Written assignment

Problem 1.

Suppose the transformation is $x' = Tx$, where x is the origin point, x' is the corresponding transformed point, T is the transform matrix.

$$T = T_3T_2T_1$$

T_1 is the scale transform matrix, it doubles the length of the edge.

T_2 is the rotate transform matrix, it rotates the square anti-clockwise by angle θ.

T_3 is the shift transform matrix, it shifts the square to target location.

I.

As length of edge is twice the length of its origin value, we have

$$T_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

II.

As the angle between the bottom of origin square and transformed square is θ, we have

$$T_2 = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

III.

After the first two transformation, the coordinate of $c_1(x_1, y_1)$ has become $c'(x', y')$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = T_2T_1 \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2(cos\theta)x_1 - 2(sin\theta)y_1 \\ 2(sin\theta)x_1 + 2(cos\theta)y_1 \\ 1 \end{bmatrix}$$

And the shift transformation moves the point $c'(x', y')$ to $c_2(x_2, y_2)$, so we have

$$T_3 = \begin{bmatrix} 1 & 0 & x_2 - (2(cos\theta)x_1 - 2(sin\theta)y_1) \\ 0 & 1 & y_2 - (2(sin\theta)x_1 + 2(cos\theta)y_1) \\ 0 & 0 & 1 \end{bmatrix}$$

A brief summary, $T = T_3T_2T_1$.

T_1 is the scale matrix, $T_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

T_2 is the rotate matrix, $T_2 = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

T_3 is the shift matrix, $T_3 = \begin{bmatrix} 1 & 0 & x_2 - (2(cos\theta)x_1 - 2(sin\theta)y_1) \\ 0 & 1 & y_2 - (2(sin\theta)x_1 + 2(cos\theta)y_1) \\ 0 & 0 & 1 \end{bmatrix}$