

# Written assignments

For a fixed point  $(x_0, y_0)$  in the  $(x, y)$ -image space, the line equation is

$$x_0 \sin(\theta) - y_0 \cos(\theta) + \rho = 0$$

$$\rho = y_0 \cos(\theta) - x_0 \sin(\theta)$$

$$\rho = \sqrt{y_0^2 + x_0^2} \left( \frac{y_0}{\sqrt{y_0^2 + x_0^2}} \cos(\theta) - \frac{x_0}{\sqrt{y_0^2 + x_0^2}} \sin(\theta) \right)$$

$$\rho = \sqrt{y_0^2 + x_0^2} \cos(\theta + \varphi) = \sqrt{y_0^2 + x_0^2} \sin\left(\theta + \varphi + \frac{\pi}{2}\right)$$

Where  $\cos(\varphi) = \frac{y_0}{\sqrt{y_0^2 + x_0^2}}$  and  $\sin(\varphi) = \frac{x_0}{\sqrt{y_0^2 + x_0^2}}$ .

Thus a point in image space results in a sinusoid in Hough space.

The amplitude is  $\sqrt{y_0^2 + x_0^2}$  and the phase is  $\varphi + \frac{\pi}{2}$  (where  $\cos(\varphi) = \frac{y_0}{\sqrt{y_0^2 + x_0^2}}$  and  $\sin(\varphi) = \frac{x_0}{\sqrt{y_0^2 + x_0^2}}$ ).

The period does not change when the position of the image point moves. In a sinuous function  $f(x) = A \sin(\omega\theta + \varphi)$  the period depends only on parameter  $\omega$ . As shown in the equation above,  $\omega$  is independent of  $x$  and  $y$  and in this case  $\omega$  is always 1, thus the period does not change.