homogeneous coordinate, can be expressed as and similarly, as .

The transformation matrix is applied as . Now that and are already given, and we need to compute .

First rewrite the equation as

Then we can solve ,

And the transformation matrix is