HW6

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# Problem 1.

$$r_0 = (x_0, y_0, z_0)$$

$$v_0 = (u, v, w)$$

$$v_i = f\frac{(r_0 \cdot z)v_0 - (v_0 \cdot z)r_0}{(r_0 \cdot z)^2} = f\frac{z_0(u, v, w) - w(x_0, y_0, z_0)}{{z_0}^2} = \frac{f}{z_0}(u - \frac{w}{z_0}x_0, v - \frac{w}{z_0}y_0, 0)$$

$$u_i = \frac{f}{z_0}(u - \frac{w}{z_0}x_0)$$

$$v_i = \frac{f}{z_0}(v - \frac{w}{z_0}y_0)$$

Thus, velocity of image point is a constant both on x-axis and y-axis, as $t$ increases, the movement is a straight line. To be specific, the coordinate of the point at time $t$ is $[f\frac{x_0}{z_0} + t\frac{f}{z_0}\left(u - \frac{w}{z_0}x_0\right), f\frac{y_0}{z_0} + t\frac{f}{z_0}\left(v - \frac{w}{z_0}y_0\right)]$

$$\frac{y_i - f\frac{y_0}{z_0}}{\left(v - \frac{w}{z_0}y_0\right)} = \frac{x_i - f\frac{x_0}{z_0}}{\left(u - \frac{w}{z_0}x_0\right)}$$

$$\left(v - \frac{w}{z_0}y_0\right)x_i - \left(u - \frac{w}{z_0}x_0\right)y_i + \left[f\frac{y_0}{z_0}\left(u - \frac{w}{z_0}x_0\right) - f\frac{x_0}{z_0}\left(v - \frac{w}{z_0}y_0\right)\right] = 0$$

Here, $\left(v - \frac{w}{z_0}y_0\right)$, $\left(u - \frac{w}{z_0}x_0\right)$ and $\left[f\frac{y_0}{z_0}\left(u - \frac{w}{z_0}x_0\right) - f\frac{x_0}{z_0}\left(v - \frac{w}{z_0}y_0\right)\right]$ are all constants. Thus the movement is a straight line.

# Problem 2.

Suppose now we double the frame rate, and rewrite the origin assumption in lecture as follows:

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t)$$

$$\rightarrow I(x + \delta x, y + \delta y, t + \delta t) = I(x + \delta x/2, y + \delta y/2, t + \delta t/2) = I(x, y, t)$$

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + I_x\delta x + I_y\delta y + I_t\delta t$$

$$\rightarrow I(x + \delta x/2, y + \delta y/2, t + \delta t/2) = I(x, y, t) + I_x\delta x/2 + I_y\delta y/2 + I_t\delta t/2$$

$$\rightarrow I(x + \delta x, y + \delta y, t + \delta t) = I(x + \delta x/2, y + \delta y/2, t + \delta t/2) + I_x\delta x/2 + I_y\delta y/2 + I_t\delta t/2$$

So, now we have two constraints,

$$I_{x1}u_1 + I_{y1}v_1 + I_{t1} = 0$$

$$I_{x2}u_2 + I_{y2}v_2 + I_{t2} = 0$$

And as illumination can change much faster than the object's movement, we can rewrite

$$u_1 = u_2 = u$$

$$v_1 = v_2 = v$$

Thus, the constraints are

$$I_{x1}u + I_{y1}v + I_{t1} = 0$$

$$I_{x2}u + I_{y2}v + I_{t2} = 0$$

Now $(u, v)$ lies on two lines, and it is on the intersection of the two lines, and we have a unique solution of optical flow $(u, v)$.