COMS 4731 HW2

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Written Assigmments

Problem 1

i)

Calculate eigenvalues of matrix first, let λ denote for eigenvalues, we have

$$|E - \lambda I| = 0$$

$$\begin{vmatrix} a-\lambda & b/2 \\ b/2 & c-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)(c-\lambda) - \frac{b^2}{4} = 0$$

$$\lambda^2 - (a+c)\lambda + \left(ac - \frac{b^2}{4}\right) = 0$$

$$\lambda = \frac{(a+c) \pm \sqrt{(a-c)^2 + b^2}}{2}$$

Thus, we have two eigenvalues of the matrix.

Then, rewrite E as follows

$$E = asin^2\theta - bsin\theta cos\theta + ccos^2\theta$$

$$E = asin^2\theta + acos^2\theta - bsin\theta cos\theta + ccos^2\theta - acos^2\theta$$

$$E = a - bsin\theta cos\theta + (c-a)(cos^2\theta - \frac{1}{2}) + \frac{c-a}{2}$$

$$E = \frac{a+c}{2} + \frac{c-a}{2}cos2\theta - \frac{b}{2}sin2\theta$$

$$E = \frac{a+c}{2} + \frac{1}{2}\sqrt{(a-c)^2 + b^2}\left(\frac{c-a}{\sqrt{(a-c)^2 + b^2}}cos2\theta - \frac{b}{\sqrt{(a-c)^2 + b^2}}sin2\theta\right)$$

$$E = \frac{a+c}{2} + \frac{1}{2}\sqrt{(a-c)^2 + b^2}\cos(2\theta + \varphi)$$

Where $\varphi = \text{acos}\left(\frac{c-a}{\sqrt{(a-c)^2+b^2}}\right)$

As $\cos(2\theta + \varphi)$ lies in $[-1, 1]$, it is obvious that the minimum value of E is $\frac{a+c}{2} - \frac{1}{2}\sqrt{(a-c)^2 + b^2}$ and the maximum value of E is $\frac{a+c}{2} + \frac{1}{2}\sqrt{(a-c)^2 + b^2}$, which are equal to eigenvalue of the matrix respectively.

ii)

Let us examine the origin definition of E, $E = \iint_I \; r^2 b(x,y)dxdy$

It is obvious that r^2 is both real and non-negative, hence E is also real and non-negative.

As E is non-negative, the minimum value of E should be non-negative as well. As given in (i), $Emin = \frac{a+c}{2} - \frac{1}{2}\sqrt{(a-c)^2 + b^2}$. Thus, $\frac{a+c}{2} - \frac{1}{2}\sqrt{(a-c)^2 + b^2} \geq 0$

$$a + c \geq \sqrt{(a-c)^2 + b^2}$$

$$(a+c)^2 \geq (a-c)^2 + b^2$$

$$4ac \geq b^2$$

iii)

According to the definition of E, 1-D objects has zero E all the time and 2-D objects could have zero E only if the projection of this object is zero.